# Computer Algebra Independent Integration Tests

Summer 2024

4-Trig-functions/4.2-Cosine/199-4.2.1.2

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May 18, 2024 Compiled on May 18, 2024 at 12:40am

# Contents

# CHAPTER 1

## Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [ **89** ]. This is test number [ 199 ].

## 1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

## 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

| System | % solved | % Failed |
|---|---|---|
| Rubi | 100.00 ( 89 ) | 0.00 ( 0 ) |
| Mathematica | 100.00 ( 89 ) | 0.00 ( 0 ) |
| Maple | 98.88 ( 88 ) | 1.12 ( 1 ) |
| Fricas | 64.04 ( 57 ) | 35.96 ( 32 ) |
| Mupad | 38.20 ( 34 ) | 61.80 ( 55 ) |
| Giac | 35.96 ( 32 ) | 64.04 ( 57 ) |
| Reduce | 35.96 ( 32 ) | 64.04 ( 57 ) |
| Maxima | 30.34 ( 27 ) | 69.66 ( 62 ) |
| Sympy | 25.84 ( 23 ) | 74.16 ( 66 ) |

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

| grade | description |
|---|---|
| A | Integral was solved and antiderivative is optimal in quality and leaf size. |
| B | Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size. |
| C | Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons<br>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.<br>2. antiderivative contains a special function and the optimal antiderivative does not.<br>3. antiderivative contains the imaginary unit and the optimal antiderivative does not. |
| F | Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised. |

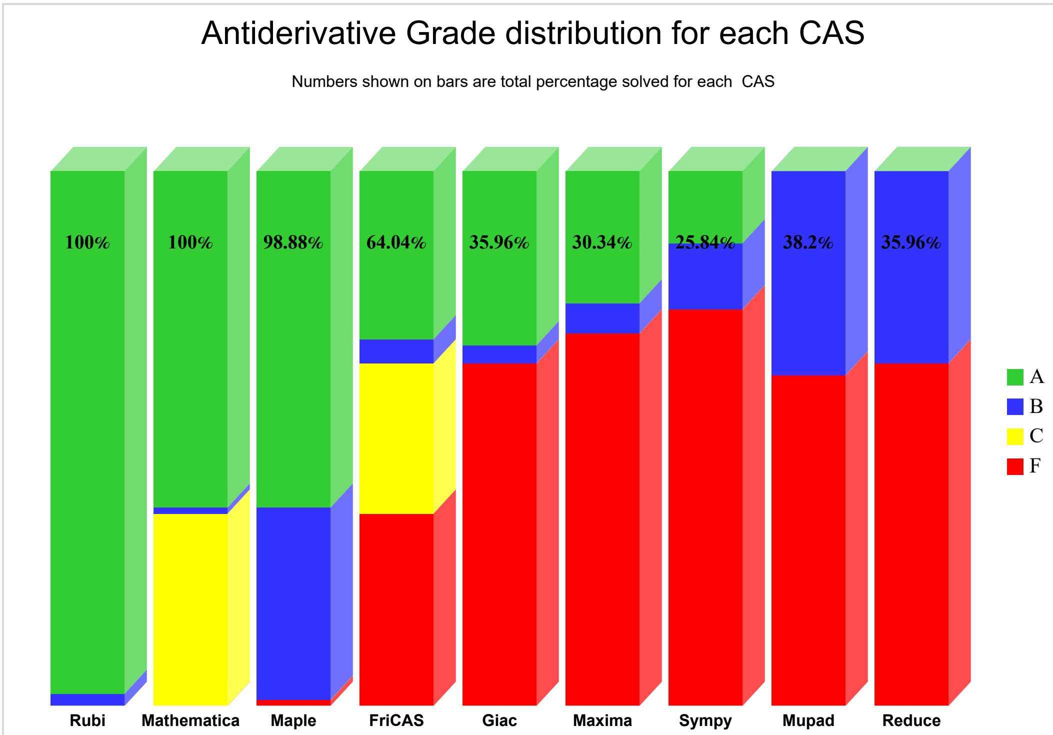
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

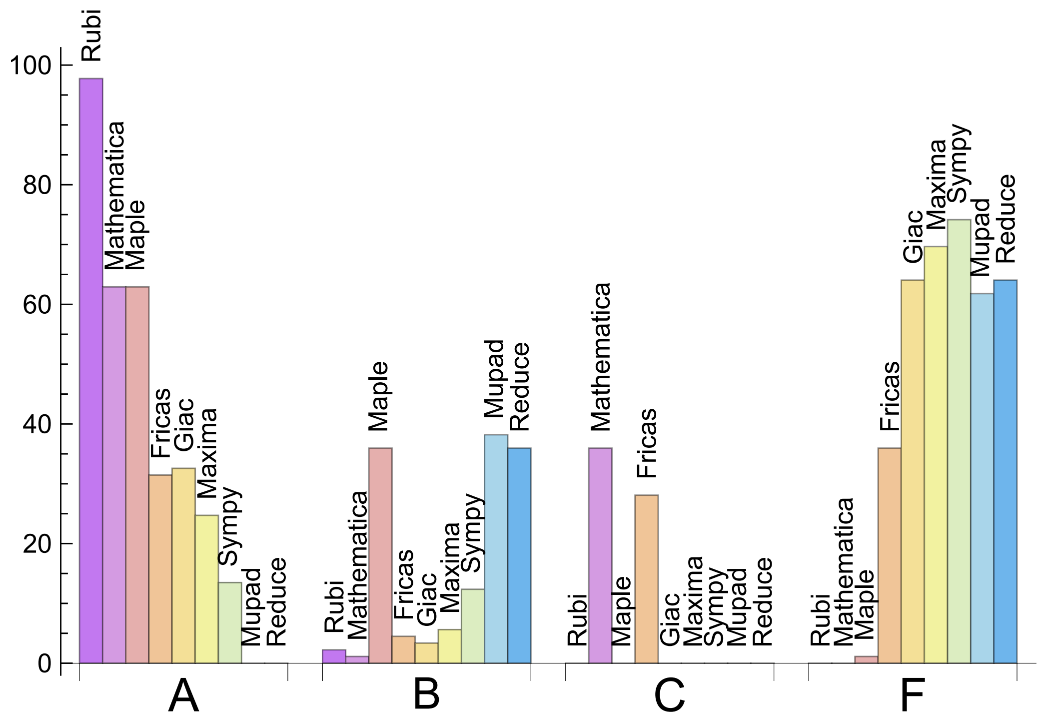
| System | % A grade | % B grade | % C grade | % F grade |
|---|---|---|---|---|
| Rubi | 97.753 | 2.247 | 0.000 | 0.000 |
| Mathematica | 62.921 | 1.124 | 35.955 | 0.000 |
| Maple | 62.921 | 35.955 | 0.000 | 1.124 |
| Giac | 32.584 | 3.371 | 0.000 | 64.045 |
| Fricas | 31.461 | 4.494 | 28.090 | 35.955 |
| Maxima | 24.719 | 5.618 | 0.000 | 69.663 |
| Sympy | 13.483 | 12.360 | 0.000 | 74.157 |
| Mupad | 0.000 | 38.202 | 0.000 | 61.798 |
| Reduce | 0.000 | 35.955 | 0.000 | 64.045 |

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.




The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

| System | Number failed | Percentage normal failure | Percentage timeout failure | Percentage exception failure |
|---|---|---|---|---|
| Rubi | 0 | 0.00 | 0.00 | 0.00 |
| Mathematica | 0 | 0.00 | 0.00 | 0.00 |
| Maple | 1 | 100.00 | 0.00 | 0.00 |
| Fricas | 32 | 12.50 | 87.50 | 0.00 |
| Mupad | 55 | 0.00 | 100.00 | 0.00 |
| Giac | 57 | 100.00 | 0.00 | 0.00 |
| Reduce | 57 | 100.00 | 0.00 | 0.00 |
| Maxima | 62 | 69.35 | 22.58 | 8.06 |
| Sympy | 66 | 50.00 | 50.00 | 0.00 |

Table 1.4: Failure statistics for each CAS

## 1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

| System | Mean time (sec) |
|---|---|
| Maxima | 0.04 |
| Fricas | 0.09 |
| Giac | 0.11 |
| Reduce | 0.19 |
| Rubi | 1.05 |
| Mathematica | 5.16 |
| Sympy | 9.12 |
| Maple | 9.41 |
| Mupad | 31.66 |

Table 1.5: Time performance for each CAS

| System | Mean size | Normalized mean | Median size | Normalized median |
|---|---|---|---|---|
| Maxima | 27.85 | 1.32 | 14.00 | 1.00 |
| Giac | 40.88 | 1.22 | 14.00 | 1.00 |
| Reduce | 40.94 | 1.47 | 20.00 | 1.12 |
| Mupad | 80.32 | 1.44 | 13.00 | 0.93 |
| Fricas | 103.53 | 1.37 | 99.00 | 1.14 |
| Sympy | 149.30 | 4.84 | 15.00 | 2.00 |
| Rubi | 220.89 | 1.06 | 150.00 | 1.00 |
| Mathematica | 423.39 | 1.39 | 102.00 | 1.02 |
| Maple | 671.97 | 2.00 | 227.50 | 1.59 |

Table 1.6: Leaf size performance for each CAS

## 1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the $y$ axis is the percentage solved which Rubi itself needed the number of rules given the $x$ axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

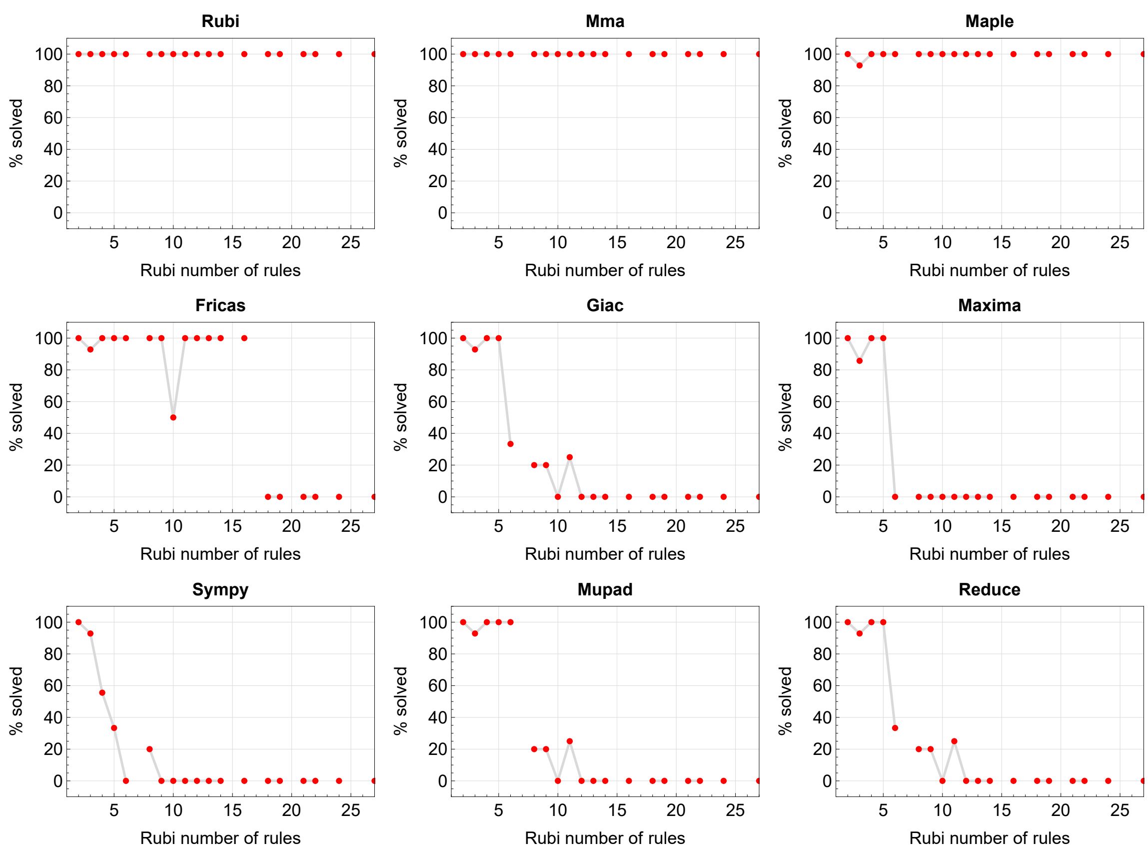



Figure 1.1: Solving statistics per number of Rubi rules used

## 1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

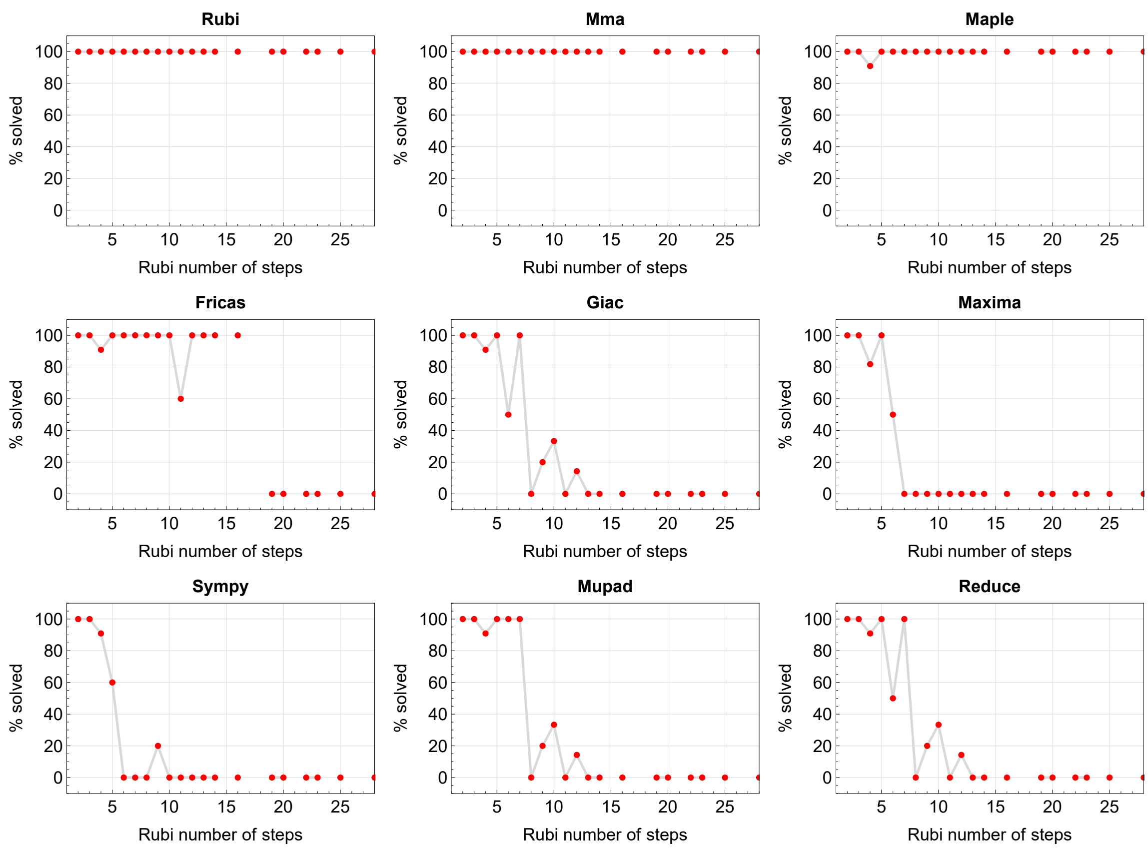



Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

## 1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

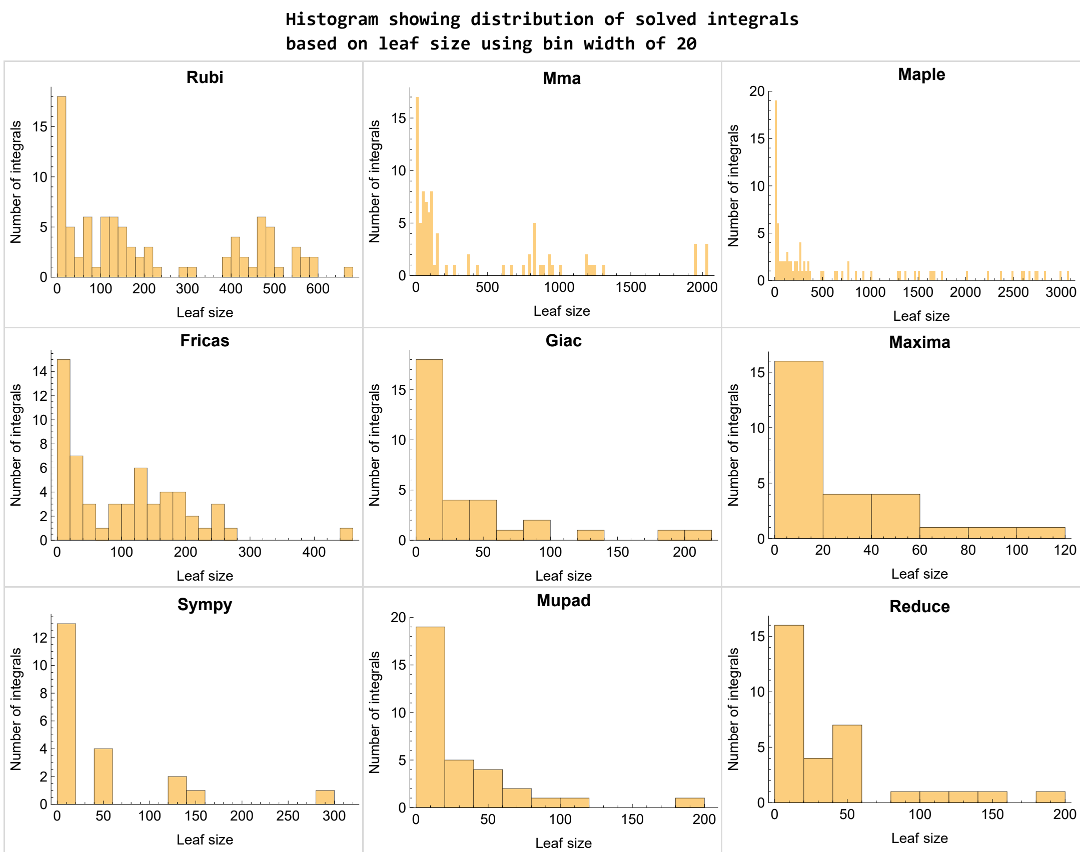



Figure 1.3: Solved integrals based on leaf size distribution

## 1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

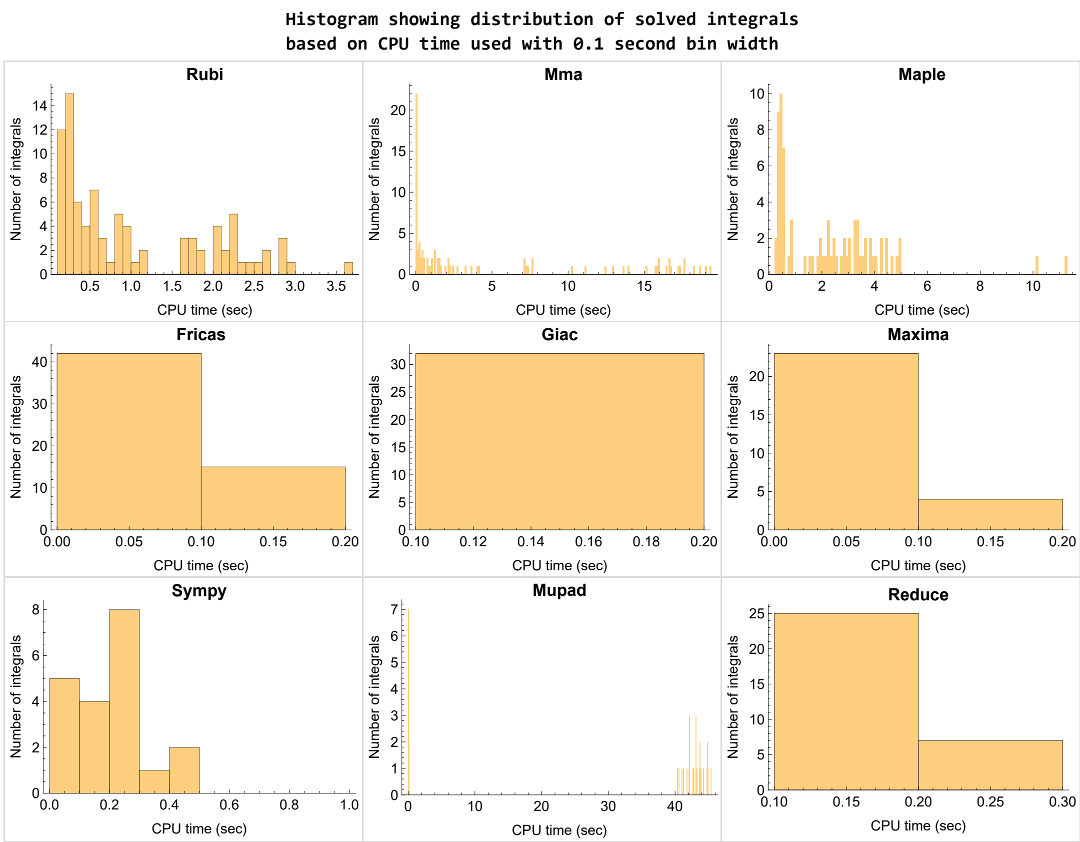



Figure 1.4: Solved integrals histogram based on CPU time used

## 1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

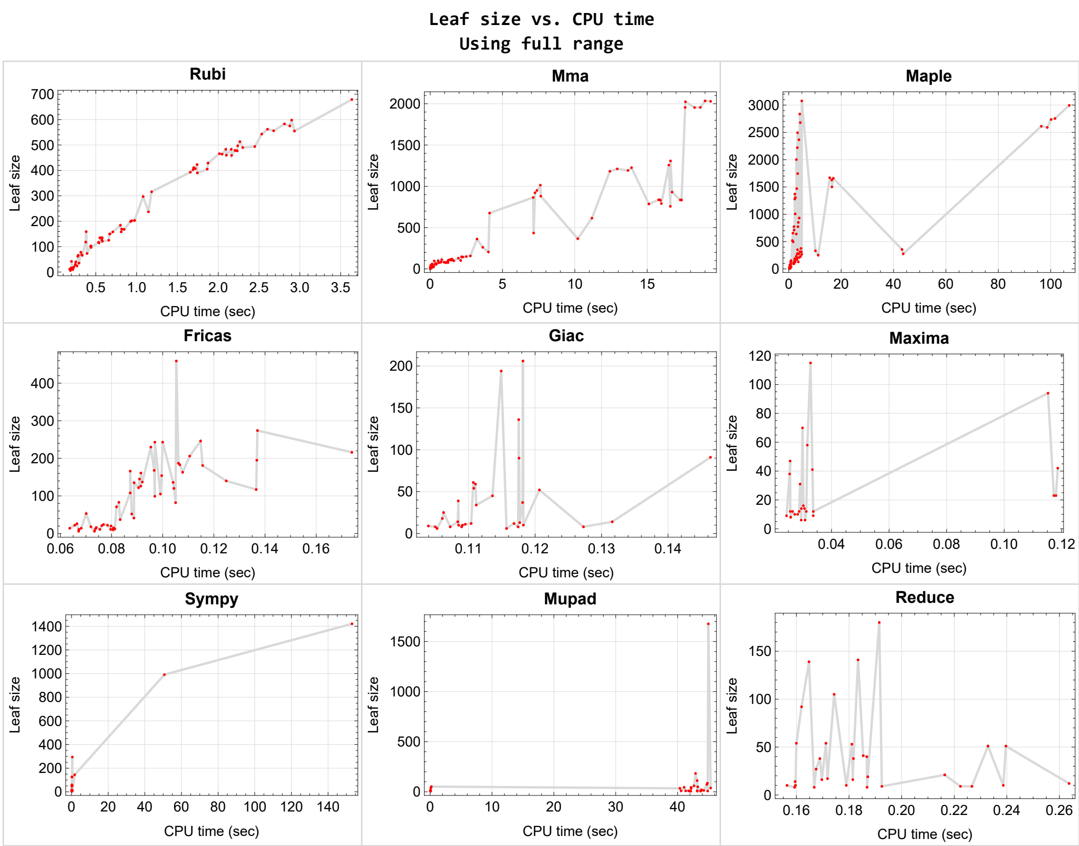



Figure 1.5: Leaf size vs. CPU time. Full range

## 1.9 list of integrals with no known antiderivative

{}

## 1.10 List of integrals solved by CAS but has no known antiderivative

**Rubi** {}

**Mathematica** {}

**Maple** {}

**Maxima** {}

**Fricas** {}

**Sympy** {}

**Giac** {}

**Reduce** {}

**Mupad** {}

## 1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

**Rubi** {58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88}

**Mathematica** {58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89}

**Maple** $\{58, 66, 68, 69, 70, 76, 77, 78, 79, 80, 81, 84, 85, 86, 87, 88\}$

**Maxima** Verification phase not currently implemented.

**Fricas** Verification phase not currently implemented.

**Sympy** Verification phase not currently implemented.

**Giac** Verification phase not currently implemented.

**Reduce** Verification phase not currently implemented.

**Mupad** Verification phase not currently implemented.

## 1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

## 1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

# 1.14 Important notes about some of the results

## Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

## Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

## Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

## Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

# 1.15 Current tree layout of integration tests

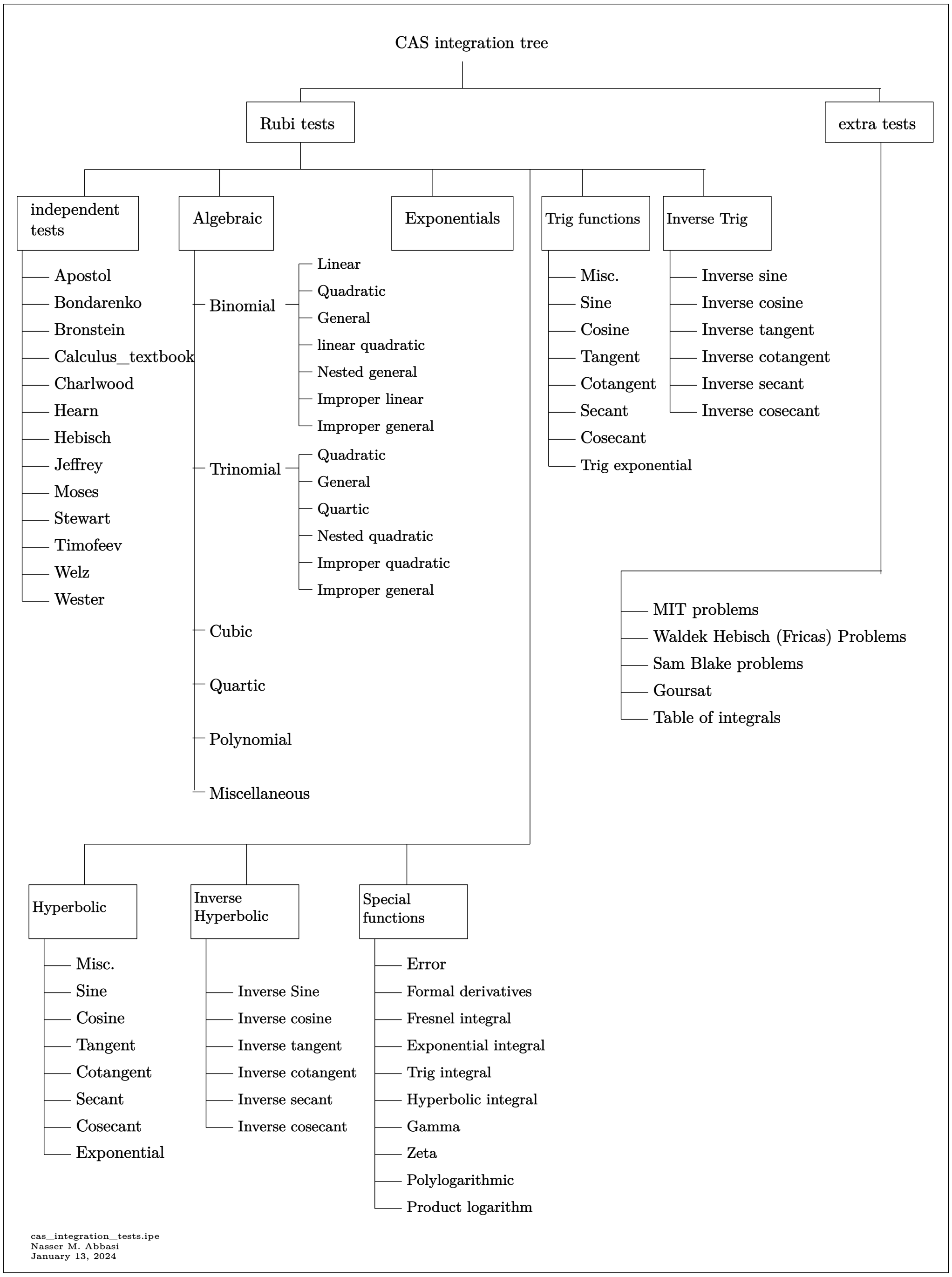
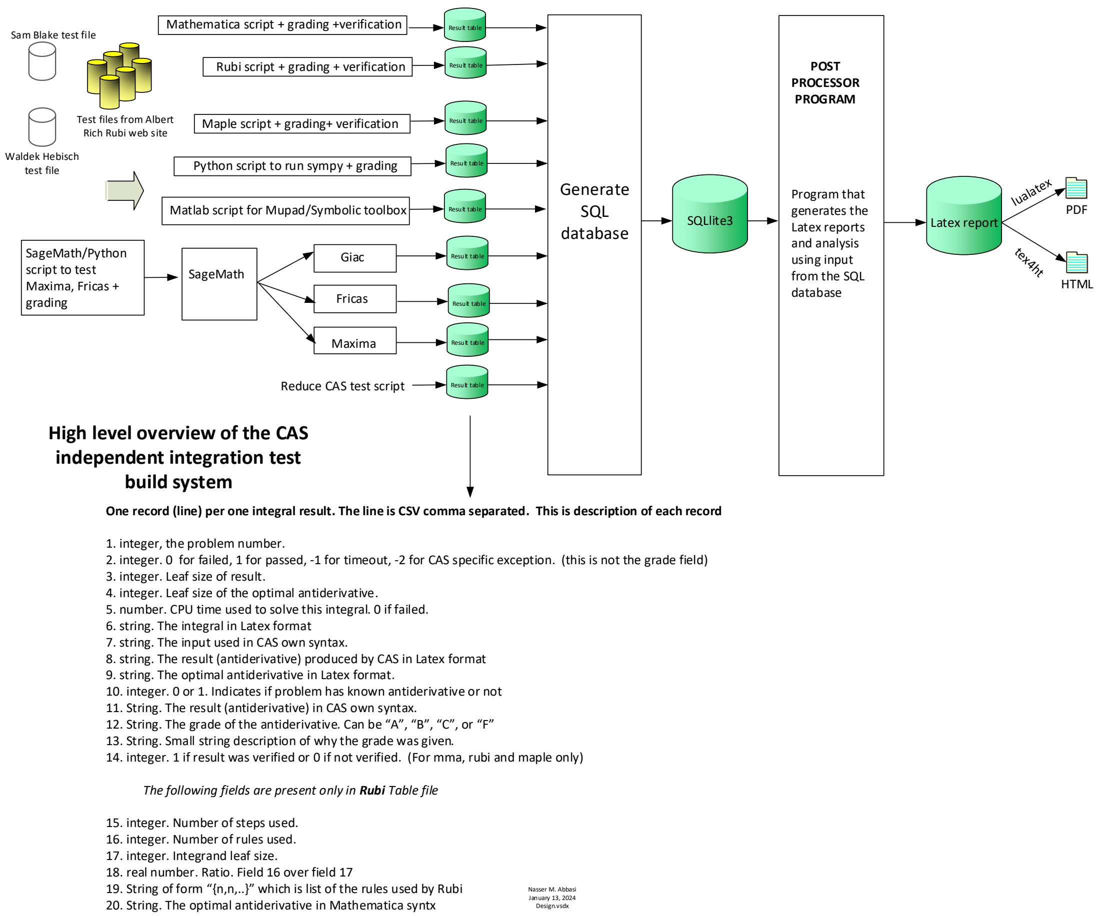



Figure 1.6: CAS integration tests tree

## 1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



# CHAPTER 2

## DETAILED SUMMARY TABLES OF RESULTS

## 2.1 List of integrals sorted by grade for each CAS



### Rubi

**A grade** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89 }

**B grade** { 10, 11 }

**C grade** { }

**F normal fail** { }

**F(-1) timedout fail** { }

**F(-2) exception fail** { }

### Mma

**A grade** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57 }

**B grade** { 89 }

**C grade** { 15, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88 }

**F normal fail** { }

**F(-1) timedout fail** { }

**F(-2) exception fail** { }

## Maple

**A grade** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 43, 45, 47, 49, 50, 51, 53, 55, 57, 58, 60, 61, 62, 63, 64, 66 }

**B grade** { 11, 42, 44, 46, 48, 52, 54, 56, 59, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88 }

**C grade** { }

**F normal fail** { 89 }

**F(-1) timedout fail** { }

**F(-2) exception fail** { }

## Fricas

**A grade** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30 }

**B grade** { 10, 11, 31, 32 }

**C grade** { 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57 }

**F normal fail** { 63, 66, 74, 89 }

**F(-1) timedout fail** { 58, 59, 60, 61, 62, 64, 65, 67, 68, 69, 70, 71, 72, 73, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88 }

**F(-2) exception fail** { }

## Maxima

**A grade** { 2, 4, 5, 6, 8, 10, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 27, 29, 31 }

**B grade** { 1, 3, 7, 9, 11 }

**C grade** { }

**F normal fail** { 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 68, 69, 70, 71, 72, 73, 81, 83, 84, 89 }

**F(-1) timedout fail** { 66, 67, 74, 75, 76, 77, 78, 79, 80, 82, 85, 86, 87, 88 }

**F(-2) exception fail** { 24, 26, 28, 30, 32 }

## Giac

**A grade** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 29, 30, 31 }

**B grade** { 11, 24, 32 }

**C grade** { }

**F normal fail** { 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89 }

**F(-1) timedout fail** { }

**F(-2) exception fail** { }

## Mupad

**A grade** { }

**B grade** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 36, 37 }

**C grade** { }

**F normal fail** { }

**F(-1) timedout fail** { 33, 34, 35, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89 }

**F(-2) exception fail** { }

## Sympy

**A grade** { 4, 5, 10, 12, 13, 14, 15, 18, 19, 20, 21, 27 }

**B grade** { 1, 2, 3, 11, 16, 17, 22, 23, 25, 26, 28 }

**C grade** { }

**F normal fail** { 6, 7, 8, 9, 29, 30, 31, 32, 34, 35, 36, 37, 38, 39, 43, 44, 45, 46, 47, 51, 52, 53, 54, 55, 62, 63, 64, 65, 66, 73, 74, 75, 89 }

**F(-1) timedout fail** { 24, 33, 40, 41, 42, 48, 49, 50, 56, 57, 58, 59, 60, 61, 67, 68, 69, 70, 71, 72, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88 }

**F(-2) exception fail** { }

## Reduce

**A grade** { }

**B grade** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32 }

**C grade** { }

**F normal fail** { 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89 }

**F(-1) timedout fail** { }

**F(-2) exception fail** { }

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

| Problem 1 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 30 | 25 | 24 | 94 | 24 | 294 | 45 | 21 | 34 |
| N.S. | 1 | 0.97 | 0.81 | 0.77 | 3.03 | 0.77 | 9.48 | 1.45 | 0.68 | 1.10 |
| time (sec) | N/A | 0.249 | 0.240 | 0.570 | 0.115 | 0.077 | 0.452 | 0.114 | 0.216 | 42.175 |

| Problem 2 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 13 | 16 | 14 | 14 | 51 | 14 | 17 | 11 |
| N.S. | 1 | 1.00 | 0.81 | 1.00 | 0.88 | 0.88 | 3.19 | 0.88 | 1.06 | 0.69 |
| time (sec) | N/A | 0.210 | 0.022 | 0.430 | 0.030 | 0.081 | 0.269 | 0.108 | 0.172 | 0.045 |

| Problem 3 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 13 | 13 | 17 | 11 | 42 | 10 | 46 | 25 | 10 | 10 |
| N.S. | 1 | 1.00 | 1.31 | 0.85 | 3.23 | 0.77 | 3.54 | 1.92 | 0.77 | 0.77 |
| time (sec) | N/A | 0.209 | 0.049 | 0.379 | 0.119 | 0.081 | 0.174 | 0.106 | 0.160 | 41.297 |

| Problem 4 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 10 | 12 | 12 | 12 | 12 | 12 | 8 | 10 | 10 | 10 |
| N.S. | 1 | 1.20 | 1.20 | 1.20 | 1.20 | 1.20 | 0.80 | 1.00 | 1.00 | 1.00 |
| time (sec) | N/A | 0.191 | 0.011 | 0.303 | 0.031 | 0.074 | 0.072 | 0.109 | 0.156 | 41.780 |

| Problem 5 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 11 | 11 | 10 | 9 | 12 | 11 | 5 | 8 | 8 | 8 |
| N.S. | 1 | 1.00 | 0.91 | 0.82 | 1.09 | 1.00 | 0.45 | 0.73 | 0.73 | 0.73 |
| time (sec) | N/A | 0.181 | 0.008 | 0.250 | 0.029 | 0.067 | 0.097 | 0.107 | 0.159 | 43.183 |

| Problem 6 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 29 | 42 | 20 | 31 | 37 | 0 | 34 | 19 | 20 |
| N.S. | 1 | 1.26 | 1.83 | 0.87 | 1.35 | 1.61 | 0.00 | 1.48 | 0.83 | 0.87 |
| time (sec) | N/A | 0.248 | 0.038 | 0.457 | 0.029 | 0.083 | 0.000 | 0.111 | 0.187 | 43.856 |

| Problem 7 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 30 | 25 | 41 | 26 | 0 | 37 | 27 | 35 |
| N.S. | 1 | 1.00 | 1.25 | 1.04 | 1.71 | 1.08 | 0.00 | 1.54 | 1.12 | 1.46 |
| time (sec) | N/A | 0.270 | 0.219 | 0.458 | 0.033 | 0.066 | 0.000 | 0.118 | 0.167 | 40.364 |

| Problem 8 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 55 | 62 | 60 | 36 | 58 | 83 | 0 | 52 | 40 | 45 |
| N.S. | 1 | 1.13 | 1.09 | 0.65 | 1.05 | 1.51 | 0.00 | 0.95 | 0.73 | 0.82 |
| time (sec) | N/A | 0.280 | 0.133 | 0.562 | 0.032 | 0.083 | 0.000 | 0.121 | 0.187 | 41.055 |

| Problem 9 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 37 | 35 | 38 | 43 | 70 | 53 | 0 | 59 | 41 | 45 |
| N.S. | 1 | 0.95 | 1.03 | 1.16 | 1.89 | 1.43 | 0.00 | 1.59 | 1.11 | 1.22 |
| time (sec) | N/A | 0.294 | 0.219 | 0.519 | 0.030 | 0.070 | 0.000 | 0.111 | 0.185 | 42.171 |

| Problem 10 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | B | A | A | A | B | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 5 | 11 | 5 | 8 | 9 | 11 | 10 | 9 | 9 | 7 |
| N.S. | 1 | 2.20 | 1.00 | 1.60 | 1.80 | 2.20 | 2.00 | 1.80 | 1.80 | 1.40 |
| time (sec) | N/A | 0.193 | 0.011 | 0.454 | 0.034 | 0.075 | 0.053 | 0.104 | 0.192 | 0.062 |

| Problem 11 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | B | A | B | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 3 | 13 | 3 | 12 | 9 | 11 | 8 | 11 | 9 | 9 |
| N.S. | 1 | 4.33 | 1.00 | 4.00 | 3.00 | 3.67 | 2.67 | 3.67 | 3.00 | 3.00 |
| time (sec) | N/A | 0.193 | 0.005 | 0.411 | 0.024 | 0.080 | 0.060 | 0.109 | 0.227 | 0.091 |

| Problem 12 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 6 | 6 | 12 | 7 | 6 | 6 | 5 | 6 | 10 | 6 |
| N.S. | 1 | 1.00 | 2.00 | 1.17 | 1.00 | 1.00 | 0.83 | 1.00 | 1.67 | 1.00 |
| time (sec) | N/A | 0.189 | 0.015 | 0.316 | 0.029 | 0.067 | 0.162 | 0.105 | 0.239 | 42.008 |

| Problem 13 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 10 | 10 | 12 | 9 | 6 | 6 | 5 | 6 | 9 | 6 |
| N.S. | 1 | 1.00 | 1.20 | 0.90 | 0.60 | 0.60 | 0.50 | 0.60 | 0.90 | 0.60 |
| time (sec) | N/A | 0.188 | 0.014 | 0.315 | 0.031 | 0.073 | 0.165 | 0.116 | 0.222 | 42.127 |

| Problem 14 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 14 | 14 | 18 | 11 | 23 | 18 | 7 | 10 | 10 | 10 |
| N.S. | 1 | 1.00 | 1.29 | 0.79 | 1.64 | 1.29 | 0.50 | 0.71 | 0.71 | 0.71 |
| time (sec) | N/A | 0.199 | 0.025 | 0.363 | 0.117 | 0.072 | 0.196 | 0.108 | 0.179 | 40.564 |

| Problem 15 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 26 | 11 | 23 | 16 | 8 | 12 | 16 | 10 |
| N.S. | 1 | 1.00 | 1.62 | 0.69 | 1.44 | 1.00 | 0.50 | 0.75 | 1.00 | 0.62 |
| time (sec) | N/A | 0.202 | 0.029 | 0.408 | 0.118 | 0.074 | 0.321 | 0.117 | 0.181 | 43.160 |

| Problem 16 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 10 | 10 | 13 | 11 | 10 | 12 | 58 | 10 | 38 | 10 |
| N.S. | 1 | 1.00 | 1.30 | 1.10 | 1.00 | 1.20 | 5.80 | 1.00 | 3.80 | 1.00 |
| time (sec) | N/A | 0.218 | 0.032 | 0.444 | 0.027 | 0.081 | 0.209 | 0.118 | 0.169 | 0.045 |

| Problem 17 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 12 | 12 | 13 | 11 | 10 | 12 | 58 | 12 | 38 | 10 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.83 | 1.00 | 4.83 | 1.00 | 3.17 | 0.83 |
| time (sec) | N/A | 0.214 | 0.022 | 0.543 | 0.028 | 0.081 | 0.202 | 0.110 | 0.182 | 43.750 |

| Problem 18 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 10 | 10 | 12 | 9 | 8 | 14 | 14 | 8 | 14 | 8 |
| N.S. | 1 | 1.00 | 1.20 | 0.90 | 0.80 | 1.40 | 1.40 | 0.80 | 1.40 | 0.80 |
| time (sec) | N/A | 0.184 | 0.012 | 0.363 | 0.026 | 0.068 | 0.262 | 0.105 | 0.159 | 0.041 |

| Problem 19 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 12 | 12 | 12 | 11 | 8 | 14 | 15 | 8 | 16 | 8 |
| N.S. | 1 | 1.00 | 1.00 | 0.92 | 0.67 | 1.17 | 1.25 | 0.67 | 1.33 | 0.67 |
| time (sec) | N/A | 0.187 | 0.015 | 0.376 | 0.026 | 0.064 | 0.247 | 0.127 | 0.170 | 43.694 |

| Problem 20 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 14 | 14 | 12 | 9 | 12 | 20 | 7 | 8 | 8 | 8 |
| N.S. | 1 | 1.00 | 0.86 | 0.64 | 0.86 | 1.43 | 0.50 | 0.57 | 0.57 | 0.57 |
| time (sec) | N/A | 0.194 | 0.094 | 0.346 | 0.034 | 0.080 | 0.294 | 0.109 | 0.187 | 43.733 |

| Problem 21 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 12 | 9 | 12 | 22 | 10 | 8 | 8 | 8 |
| N.S. | 1 | 1.00 | 0.75 | 0.56 | 0.75 | 1.38 | 0.62 | 0.50 | 0.50 | 0.50 |
| time (sec) | N/A | 0.197 | 0.103 | 0.401 | 0.026 | 0.066 | 0.470 | 0.117 | 0.167 | 44.861 |

| Problem 22 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 14 | 14 | 18 | 15 | 14 | 21 | 126 | 14 | 54 | 14 |
| N.S. | 1 | 1.00 | 1.29 | 1.07 | 1.00 | 1.50 | 9.00 | 1.00 | 3.86 | 1.00 |
| time (sec) | N/A | 0.220 | 0.014 | 0.493 | 0.031 | 0.076 | 0.250 | 0.132 | 0.171 | 44.224 |

| Problem 23 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 20 | 17 | 16 | 22 | 126 | 18 | 54 | 16 |
| N.S. | 1 | 1.00 | 1.00 | 0.85 | 0.80 | 1.10 | 6.30 | 0.90 | 2.70 | 0.80 |
| time (sec) | N/A | 0.231 | 0.012 | 0.591 | 0.030 | 0.078 | 0.254 | 0.106 | 0.160 | 0.050 |

| Problem 24 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | A | F(-1) | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 104 | 125 | 96 | 152 | 0 | 243 | 0 | 194 | 139 | 1677 |
| N.S. | 1 | 1.20 | 0.92 | 1.46 | 0.00 | 2.34 | 0.00 | 1.87 | 1.34 | 16.12 |
| time (sec) | N/A | 0.659 | 0.285 | 0.852 | 0.000 | 0.100 | 0.000 | 0.115 | 0.165 | 44.970 |

| Problem 25 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 40 | 40 | 40 | 39 | 38 | 41 | 1421 | 39 | 105 | 38 |
| N.S. | 1 | 1.00 | 1.00 | 0.98 | 0.95 | 1.02 | 35.52 | 0.98 | 2.62 | 0.95 |
| time (sec) | N/A | 0.254 | 0.094 | 0.891 | 0.025 | 0.089 | 153.079 | 0.108 | 0.174 | 0.101 |

| Problem 26 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 59 | 73 | 54 | 78 | 0 | 154 | 991 | 90 | 53 | 74 |
| N.S. | 1 | 1.24 | 0.92 | 1.32 | 0.00 | 2.61 | 16.80 | 1.53 | 0.90 | 1.25 |
| time (sec) | N/A | 0.393 | 0.136 | 0.461 | 0.000 | 0.100 | 50.680 | 0.118 | 0.181 | 44.729 |

| Problem 27 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 12 | 12 | 12 | 13 | 12 | 15 | 17 | 13 | 12 | 12 |
| N.S. | 1 | 1.00 | 1.00 | 1.08 | 1.00 | 1.25 | 1.42 | 1.08 | 1.00 | 1.00 |
| time (sec) | N/A | 0.196 | 0.038 | 0.358 | 0.026 | 0.081 | 0.079 | 0.118 | 0.264 | 0.046 |

| Problem 28 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 42 | 42 | 41 | 36 | 0 | 137 | 144 | 61 | 51 | 38 |
| N.S. | 1 | 1.00 | 0.98 | 0.86 | 0.00 | 3.26 | 3.43 | 1.45 | 1.21 | 0.90 |
| time (sec) | N/A | 0.202 | 0.075 | 0.236 | 0.000 | 0.092 | 1.699 | 0.111 | 0.233 | 45.336 |

| Problem 29 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 53 | 66 | 50 | 44 | 47 | 52 | 0 | 54 | 51 | 52 |
| N.S. | 1 | 1.25 | 0.94 | 0.83 | 0.89 | 0.98 | 0.00 | 1.02 | 0.96 | 0.98 |
| time (sec) | N/A | 0.285 | 0.085 | 0.582 | 0.026 | 0.088 | 0.000 | 0.111 | 0.240 | 0.193 |

| Problem 30 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 67 | 78 | 66 | 78 | 0 | 230 | 0 | 91 | 92 | 86 |
| N.S. | 1 | 1.16 | 0.99 | 1.16 | 0.00 | 3.43 | 0.00 | 1.36 | 1.37 | 1.28 |
| time (sec) | N/A | 0.317 | 0.404 | 0.523 | 0.000 | 0.095 | 0.000 | 0.146 | 0.162 | 44.811 |

| Problem 31 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 100 | 118 | 99 | 91 | 115 | 181 | 0 | 136 | 141 | 112 |
| N.S. | 1 | 1.18 | 0.99 | 0.91 | 1.15 | 1.81 | 0.00 | 1.36 | 1.41 | 1.12 |
| time (sec) | N/A | 0.377 | 0.547 | 0.832 | 0.033 | 0.116 | 0.000 | 0.118 | 0.183 | 43.159 |

| Problem 32 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | B | F | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 110 | 135 | 112 | 127 | 0 | 459 | 0 | 206 | 180 | 184 |
| N.S. | 1 | 1.23 | 1.02 | 1.15 | 0.00 | 4.17 | 0.00 | 1.87 | 1.64 | 1.67 |
| time (sec) | N/A | 0.579 | 0.805 | 0.716 | 0.000 | 0.105 | 0.000 | 0.118 | 0.191 | 42.892 |

| Problem 33 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | C | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 129 | 135 | 108 | 127 | 0 | 136 | 0 | 0 | 51 | 0 |
| N.S. | 1 | 1.05 | 0.84 | 0.98 | 0.00 | 1.05 | 0.00 | 0.00 | 0.40 | 0.00 |
| time (sec) | N/A | 0.562 | 1.295 | 3.638 | 0.000 | 0.104 | 0.000 | 0.000 | 0.167 | 0.000 |

| Problem 34 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 100 | 101 | 80 | 171 | 0 | 120 | 0 | 0 | 51 | 0 |
| N.S. | 1 | 1.01 | 0.80 | 1.71 | 0.00 | 1.20 | 0.00 | 0.00 | 0.51 | 0.00 |
| time (sec) | N/A | 0.437 | 0.962 | 3.095 | 0.000 | 0.104 | 0.000 | 0.000 | 0.163 | 0.000 |

| Problem 35 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 100 | 101 | 80 | 116 | 0 | 99 | 0 | 0 | 47 | 0 |
| N.S. | 1 | 1.01 | 0.80 | 1.16 | 0.00 | 0.99 | 0.00 | 0.00 | 0.47 | 0.00 |
| time (sec) | N/A | 0.437 | 0.720 | 1.847 | 0.000 | 0.097 | 0.000 | 0.000 | 0.168 | 0.000 |

| Problem 36 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | C | F | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 68 | 68 | 60 | 117 | 0 | 82 | 0 | 0 | 37 | 60 |
| N.S. | 1 | 1.00 | 0.88 | 1.72 | 0.00 | 1.21 | 0.00 | 0.00 | 0.54 | 0.88 |
| time (sec) | N/A | 0.328 | 0.344 | 2.136 | 0.000 | 0.105 | 0.000 | 0.000 | 0.192 | 42.657 |

| Problem 37 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | C | F | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 66 | 66 | 54 | 92 | 0 | 71 | 0 | 0 | 41 | 50 |
| N.S. | 1 | 1.00 | 0.82 | 1.39 | 0.00 | 1.08 | 0.00 | 0.00 | 0.62 | 0.76 |
| time (sec) | N/A | 0.333 | 0.416 | 1.917 | 0.000 | 0.082 | 0.000 | 0.000 | 0.194 | 43.254 |

| Problem 38 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 96 | 97 | 58 | 153 | 0 | 108 | 0 | 0 | 55 | 0 |
| N.S. | 1 | 1.01 | 0.60 | 1.59 | 0.00 | 1.12 | 0.00 | 0.00 | 0.57 | 0.00 |
| time (sec) | N/A | 0.442 | 0.358 | 2.221 | 0.000 | 0.087 | 0.000 | 0.000 | 0.177 | 0.000 |

| Problem 39 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 102 | 103 | 59 | 124 | 0 | 122 | 0 | 0 | 59 | 0 |
| N.S. | 1 | 1.01 | 0.58 | 1.22 | 0.00 | 1.20 | 0.00 | 0.00 | 0.58 | 0.00 |
| time (sec) | N/A | 0.441 | 0.430 | 2.416 | 0.000 | 0.091 | 0.000 | 0.000 | 0.164 | 0.000 |

| Problem 40 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | C | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 131 | 135 | 74 | 187 | 0 | 166 | 0 | 0 | 59 | 0 |
| N.S. | 1 | 1.03 | 0.56 | 1.43 | 0.00 | 1.27 | 0.00 | 0.00 | 0.45 | 0.00 |
| time (sec) | N/A | 0.551 | 0.596 | 2.249 | 0.000 | 0.087 | 0.000 | 0.000 | 0.184 | 0.000 |

| Problem 41 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | C | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 193 | 184 | 157 | 252 | 0 | 195 | 0 | 0 | 86 | 0 |
| N.S. | 1 | 0.95 | 0.81 | 1.31 | 0.00 | 1.01 | 0.00 | 0.00 | 0.45 | 0.00 |
| time (sec) | N/A | 0.801 | 2.781 | 11.235 | 0.000 | 0.137 | 0.000 | 0.000 | 0.193 | 0.000 |

| Problem 42 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | C | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 154 | 150 | 116 | 332 | 0 | 168 | 0 | 0 | 86 | 0 |
| N.S. | 1 | 0.97 | 0.75 | 2.16 | 0.00 | 1.09 | 0.00 | 0.00 | 0.56 | 0.00 |
| time (sec) | N/A | 0.669 | 1.426 | 10.173 | 0.000 | 0.097 | 0.000 | 0.000 | 0.173 | 0.000 |

| Problem 43 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 154 | 150 | 117 | 229 | 0 | 145 | 0 | 0 | 80 | 0 |
| N.S. | 1 | 0.97 | 0.76 | 1.49 | 0.00 | 0.94 | 0.00 | 0.00 | 0.52 | 0.00 |
| time (sec) | N/A | 0.673 | 1.517 | 2.834 | 0.000 | 0.091 | 0.000 | 0.000 | 0.192 | 0.000 |

| Problem 44 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 114 | 117 | 83 | 272 | 0 | 117 | 0 | 0 | 64 | 0 |
| N.S. | 1 | 1.03 | 0.73 | 2.39 | 0.00 | 1.03 | 0.00 | 0.00 | 0.56 | 0.00 |
| time (sec) | N/A | 0.533 | 0.755 | 3.210 | 0.000 | 0.137 | 0.000 | 0.000 | 0.173 | 0.000 |

| Problem 45 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 114 | 115 | 79 | 170 | 0 | 105 | 0 | 0 | 75 | 0 |
| N.S. | 1 | 1.01 | 0.69 | 1.49 | 0.00 | 0.92 | 0.00 | 0.00 | 0.66 | 0.00 |
| time (sec) | N/A | 0.539 | 1.083 | 2.741 | 0.000 | 0.099 | 0.000 | 0.000 | 0.183 | 0.000 |

| Problem 46 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 118 | 121 | 75 | 283 | 0 | 135 | 0 | 0 | 95 | 0 |
| N.S. | 1 | 1.03 | 0.64 | 2.40 | 0.00 | 1.14 | 0.00 | 0.00 | 0.81 | 0.00 |
| time (sec) | N/A | 0.581 | 1.082 | 3.360 | 0.000 | 0.089 | 0.000 | 0.000 | 0.190 | 0.000 |

| Problem 47 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 124 | 125 | 76 | 202 | 0 | 161 | 0 | 0 | 102 | 0 |
| N.S. | 1 | 1.01 | 0.61 | 1.63 | 0.00 | 1.30 | 0.00 | 0.00 | 0.82 | 0.00 |
| time (sec) | N/A | 0.574 | 1.232 | 3.218 | 0.000 | 0.091 | 0.000 | 0.000 | 0.171 | 0.000 |

| Problem 48 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | C | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 165 | 158 | 109 | 351 | 0 | 216 | 0 | 0 | 102 | 0 |
| N.S. | 1 | 0.96 | 0.66 | 2.13 | 0.00 | 1.31 | 0.00 | 0.00 | 0.62 | 0.00 |
| time (sec) | N/A | 0.708 | 1.539 | 3.337 | 0.000 | 0.174 | 0.000 | 0.000 | 0.167 | 0.000 |

| Problem 49 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | C | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 242 | 237 | 205 | 276 | 0 | 246 | 0 | 0 | 121 | 0 |
| N.S. | 1 | 0.98 | 0.85 | 1.14 | 0.00 | 1.02 | 0.00 | 0.00 | 0.50 | 0.00 |
| time (sec) | N/A | 1.145 | 4.026 | 43.706 | 0.000 | 0.115 | 0.000 | 0.000 | 0.202 | 0.000 |

| Problem 50 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | C | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 202 | 203 | 149 | 356 | 0 | 206 | 0 | 0 | 121 | 0 |
| N.S. | 1 | 1.00 | 0.74 | 1.76 | 0.00 | 1.02 | 0.00 | 0.00 | 0.60 | 0.00 |
| time (sec) | N/A | 0.974 | 2.491 | 43.233 | 0.000 | 0.111 | 0.000 | 0.000 | 0.221 | 0.000 |

| Problem 51 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 202 | 203 | 147 | 272 | 0 | 183 | 0 | 0 | 113 | 0 |
| N.S. | 1 | 1.00 | 0.73 | 1.35 | 0.00 | 0.91 | 0.00 | 0.00 | 0.56 | 0.00 |
| time (sec) | N/A | 0.975 | 2.171 | 3.826 | 0.000 | 0.107 | 0.000 | 0.000 | 0.210 | 0.000 |

| Problem 52 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 161 | 169 | 105 | 314 | 0 | 140 | 0 | 0 | 88 | 0 |
| N.S. | 1 | 1.05 | 0.65 | 1.95 | 0.00 | 0.87 | 0.00 | 0.00 | 0.55 | 0.00 |
| time (sec) | N/A | 0.818 | 1.446 | 4.272 | 0.000 | 0.125 | 0.000 | 0.000 | 0.265 | 0.000 |

| Problem 53 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 157 | 158 | 98 | 210 | 0 | 126 | 0 | 0 | 110 | 0 |
| N.S. | 1 | 1.01 | 0.62 | 1.34 | 0.00 | 0.80 | 0.00 | 0.00 | 0.70 | 0.00 |
| time (sec) | N/A | 0.811 | 1.630 | 4.012 | 0.000 | 0.091 | 0.000 | 0.000 | 0.270 | 0.000 |

| Problem 54 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 165 | 168 | 101 | 313 | 0 | 163 | 0 | 0 | 141 | 0 |
| N.S. | 1 | 1.02 | 0.61 | 1.90 | 0.00 | 0.99 | 0.00 | 0.00 | 0.85 | 0.00 |
| time (sec) | N/A | 0.842 | 1.234 | 4.810 | 0.000 | 0.108 | 0.000 | 0.000 | 0.249 | 0.000 |

| Problem 55 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 169 | 168 | 102 | 226 | 0 | 187 | 0 | 0 | 145 | 0 |
| N.S. | 1 | 0.99 | 0.60 | 1.34 | 0.00 | 1.11 | 0.00 | 0.00 | 0.86 | 0.00 |
| time (sec) | N/A | 0.847 | 2.109 | 4.484 | 0.000 | 0.106 | 0.000 | 0.000 | 0.165 | 0.000 |

| Problem 56 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | C | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 192 | 199 | 130 | 375 | 0 | 243 | 0 | 0 | 145 | 0 |
| N.S. | 1 | 1.04 | 0.68 | 1.95 | 0.00 | 1.27 | 0.00 | 0.00 | 0.76 | 0.00 |
| time (sec) | N/A | 0.926 | 1.952 | 4.681 | 0.000 | 0.097 | 0.000 | 0.000 | 0.173 | 0.000 |

| Problem 57 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | C | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 193 | 202 | 144 | 265 | 0 | 274 | 0 | 0 | 145 | 0 |
| N.S. | 1 | 1.05 | 0.75 | 1.37 | 0.00 | 1.42 | 0.00 | 0.00 | 0.75 | 0.00 |
| time (sec) | N/A | 0.939 | 2.258 | 4.946 | 0.000 | 0.137 | 0.000 | 0.000 | 0.182 | 0.000 |

| Problem 58 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F | F(-1) | F(-1) | F | F | F(-1) |
| verified | N/A | No | No | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 544 | 555 | 2035 | 931 | 0 | 0 | 0 | 0 | 36 | 0 |
| N.S. | 1 | 1.02 | 3.74 | 1.71 | 0.00 | 0.00 | 0.00 | 0.00 | 0.07 | 0.00 |
| time (sec) | N/A | 2.934 | 19.001 | 3.987 | 0.000 | 0.000 | 0.000 | 0.000 | 0.158 | 0.000 |

| Problem 59 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | F | F(-1) | F(-1) | F | F | F(-1) |
| verified | N/A | No | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 461 | 459 | 834 | 851 | 0 | 0 | 0 | 0 | 36 | 0 |
| N.S. | 1 | 1.00 | 1.81 | 1.85 | 0.00 | 0.00 | 0.00 | 0.00 | 0.08 | 0.00 |
| time (sec) | N/A | 2.164 | 17.375 | 3.635 | 0.000 | 0.000 | 0.000 | 0.000 | 0.176 | 0.000 |

| Problem 60 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F | F(-1) | F(-1) | F | F | F(-1) |
| verified | N/A | No | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 474 | 478 | 1955 | 773 | 0 | 0 | 0 | 0 | 36 | 0 |
| N.S. | 1 | 1.01 | 4.12 | 1.63 | 0.00 | 0.00 | 0.00 | 0.00 | 0.08 | 0.00 |
| time (sec) | N/A | 2.214 | 17.626 | 3.506 | 0.000 | 0.000 | 0.000 | 0.000 | 0.162 | 0.000 |

| Problem 61 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F | F(-1) | F(-1) | F | F | F(-1) |
| verified | N/A | No | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 399 | 390 | 757 | 639 | 0 | 0 | 0 | 0 | 36 | 0 |
| N.S. | 1 | 0.98 | 1.90 | 1.60 | 0.00 | 0.00 | 0.00 | 0.00 | 0.09 | 0.00 |
| time (sec) | N/A | 1.745 | 16.604 | 2.871 | 0.000 | 0.000 | 0.000 | 0.000 | 0.160 | 0.000 |

| Problem 62 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F | F(-1) | F | F | F | F(-1) |
| verified | N/A | No | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 410 | 407 | 434 | 655 | 0 | 0 | 0 | 0 | 32 | 0 |
| N.S. | 1 | 0.99 | 1.06 | 1.60 | 0.00 | 0.00 | 0.00 | 0.00 | 0.08 | 0.00 |
| time (sec) | N/A | 1.719 | 7.150 | 1.570 | 0.000 | 0.000 | 0.000 | 0.000 | 0.196 | 0.000 |

| Problem 63 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F | F | F | F | F | F(-1) |
| verified | N/A | No | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 302 | 297 | 361 | 496 | 0 | 0 | 0 | 0 | 25 | 0 |
| N.S. | 1 | 0.98 | 1.20 | 1.64 | 0.00 | 0.00 | 0.00 | 0.00 | 0.08 | 0.00 |
| time (sec) | N/A | 1.080 | 3.239 | 1.666 | 0.000 | 0.000 | 0.000 | 0.000 | 0.156 | 0.000 |

| Problem 64 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F | F(-1) | F | F | F | F(-1) |
| verified | N/A | No | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 307 | 316 | 261 | 519 | 0 | 0 | 0 | 0 | 41 | 0 |
| N.S. | 1 | 1.03 | 0.85 | 1.69 | 0.00 | 0.00 | 0.00 | 0.00 | 0.13 | 0.00 |
| time (sec) | N/A | 1.183 | 3.638 | 1.371 | 0.000 | 0.000 | 0.000 | 0.000 | 0.179 | 0.000 |

| Problem 65 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | F | F(-1) | F | F | F | F(-1) |
| verified | N/A | No | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 426 | 405 | 791 | 774 | 0 | 0 | 0 | 0 | 45 | 0 |
| N.S. | 1 | 0.95 | 1.86 | 1.82 | 0.00 | 0.00 | 0.00 | 0.00 | 0.11 | 0.00 |
| time (sec) | N/A | 1.865 | 15.982 | 1.968 | 0.000 | 0.000 | 0.000 | 0.000 | 0.168 | 0.000 |

| Problem 66 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F(-1) | F | F | F | F | F(-1) |
| verified | N/A | No | No | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 447 | 429 | 1192 | 711 | 0 | 0 | 0 | 0 | 45 | 0 |
| N.S. | 1 | 0.96 | 2.67 | 1.59 | 0.00 | 0.00 | 0.00 | 0.00 | 0.10 | 0.00 |
| time (sec) | N/A | 1.875 | 13.674 | 2.034 | 0.000 | 0.000 | 0.000 | 0.000 | 0.163 | 0.000 |

| Problem 67 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | F(-1) | F(-1) | F(-1) | F | F | F(-1) |
| verified | N/A | No | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 501 | 494 | 881 | 1007 | 0 | 0 | 0 | 0 | 45 | 0 |
| N.S. | 1 | 0.99 | 1.76 | 2.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.09 | 0.00 |
| time (sec) | N/A | 2.448 | 7.647 | 2.453 | 0.000 | 0.000 | 0.000 | 0.000 | 0.191 | 0.000 |

| Problem 68 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | F | F(-1) | F(-1) | F | F | F(-1) |
| verified | N/A | No | No | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 557 | 556 | 2029 | 1659 | 0 | 0 | 0 | 0 | 52 | 0 |
| N.S. | 1 | 1.00 | 3.64 | 2.98 | 0.00 | 0.00 | 0.00 | 0.00 | 0.09 | 0.00 |
| time (sec) | N/A | 2.679 | 19.387 | 17.014 | 0.000 | 0.000 | 0.000 | 0.000 | 0.162 | 0.000 |

| Problem 69 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | F | F(-1) | F(-1) | F | F | F(-1) |
| verified | N/A | No | No | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 473 | 460 | 835 | 1628 | 0 | 0 | 0 | 0 | 52 | 0 |
| N.S. | 1 | 0.97 | 1.77 | 3.44 | 0.00 | 0.00 | 0.00 | 0.00 | 0.11 | 0.00 |
| time (sec) | N/A | 2.100 | 17.292 | 16.414 | 0.000 | 0.000 | 0.000 | 0.000 | 0.162 | 0.000 |

| Problem 70 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | F | F(-1) | F(-1) | F | F | F(-1) |
| verified | N/A | No | No | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 487 | 479 | 1956 | 1501 | 0 | 0 | 0 | 0 | 52 | 0 |
| N.S. | 1 | 0.98 | 4.02 | 3.08 | 0.00 | 0.00 | 0.00 | 0.00 | 0.11 | 0.00 |
| time (sec) | N/A | 2.205 | 18.676 | 16.473 | 0.000 | 0.000 | 0.000 | 0.000 | 0.170 | 0.000 |

| Problem 71 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | F | F(-1) | F(-1) | F | F | F(-1) |
| verified | N/A | No | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 404 | 393 | 366 | 1668 | 0 | 0 | 0 | 0 | 52 | 0 |
| N.S. | 1 | 0.97 | 0.91 | 4.13 | 0.00 | 0.00 | 0.00 | 0.00 | 0.13 | 0.00 |
| time (sec) | N/A | 1.658 | 10.204 | 15.635 | 0.000 | 0.000 | 0.000 | 0.000 | 0.173 | 0.000 |

| Problem 72 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | F | F(-1) | F(-1) | F | F | F(-1) |
| verified | N/A | No | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 418 | 411 | 614 | 1371 | 0 | 0 | 0 | 0 | 129 | 0 |
| N.S. | 1 | 0.98 | 1.47 | 3.28 | 0.00 | 0.00 | 0.00 | 0.00 | 0.31 | 0.00 |
| time (sec) | N/A | 1.699 | 11.180 | 2.316 | 0.000 | 0.000 | 0.000 | 0.000 | 0.200 | 0.000 |

| Problem 73 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | F | F(-1) | F | F | F | F(-1) |
| verified | N/A | No | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 438 | 403 | 786 | 1306 | 0 | 0 | 0 | 0 | 41 | 0 |
| N.S. | 1 | 0.92 | 1.79 | 2.98 | 0.00 | 0.00 | 0.00 | 0.00 | 0.09 | 0.00 |
| time (sec) | N/A | 1.692 | 15.118 | 2.550 | 0.000 | 0.000 | 0.000 | 0.000 | 0.171 | 0.000 |

| Problem 74 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | F(-1) | F | F | F | F | F(-1) |
| verified | N/A | No | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 445 | 423 | 1182 | 1280 | 0 | 0 | 0 | 0 | 63 | 0 |
| N.S. | 1 | 0.95 | 2.66 | 2.88 | 0.00 | 0.00 | 0.00 | 0.00 | 0.14 | 0.00 |
| time (sec) | N/A | 1.741 | 12.422 | 2.279 | 0.000 | 0.000 | 0.000 | 0.000 | 0.176 | 0.000 |

| Problem 75 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | F(-1) | F(-1) | F | F | F | F(-1) |
| verified | N/A | No | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 507 | 490 | 865 | 2002 | 0 | 0 | 0 | 0 | 69 | 0 |
| N.S. | 1 | 0.97 | 1.71 | 3.95 | 0.00 | 0.00 | 0.00 | 0.00 | 0.14 | 0.00 |
| time (sec) | N/A | 2.300 | 7.123 | 2.915 | 0.000 | 0.000 | 0.000 | 0.000 | 0.175 | 0.000 |

| Problem 76 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | F(-1) | F(-1) | F(-1) | F | F | F(-1) |
| verified | N/A | No | No | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 530 | 513 | 1257 | 1470 | 0 | 0 | 0 | 0 | 69 | 0 |
| N.S. | 1 | 0.97 | 2.37 | 2.77 | 0.00 | 0.00 | 0.00 | 0.00 | 0.13 | 0.00 |
| time (sec) | N/A | 2.265 | 16.500 | 3.051 | 0.000 | 0.000 | 0.000 | 0.000 | 0.168 | 0.000 |

| Problem 77 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | F(-1) | F(-1) | F(-1) | F | F | F(-1) |
| verified | N/A | No | No | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 590 | 583 | 950 | 1745 | 0 | 0 | 0 | 0 | 69 | 0 |
| N.S. | 1 | 0.99 | 1.61 | 2.96 | 0.00 | 0.00 | 0.00 | 0.00 | 0.12 | 0.00 |
| time (sec) | N/A | 2.811 | 7.382 | 3.401 | 0.000 | 0.000 | 0.000 | 0.000 | 0.191 | 0.000 |

| Problem 78 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | F(-1) | F(-1) | F(-1) | F | F | F(-1) |
| verified | N/A | No | No | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 590 | 543 | 930 | 2995 | 0 | 0 | 0 | 0 | 68 | 0 |
| N.S. | 1 | 0.92 | 1.58 | 5.08 | 0.00 | 0.00 | 0.00 | 0.00 | 0.12 | 0.00 |
| time (sec) | N/A | 2.533 | 16.722 | 107.099 | 0.000 | 0.000 | 0.000 | 0.000 | 0.179 | 0.000 |

| Problem 79 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | F(-1) | F(-1) | F(-1) | F | F | F(-1) |
| verified | N/A | No | No | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 604 | 562 | 2024 | 2754 | 0 | 0 | 0 | 0 | 68 | 0 |
| N.S. | 1 | 0.93 | 3.35 | 4.56 | 0.00 | 0.00 | 0.00 | 0.00 | 0.11 | 0.00 |
| time (sec) | N/A | 2.603 | 17.645 | 101.655 | 0.000 | 0.000 | 0.000 | 0.000 | 0.168 | 0.000 |

| Problem 80 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | F(-1) | F(-1) | F(-1) | F | F | F(-1) |
| verified | N/A | No | No | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 498 | 466 | 837 | 2736 | 0 | 0 | 0 | 0 | 68 | 0 |
| N.S. | 1 | 0.94 | 1.68 | 5.49 | 0.00 | 0.00 | 0.00 | 0.00 | 0.14 | 0.00 |
| time (sec) | N/A | 2.015 | 15.857 | 100.149 | 0.000 | 0.000 | 0.000 | 0.000 | 0.177 | 0.000 |

| Problem 81 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | F | F(-1) | F(-1) | F | F | F(-1) |
| verified | N/A | No | No | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 512 | 483 | 1954 | 2590 | 0 | 0 | 0 | 0 | 68 | 0 |
| N.S. | 1 | 0.94 | 3.82 | 5.06 | 0.00 | 0.00 | 0.00 | 0.00 | 0.13 | 0.00 |
| time (sec) | N/A | 2.160 | 18.276 | 98.709 | 0.000 | 0.000 | 0.000 | 0.000 | 0.176 | 0.000 |

| Problem 82 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | F(-1) | F(-1) | F(-1) | F | F | F(-1) |
| verified | N/A | No | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 520 | 464 | 831 | 2612 | 0 | 0 | 0 | 0 | 68 | 0 |
| N.S. | 1 | 0.89 | 1.60 | 5.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.13 | 0.00 |
| time (sec) | N/A | 2.047 | 15.921 | 96.435 | 0.000 | 0.000 | 0.000 | 0.000 | 0.246 | 0.000 |

| Problem 83 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | F | F(-1) | F(-1) | F | F | F(-1) |
| verified | N/A | No | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 534 | 483 | 1211 | 2494 | 0 | 0 | 0 | 0 | 64 | 0 |
| N.S. | 1 | 0.90 | 2.27 | 4.67 | 0.00 | 0.00 | 0.00 | 0.00 | 0.12 | 0.00 |
| time (sec) | N/A | 2.093 | 12.932 | 3.362 | 0.000 | 0.000 | 0.000 | 0.000 | 0.245 | 0.000 |

| Problem 84 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | F | F(-1) | F(-1) | F | F | F(-1) |
| verified | N/A | No | No | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 529 | 477 | 837 | 2365 | 0 | 0 | 0 | 0 | 57 | 0 |
| N.S. | 1 | 0.90 | 1.58 | 4.47 | 0.00 | 0.00 | 0.00 | 0.00 | 0.11 | 0.00 |
| time (sec) | N/A | 2.233 | 15.794 | 3.889 | 0.000 | 0.000 | 0.000 | 0.000 | 0.234 | 0.000 |

| Problem 85 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | F(-1) | F(-1) | F(-1) | F | F | F(-1) |
| verified | N/A | No | No | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 535 | 497 | 1226 | 2220 | 0 | 0 | 0 | 0 | 85 | 0 |
| N.S. | 1 | 0.93 | 2.29 | 4.15 | 0.00 | 0.00 | 0.00 | 0.00 | 0.16 | 0.00 |
| time (sec) | N/A | 2.239 | 13.919 | 3.271 | 0.000 | 0.000 | 0.000 | 0.000 | 0.161 | 0.000 |

| Problem 86 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | F(-1) | F(-1) | F(-1) | F | F | F(-1) |
| verified | N/A | No | No | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 611 | 575 | 922 | 2836 | 0 | 0 | 0 | 0 | 93 | 0 |
| N.S. | 1 | 0.94 | 1.51 | 4.64 | 0.00 | 0.00 | 0.00 | 0.00 | 0.15 | 0.00 |
| time (sec) | N/A | 2.877 | 7.245 | 4.200 | 0.000 | 0.000 | 0.000 | 0.000 | 0.180 | 0.000 |

| Problem 87 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | F(-1) | F(-1) | F(-1) | F | F | F(-1) |
| verified | N/A | No | No | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 629 | 598 | 1308 | 2679 | 0 | 0 | 0 | 0 | 93 | 0 |
| N.S. | 1 | 0.95 | 2.08 | 4.26 | 0.00 | 0.00 | 0.00 | 0.00 | 0.15 | 0.00 |
| time (sec) | N/A | 2.900 | 16.612 | 4.406 | 0.000 | 0.000 | 0.000 | 0.000 | 0.180 | 0.000 |

| Problem 88 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | F(-1) | F(-1) | F(-1) | F | F | F(-1) |
| verified | N/A | No | No | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 700 | 679 | 1014 | 3077 | 0 | 0 | 0 | 0 | 93 | 0 |
| N.S. | 1 | 0.97 | 1.45 | 4.40 | 0.00 | 0.00 | 0.00 | 0.00 | 0.13 | 0.00 |
| time (sec) | N/A | 3.637 | 7.617 | 4.959 | 0.000 | 0.000 | 0.000 | 0.000 | 0.172 | 0.000 |

| Problem 89 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | No | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 159 | 159 | 676 | 0 | 0 | 0 | 0 | 0 | 27 | 0 |
| N.S. | 1 | 1.00 | 4.25 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.17 | 0.00 |
| time (sec) | N/A | 0.382 | 4.105 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.183 | 0.000 |

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [88] had the largest ratio of [1.0800000000000007]

Table 2.1: Rubi specific breakdown of results for each integral

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 1 | A | 5 | 5 | 0.97 | 13 | 0.385 |
| 2 | A | 4 | 3 | 1.00 | 13 | 0.231 |
| 3 | A | 3 | 3 | 1.00 | 13 | 0.231 |
| 4 | A | 4 | 3 | 1.20 | 11 | 0.273 |
| 5 | A | 2 | 2 | 1.00 | 8 | 0.250 |
| 6 | A | 5 | 4 | 1.26 | 11 | 0.364 |
| 7 | A | 6 | 5 | 1.00 | 13 | 0.385 |
| 8 | A | 5 | 4 | 1.13 | 13 | 0.308 |
| 9 | A | 6 | 5 | 0.95 | 13 | 0.385 |
| 10 | B | 4 | 3 | 2.20 | 13 | 0.231 |
| 11 | B | 4 | 3 | 4.33 | 15 | 0.200 |
| 12 | A | 4 | 3 | 1.00 | 9 | 0.333 |
| 13 | A | 4 | 3 | 1.00 | 11 | 0.273 |
| 14 | A | 3 | 3 | 1.00 | 11 | 0.273 |
| 15 | A | 3 | 3 | 1.00 | 13 | 0.231 |
| 16 | A | 5 | 4 | 1.00 | 11 | 0.364 |
| 17 | A | 5 | 4 | 1.00 | 13 | 0.308 |
| 18 | A | 4 | 3 | 1.00 | 9 | 0.333 |
| 19 | A | 4 | 3 | 1.00 | 11 | 0.273 |
| 20 | A | 2 | 2 | 1.00 | 11 | 0.182 |
| 21 | A | 2 | 2 | 1.00 | 13 | 0.154 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 22 | A | 5 | 4 | 1.00 | 11 | 0.364 |
| 23 | A | 5 | 4 | 1.00 | 13 | 0.308 |
| 24 | A | 12 | 11 | 1.20 | 13 | 0.846 |
| 25 | A | 5 | 4 | 1.00 | 13 | 0.308 |
| 26 | A | 9 | 8 | 1.24 | 13 | 0.615 |
| 27 | A | 4 | 3 | 1.00 | 11 | 0.273 |
| 28 | A | 4 | 3 | 1.00 | 8 | 0.375 |
| 29 | A | 5 | 4 | 1.25 | 11 | 0.364 |
| 30 | A | 7 | 6 | 1.16 | 13 | 0.462 |
| 31 | A | 5 | 4 | 1.18 | 13 | 0.308 |
| 32 | A | 10 | 9 | 1.23 | 13 | 0.692 |
| 33 | A | 10 | 10 | 1.05 | 23 | 0.435 |
| 34 | A | 8 | 8 | 1.01 | 23 | 0.348 |
| 35 | A | 8 | 8 | 1.01 | 23 | 0.348 |
| 36 | A | 6 | 6 | 1.00 | 23 | 0.261 |
| 37 | A | 6 | 6 | 1.00 | 23 | 0.261 |
| 38 | A | 8 | 8 | 1.01 | 23 | 0.348 |
| 39 | A | 8 | 8 | 1.01 | 23 | 0.348 |
| 40 | A | 10 | 10 | 1.03 | 23 | 0.435 |
| 41 | A | 13 | 13 | 0.95 | 25 | 0.520 |
| 42 | A | 11 | 11 | 0.97 | 25 | 0.440 |
| 43 | A | 11 | 11 | 0.97 | 25 | 0.440 |
| 44 | A | 9 | 9 | 1.03 | 25 | 0.360 |
| 45 | A | 9 | 9 | 1.01 | 25 | 0.360 |
| 46 | A | 9 | 9 | 1.03 | 25 | 0.360 |
| 47 | A | 9 | 9 | 1.01 | 25 | 0.360 |
| 48 | A | 11 | 11 | 0.96 | 25 | 0.440 |
| 49 | A | 16 | 16 | 0.98 | 25 | 0.640 |
| 50 | A | 14 | 14 | 1.00 | 25 | 0.560 |
| 51 | A | 14 | 14 | 1.00 | 25 | 0.560 |
| 52 | A | 12 | 12 | 1.05 | 25 | 0.480 |
| 53 | A | 12 | 12 | 1.01 | 25 | 0.480 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 54 | A | 12 | 12 | 1.02 | 25 | 0.480 |
| 55 | A | 12 | 12 | 0.99 | 25 | 0.480 |
| 56 | A | 12 | 12 | 1.04 | 25 | 0.480 |
| 57 | A | 12 | 12 | 1.05 | 25 | 0.480 |
| 58 | A | 25 | 24 | 1.02 | 25 | 0.960 |
| 59 | A | 22 | 21 | 1.00 | 25 | 0.840 |
| 60 | A | 22 | 21 | 1.01 | 25 | 0.840 |
| 61 | A | 19 | 18 | 0.98 | 25 | 0.720 |
| 62 | A | 19 | 18 | 0.99 | 25 | 0.720 |
| 63 | A | 11 | 10 | 0.98 | 25 | 0.400 |
| 64 | A | 11 | 10 | 1.03 | 25 | 0.400 |
| 65 | A | 19 | 18 | 0.95 | 25 | 0.720 |
| 66 | A | 19 | 18 | 0.96 | 25 | 0.720 |
| 67 | A | 22 | 21 | 0.99 | 25 | 0.840 |
| 68 | A | 25 | 24 | 1.00 | 25 | 0.960 |
| 69 | A | 22 | 21 | 0.97 | 25 | 0.840 |
| 70 | A | 22 | 21 | 0.98 | 25 | 0.840 |
| 71 | A | 19 | 18 | 0.97 | 25 | 0.720 |
| 72 | A | 20 | 19 | 0.98 | 25 | 0.760 |
| 73 | A | 19 | 18 | 0.92 | 25 | 0.720 |
| 74 | A | 19 | 18 | 0.95 | 25 | 0.720 |
| 75 | A | 22 | 21 | 0.97 | 25 | 0.840 |
| 76 | A | 22 | 21 | 0.97 | 25 | 0.840 |
| 77 | A | 25 | 24 | 0.99 | 25 | 0.960 |
| 78 | A | 25 | 24 | 0.92 | 25 | 0.960 |
| 79 | A | 25 | 24 | 0.93 | 25 | 0.960 |
| 80 | A | 22 | 21 | 0.94 | 25 | 0.840 |
| 81 | A | 22 | 21 | 0.94 | 25 | 0.840 |
| 82 | A | 22 | 21 | 0.89 | 25 | 0.840 |
| 83 | A | 23 | 22 | 0.90 | 25 | 0.880 |
| 84 | A | 22 | 21 | 0.90 | 25 | 0.840 |
| 85 | A | 22 | 21 | 0.93 | 25 | 0.840 |

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| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 86 | A | 25 | 24 | 0.94 | 25 | 0.960 |
| 87 | A | 25 | 24 | 0.95 | 25 | 0.960 |
| 88 | A | 28 | 27 | 0.97 | 25 | 1.080 |
| 89 | A | 4 | 3 | 1.00 | 23 | 0.130 |

# CHAPTER 3

## Listing of integrals

# 3.1 $\int \frac{\sin^4(x)}{a+a\cos(x)}\,dx$



## Optimal result

Integrand size = 13, antiderivative size = 31

$$\int \frac{\sin^4(x)}{a+a\cos(x)}\,dx=\frac{x}{2a}-\frac{\cos(x)\sin(x)}{2a}-\frac{\sin^3(x)}{3a}$$

output
```
1/2*x/a-1/2*cos(x)*sin(x)/a-1/3*sin(x)^3/a
```

## Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{\sin^4(x)}{a+a\cos(x)}\,dx=\frac{6x-3\sin(x)-3\sin(2x)+\sin(3x)}{12a}$$

input
```
Integrate[Sin[x]^4/(a + a*Cos[x]),x]
```

output
```
(6*x - 3*Sin[x] - 3*Sin[2*x] + Sin[3*x])/(12*a)
```

## Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3042, 3161, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(x)}{a\cos(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos\left(x-\frac{\pi}{2}\right)^4}{a-a\sin\left(x-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3161}$$

$$\frac{\int \sin^2(x)dx}{a}-\frac{\sin^3(x)}{3a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sin(x)^2dx}{a}-\frac{\sin^3(x)}{3a}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{\int 1dx}{2}-\frac{1}{2}\sin(x)\cos(x)}{a}-\frac{\sin^3(x)}{3a}$$

$$\Big\downarrow \text{24}$$

$$\frac{\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)}{a}-\frac{\sin^3(x)}{3a}$$

input
```
Int[Sin[x]^4/(a + a*Cos[x]),x]
```

output
```
-1/3*Sin[x]^3/a + (x/2 - (Cos[x]*Sin[x])/2)/a
```

### Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3161
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Si
mp[g^2/a   Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x
] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

## Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

| method | result | size |
|---|---|---|
| parallelrisch | $\frac{6x-3\sin(x)+\sin(3x)-3\sin(2x)}{12a}$ | 24 |
| risch | $\frac{x}{2a}-\frac{\sin(x)}{4a}+\frac{\sin(3x)}{12a}-\frac{\sin(2x)}{4a}$ | 33 |
| default | $\frac{\frac{16\left(\frac{\tan\left(\frac{x}{2}\right)^{5}}{16}-\frac{\tan\left(\frac{x}{2}\right)^{3}}{6}-\frac{\tan\left(\frac{x}{2}\right)}{16}\right)}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)^{3}}+\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{a}$ | 48 |
| norman | $\frac{\frac{\tan\left(\frac{x}{2}\right)^{7}}{a}-\frac{\tan\left(\frac{x}{2}\right)}{a}-\frac{11\tan\left(\frac{x}{2}\right)^{3}}{3a}-\frac{5\tan\left(\frac{x}{2}\right)^{5}}{3a}+\frac{x}{2a}+\frac{2x\tan\left(\frac{x}{2}\right)^{2}}{a}+\frac{3x\tan\left(\frac{x}{2}\right)^{4}}{a}+\frac{2x\tan\left(\frac{x}{2}\right)^{6}}{a}+\frac{x\tan\left(\frac{x}{2}\right)^{8}}{2a}}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)^{4}}$ | 108 |

input
```
int(sin(x)^4/(a+a*cos(x)),x,method=_RETURNVERBOSE)
```

output
```
1/12*(6*x-3*sin(x)+sin(3*x)-3*sin(2*x))/a
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{\sin^4(x)}{a+a\cos(x)}\,dx = \frac{\left(2\cos(x)^2-3\cos(x)-2\right)\sin(x)+3x}{6a}$$

input
```
integrate(sin(x)^4/(a+a*cos(x)),x, algorithm="fricas")
```

output
```
1/6*((2*cos(x)^2 - 3*cos(x) - 2)*sin(x) + 3*x)/a
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(22) = 44.

Time = 0.45 (sec) , antiderivative size = 294, normalized size of antiderivative = 9.48

$$\begin{aligned}\int \frac{\sin^4(x)}{a+a\cos(x)}\,dx &= \frac{3x\tan^6\left(\frac{x}{2}\right)}{6a\tan^6\left(\frac{x}{2}\right)+18a\tan^4\left(\frac{x}{2}\right)+18a\tan^2\left(\frac{x}{2}\right)+6a} \\ &+ \frac{9x\tan^4\left(\frac{x}{2}\right)}{6a\tan^6\left(\frac{x}{2}\right)+18a\tan^4\left(\frac{x}{2}\right)+18a\tan^2\left(\frac{x}{2}\right)+6a} \\ &+ \frac{9x\tan^2\left(\frac{x}{2}\right)}{6a\tan^6\left(\frac{x}{2}\right)+18a\tan^4\left(\frac{x}{2}\right)+18a\tan^2\left(\frac{x}{2}\right)+6a} \\ &+ \frac{3x}{6a\tan^6\left(\frac{x}{2}\right)+18a\tan^4\left(\frac{x}{2}\right)+18a\tan^2\left(\frac{x}{2}\right)+6a} \\ &+ \frac{6\tan^5\left(\frac{x}{2}\right)}{6a\tan^6\left(\frac{x}{2}\right)+18a\tan^4\left(\frac{x}{2}\right)+18a\tan^2\left(\frac{x}{2}\right)+6a} \\ &- \frac{16\tan^3\left(\frac{x}{2}\right)}{6a\tan^6\left(\frac{x}{2}\right)+18a\tan^4\left(\frac{x}{2}\right)+18a\tan^2\left(\frac{x}{2}\right)+6a} \\ &- \frac{6\tan\left(\frac{x}{2}\right)}{6a\tan^6\left(\frac{x}{2}\right)+18a\tan^4\left(\frac{x}{2}\right)+18a\tan^2\left(\frac{x}{2}\right)+6a}\end{aligned}$$

input
```
integrate(sin(x)**4/(a+a*cos(x)),x)
```

output
```
3*x*tan(x/2)**6/(6*a*tan(x/2)**6 + 18*a*tan(x/2)**4 + 18*a*tan(x/2)**2 + 6
*a) + 9*x*tan(x/2)**4/(6*a*tan(x/2)**6 + 18*a*tan(x/2)**4 + 18*a*tan(x/2)*
*2 + 6*a) + 9*x*tan(x/2)**2/(6*a*tan(x/2)**6 + 18*a*tan(x/2)**4 + 18*a*tan
(x/2)**2 + 6*a) + 3*x/(6*a*tan(x/2)**6 + 18*a*tan(x/2)**4 + 18*a*tan(x/2)*
*2 + 6*a) + 6*tan(x/2)**5/(6*a*tan(x/2)**6 + 18*a*tan(x/2)**4 + 18*a*tan(x
/2)**2 + 6*a) - 16*tan(x/2)**3/(6*a*tan(x/2)**6 + 18*a*tan(x/2)**4 + 18*a*
tan(x/2)**2 + 6*a) - 6*tan(x/2)/(6*a*tan(x/2)**6 + 18*a*tan(x/2)**4 + 18*a
*tan(x/2)**2 + 6*a)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(25) = 50$.

Time = 0.12 (sec) , antiderivative size = 94, normalized size of antiderivative = 3.03

$$\int \frac{\sin^4(x)}{a + a\cos(x)}\,dx = -\frac{\frac{3\sin(x)}{\cos(x)+1} + \frac{8\sin(x)^3}{(\cos(x)+1)^3} - \frac{3\sin(x)^5}{(\cos(x)+1)^5}}{3\left(a + \frac{3a\sin(x)^2}{(\cos(x)+1)^2} + \frac{3a\sin(x)^4}{(\cos(x)+1)^4} + \frac{a\sin(x)^6}{(\cos(x)+1)^6}\right)} + \frac{\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a}$$

input
```
integrate(sin(x)^4/(a+a*cos(x)),x, algorithm="maxima")
```

output
```
-1/3*(3*sin(x)/(cos(x) + 1) + 8*sin(x)^3/(cos(x) + 1)^3 - 3*sin(x)^5/(cos(
x) + 1)^5)/(a + 3*a*sin(x)^2/(cos(x) + 1)^2 + 3*a*sin(x)^4/(cos(x) + 1)^4
+ a*sin(x)^6/(cos(x) + 1)^6) + arctan(sin(x)/(cos(x) + 1))/a
```

## Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.45

$$\int \frac{\sin^4(x)}{a + a\cos(x)}\,dx = \frac{x}{2a} + \frac{3\tan\left(\frac{1}{2}x\right)^5 - 8\tan\left(\frac{1}{2}x\right)^3 - 3\tan\left(\frac{1}{2}x\right)}{3\left(\tan\left(\frac{1}{2}x\right)^2 + 1\right)^3 a}$$

input
```
integrate(sin(x)^4/(a+a*cos(x)),x, algorithm="giac")
```

output
```
1/2*x/a + 1/3*(3*tan(1/2*x)^5 - 8*tan(1/2*x)^3 - 3*tan(1/2*x))/((tan(1/2*x
)^2 + 1)^3*a)
```

## Mupad [B] (verification not implemented)

Time = 42.17 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

$$\int \frac{\sin^4(x)}{a+a\cos(x)}\,dx=\frac{x}{2\,a}-\frac{\sin\left(x\right)}{3\,a}+\frac{{\cos\left(x\right)}^2\,\sin\left(x\right)}{3\,a}-\frac{\cos\left(x\right)\,\sin\left(x\right)}{2\,a}$$

input
```
int(sin(x)^4/(a + a*cos(x)),x)
```

output
```
x/(2*a) - sin(x)/(3*a) + (cos(x)^2*sin(x))/(3*a) - (cos(x)*sin(x))/(2*a)
```

## Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.68

$$\int \frac{\sin^4(x)}{a+a\cos(x)}\,dx=\frac{-3\cos\left(x\right)\sin\left(x\right)-2\sin\left(x\right)^{3}+3x}{6a}$$

input
```
int(sin(x)^4/(a+a*cos(x)),x)
```

output
```
( - 3*cos(x)*sin(x) - 2*sin(x)**3 + 3*x)/(6*a)
```

# 3.2 $\int \frac{\sin^3(x)}{a+a\cos(x)}\,dx$



## Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{\sin^3(x)}{a+a\cos(x)}\,dx = \frac{(a-a\cos(x))^2}{2a^3}$$

output
```
1/2*(a-a*cos(x))^2/a^3
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{\sin^3(x)}{a+a\cos(x)}\,dx = \frac{2\sin^4\left(\frac{x}{2}\right)}{a}$$

input
```
Integrate[Sin[x]^3/(a + a*Cos[x]),x]
```

output
```
(2*Sin[x/2]^4)/a
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 3146, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(x)}{a\cos(x)+a}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\cos\left(x-\frac{\pi}{2}\right)^3}{a-a\sin\left(x-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \ 3146$$

$$-\frac{\int (a-a\cos(x))d(a\cos(x))}{a^3}$$

$$\Big\downarrow \ 17$$

$$\frac{(a-a\cos(x))^2}{2a^3}$$

input
```
Int[Sin[x]^3/(a + a*Cos[x]),x]
```

output
```
(a - a*Cos[x])^2/(2*a^3)
```

### Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

## Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\frac{\cos(x)^2}{2}-\cos(x)}{a}$ | 16 |
| default | $\frac{\frac{\cos(x)^2}{2}-\cos(x)}{a}$ | 16 |
| parallelrisch | $\frac{\cos(2x)-5-4\cos(x)}{4a}$ | 16 |
| risch | $-\frac{\cos(x)}{a}+\frac{\cos(2x)}{4a}$ | 18 |
| norman | $\frac{-\frac{2}{a}-\frac{4\tan\left(\frac{x}{2}\right)^4}{a}-\frac{6\tan\left(\frac{x}{2}\right)^2}{a}}{\left(1+\tan\left(\frac{x}{2}\right)^2\right)^3}$ | 40 |

input
```
int(sin(x)^3/(a+a*cos(x)),x,method=_RETURNVERBOSE)
```

output
```
1/a*(1/2*cos(x)^2-cos(x))
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\sin^3(x)}{a+a\cos(x)}\,dx=\frac{\cos(x)^2-2\cos(x)}{2a}$$

input
```
integrate(sin(x)^3/(a+a*cos(x)),x, algorithm="fricas")
```

output
```
1/2*(cos(x)^2 - 2*cos(x))/a
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(12) = 24$.

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 3.19

$$\int \frac{\sin^3(x)}{a + a\cos(x)}\,dx = -\frac{4\tan^2\left(\frac{x}{2}\right)}{a\tan^4\left(\frac{x}{2}\right) + 2a\tan^2\left(\frac{x}{2}\right) + a} - \frac{2}{a\tan^4\left(\frac{x}{2}\right) + 2a\tan^2\left(\frac{x}{2}\right) + a}$$

input
```
integrate(sin(x)**3/(a+a*cos(x)),x)
```

output
```
-4*tan(x/2)**2/(a*tan(x/2)**4 + 2*a*tan(x/2)**2 + a) - 2/(a*tan(x/2)**4 +
2*a*tan(x/2)**2 + a)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\sin^3(x)}{a + a\cos(x)}\,dx = \frac{\cos(x)^2 - 2\cos(x)}{2a}$$

input
```
integrate(sin(x)^3/(a+a*cos(x)),x, algorithm="maxima")
```

output
```
1/2*(cos(x)^2 - 2*cos(x))/a
```

## Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\sin^3(x)}{a + a\cos(x)}\,dx = \frac{\cos(x)^2 - 2\cos(x)}{2a}$$

input
```
integrate(sin(x)^3/(a+a*cos(x)),x, algorithm="giac")
```

output
```
1/2*(cos(x)^2 - 2*cos(x))/a
```

## Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.69

$$\int \frac{\sin^3(x)}{a+a\cos(x)}\,dx=\frac{\cos(x)\,\left(\cos(x)-2\right)}{2\,a}$$

input
```
int(sin(x)^3/(a + a*cos(x)),x)
```

output
```
(cos(x)*(cos(x) - 2))/(2*a)
```

## Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{\sin^3(x)}{a+a\cos(x)}\,dx=\frac{-2\cos(x)-\sin(x)^2+2}{2a}$$

input
```
int(sin(x)^3/(a+a*cos(x)),x)
```

output
```
( - 2*cos(x) - sin(x)**2 + 2)/(2*a)
```

# 3.3 $\int \frac{\sin^2(x)}{a+a\cos(x)}\,dx$



## Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{\sin^2(x)}{a+a\cos(x)}\,dx = \frac{x}{a} - \frac{\sin(x)}{a}$$

output
```
x/a-sin(x)/a
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{\sin^2(x)}{a+a\cos(x)}\,dx = \frac{2\left(\frac{x}{2} - \frac{\sin(x)}{2}\right)}{a}$$

input
```
Integrate[Sin[x]^2/(a + a*Cos[x]),x]
```

output
```
(2*(x/2 - Sin[x]/2))/a
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3042, 3161, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(x)}{a\cos(x)+a}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\cos\left(x-\frac{\pi}{2}\right)^2}{a-a\sin\left(x-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \ 3161$$

$$\frac{\int 1dx}{a}-\frac{\sin(x)}{a}$$

$$\Big\downarrow \ 24$$

$$\frac{x}{a}-\frac{\sin(x)}{a}$$

input
```
Int[Sin[x]^2/(a + a*Cos[x]),x]
```

output
```
x/a - Sin[x]/a
```

### Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3161
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Si
mp[g^2/a   Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x
] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

## Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

| method | result | size |
|---|---|---|
| parallelrisch | $\frac{x-\sin(x)}{a}$ | 11 |
| risch | $\frac{x}{a}-\frac{\sin(x)}{a}$ | 14 |
| default | $\frac{-\frac{2\tan\left(\frac{x}{2}\right)}{1+\tan\left(\frac{x}{2}\right)^{2}}+2\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{a}$ | 30 |
| norman | $\frac{\frac{x}{a}+\frac{x\tan\left(\frac{x}{2}\right)^{4}}{a}-\frac{2\tan\left(\frac{x}{2}\right)}{a}-\frac{2\tan\left(\frac{x}{2}\right)^{3}}{a}+\frac{2x\tan\left(\frac{x}{2}\right)^{2}}{a}}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)^{2}}$ | 61 |

input
```
int(sin(x)^2/(a+a*cos(x)),x,method=_RETURNVERBOSE)
```

output
```
(x-sin(x))/a
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{\sin^2(x)}{a+a\cos(x)}\,dx=\frac{x-\sin\left(x\right)}{a}$$

input
```
integrate(sin(x)^2/(a+a*cos(x)),x, algorithm="fricas")
```

output
```
(x - sin(x))/a
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(7) = 14.

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 3.54

$$\int \frac{\sin^2(x)}{a+a\cos(x)}\,dx = \frac{x\tan^2\left(\frac{x}{2}\right)}{a\tan^2\left(\frac{x}{2}\right)+a} + \frac{x}{a\tan^2\left(\frac{x}{2}\right)+a} - \frac{2\tan\left(\frac{x}{2}\right)}{a\tan^2\left(\frac{x}{2}\right)+a}$$

input
```
integrate(sin(x)**2/(a+a*cos(x)),x)
```

output
```
x*tan(x/2)**2/(a*tan(x/2)**2 + a) + x/(a*tan(x/2)**2 + a) - 2*tan(x/2)/(a*
tan(x/2)**2 + a)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(13) = 26.

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 3.23

$$\int \frac{\sin^2(x)}{a+a\cos(x)}\,dx = \frac{2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a} - \frac{2\sin(x)}{\left(a+\frac{a\sin(x)^2}{(\cos(x)+1)^2}\right)(\cos(x)+1)}$$

input
```
integrate(sin(x)^2/(a+a*cos(x)),x, algorithm="maxima")
```

output
```
2*arctan(sin(x)/(cos(x) + 1))/a - 2*sin(x)/((a + a*sin(x)^2/(cos(x) + 1)^2
)*(cos(x) + 1))
```

## Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.92

$$\int \frac{\sin^2(x)}{a+a\cos(x)}\,dx = \frac{x}{a} - \frac{2\tan\left(\frac{1}{2}x\right)}{\left(\tan\left(\frac{1}{2}x\right)^2+1\right)a}$$

input
```
integrate(sin(x)^2/(a+a*cos(x)),x, algorithm="giac")
```

output
```
x/a - 2*tan(1/2*x)/((tan(1/2*x)^2 + 1)*a)
```

## Mupad [B] (verification not implemented)

Time = 41.30 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{\sin^2(x)}{a+a\cos(x)}\,dx = \frac{x-\sin(x)}{a}$$

input
```
int(sin(x)^2/(a + a*cos(x)),x)
```

output
```
(x - sin(x))/a
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{\sin^2(x)}{a+a\cos(x)}\,dx = \frac{-\sin(x)+x}{a}$$

input
```
int(sin(x)^2/(a+a*cos(x)),x)
```

output
```
( - sin(x) + x)/a
```

# 3.4 $\int \frac{\sin(x)}{a+a\cos(x)}\,dx$



## Optimal result

Integrand size = 11, antiderivative size = 10

$$\int \frac{\sin(x)}{a+a\cos(x)}\,dx=-\frac{\log(1+\cos(x))}{a}$$

output
```
-ln(1+cos(x))/a
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\sin(x)}{a+a\cos(x)}\,dx=-\frac{2\log\left(\cos\left(\frac{x}{2}\right)\right)}{a}$$

input
```
Integrate[Sin[x]/(a + a*Cos[x]),x]
```

output
```
(-2*Log[Cos[x/2]])/a
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)}{a\cos(x)+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos\left(x-\frac{\pi}{2}\right)}{a-a\sin\left(x-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 3146}$$

$$-\frac{\int \frac{1}{\cos(x)a+a}d(a\cos(x))}{a}$$

$$\Big\downarrow \text{ 16}$$

$$-\frac{\log(a\cos(x)+a)}{a}$$

input
```
Int[Sin[x]/(a + a*Cos[x]),x]
```

output
```
-(Log[a + a*Cos[x]]/a)
```

### Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

## Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

| method | result | size |
|---|---|---|
| parallelrisch | $\frac{\ln\left(\sec\left(\frac{x}{2}\right)^{2}\right)}{a}$ | 12 |
| derivativedivides | $-\frac{\ln(a+a\cos(x))}{a}$ | 13 |
| default | $-\frac{\ln(a+a\cos(x))}{a}$ | 13 |
| norman | $\frac{\ln\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)}{a}$ | 14 |
| risch | $\frac{ix}{a}-\frac{2\ln\left(e^{ix}+1\right)}{a}$ | 22 |

input
```
int(sin(x)/(a+a*cos(x)),x,method=_RETURNVERBOSE)
```

output
```
ln(sec(1/2*x)^2)/a
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\sin(x)}{a+a\cos(x)}\,dx=-\frac{\log\left(\frac{1}{2}\,\cos\left(x\right)+\frac{1}{2}\right)}{a}$$

input
```
integrate(sin(x)/(a+a*cos(x)),x, algorithm="fricas")
```

output
```
-log(1/2*cos(x) + 1/2)/a
```

## Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\sin(x)}{a + a\cos(x)}\, dx = -\frac{\log\left(\cos\left(x\right) + 1\right)}{a}$$

input
```
integrate(sin(x)/(a+a*cos(x)),x)
```

output
```
-log(cos(x) + 1)/a
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\sin(x)}{a + a\cos(x)}\, dx = -\frac{\log\left(a\cos\left(x\right) + a\right)}{a}$$

input
```
integrate(sin(x)/(a+a*cos(x)),x, algorithm="maxima")
```

output
```
-log(a*cos(x) + a)/a
```

## Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{a + a\cos(x)}\, dx = -\frac{\log\left(\cos\left(x\right) + 1\right)}{a}$$

input
```
integrate(sin(x)/(a+a*cos(x)),x, algorithm="giac")
```

output
```
-log(cos(x) + 1)/a
```

## Mupad [B] (verification not implemented)

Time = 41.78 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{a + a\cos(x)}\, dx = -\frac{\ln\left(\cos\left(x\right) + 1\right)}{a}$$

input
```
int(sin(x)/(a + a*cos(x)),x)
```

output
```
-log(cos(x) + 1)/a
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{a + a\cos(x)}\, dx = -\frac{\log(\cos\left(x\right) + 1)}{a}$$

input
```
int(sin(x)/(a+a*cos(x)),x)
```

output
```
( - log(cos(x) + 1))/a
```

## 3.5 $\int \frac{1}{a+a\cos(x)}\,dx$



### Optimal result

Integrand size = 8, antiderivative size = 11

$$\int \frac{1}{a+a\cos(x)}\,dx = \frac{\sin(x)}{a+a\cos(x)}$$

output
```
sin(x)/(a+a*cos(x))
```

### Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{1}{a+a\cos(x)}\,dx = \frac{\tan\left(\frac{x}{2}\right)}{a}$$

input
```
Integrate[(a + a*Cos[x])^(-1),x]
```

output
```
Tan[x/2]/a
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a\cos(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a\sin\left(x+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{3127}$$

$$\frac{\sin(x)}{a\cos(x)+a}$$

input
```
Int[(a + a*Cos[x])^(-1),x]
```

output
```
Sin[x]/(a + a*Cos[x])
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

## Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

| method | result | size |
|---|---|---|
| default | $\frac{\tan\left(\frac{x}{2}\right)}{a}$ | 9 |
| norman | $\frac{\tan\left(\frac{x}{2}\right)}{a}$ | 9 |
| parallelrisch | $\frac{\tan\left(\frac{x}{2}\right)}{a}$ | 9 |
| risch | $\frac{2i}{\left(\mathrm{e}^{ix}+1\right)a}$ | 16 |

input
```
int(1/(a+a*cos(x)),x,method=_RETURNVERBOSE)
```

output
```
1/a*tan(1/2*x)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+a\cos(x)}\,dx = \frac{\sin\left(x\right)}{a\cos\left(x\right)+a}$$

input
```
integrate(1/(a+a*cos(x)),x, algorithm="fricas")
```

output
```
sin(x)/(a*cos(x) + a)
```

## Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.45

$$\int \frac{1}{a+a\cos(x)}\,dx = \frac{\tan\left(\frac{x}{2}\right)}{a}$$

input
```
integrate(1/(a+a*cos(x)),x)
```

output
```
tan(x/2)/a
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{1}{a+a\cos(x)}\,dx=\frac{\sin(x)}{a(\cos(x)+1)}$$

input
```
integrate(1/(a+a*cos(x)),x, algorithm="maxima")
```

output
```
sin(x)/(a*(cos(x) + 1))
```

## Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{1}{a+a\cos(x)}\,dx=\frac{\tan\left(\frac{1}{2}\,x\right)}{a}$$

input
```
integrate(1/(a+a*cos(x)),x, algorithm="giac")
```

output
```
tan(1/2*x)/a
```

## Mupad [B] (verification not implemented)

Time = 43.18 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{1}{a+a\cos(x)}\,dx=\frac{\tan\left(\frac{x}{2}\right)}{a}$$

input
```
int(1/(a + a*cos(x)),x)
```

output
```
tan(x/2)/a
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{1}{a+a\cos(x)}\,dx=\frac{\tan\left(\frac{x}{2}\right)}{a}$$

input
```
int(1/(a+a*cos(x)),x)
```

output
```
tan(x/2)/a
```

# 3.6 $\int \frac{\csc(x)}{a+a\cos(x)}\,dx$



## Optimal result

Integrand size = 11, antiderivative size = 23

$$\int \frac{\csc(x)}{a+a\cos(x)}\,dx = -\frac{\operatorname{arctanh}(\cos(x))}{2a} + \frac{1}{2(a+a\cos(x))}$$

output
```
-1/2*arctanh(cos(x))/a+1/(2*a+2*a*cos(x))
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \frac{\csc(x)}{a+a\cos(x)}\,dx = \frac{1-2\cos^2\left(\frac{x}{2}\right)\left(\log\left(\cos\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)\right)\right)}{2a(1+\cos(x))}$$

input
```
Integrate[Csc[x]/(a + a*Cos[x]),x]
```

output
```
(1 - 2*Cos[x/2]^2*(Log[Cos[x/2]] - Log[Sin[x/2]]))/(2*a*(1 + Cos[x]))
```

## Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 3146, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(x)}{a\cos(x)+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\cos\left(x-\frac{\pi}{2}\right)\left(a-a\sin\left(x-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{ 3146}$$

$$-a\int \frac{1}{(a-a\cos(x))(\cos(x)a+a)^2}d(a\cos(x))$$

$$\Big\downarrow \text{ 54}$$

$$-a\int \left(\frac{1}{2\left(a^2-a^2\cos^2(x)\right)a}+\frac{1}{2(\cos(x)a+a)^2a}\right)d(a\cos(x))$$

$$\Big\downarrow \text{ 2009}$$

$$-a\left(\frac{\text{arctanh}(\cos(x))}{2a^2}-\frac{1}{2a(a\cos(x)+a)}\right)$$

input
```
Int[Csc[x]/(a + a*Cos[x]),x]
```

output
```
-(a*(ArcTanh[Cos[x]]/(2*a^2) - 1/(2*a*(a + a*Cos[x]))))
```

### Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

## Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

| method | result | size |
|---|---|---|
| parallelrisch | $\frac{\tan\left(\frac{x}{2}\right)^{2}+2\ln\left(\tan\left(\frac{x}{2}\right)\right)}{4a}$ | 20 |
| norman | $\frac{\tan\left(\frac{x}{2}\right)^{2}}{4a}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2a}$ | 23 |
| default | $\frac{\frac{1}{2\cos(x)+2}-\frac{\ln(\cos(x)+1)}{4}+\frac{\ln(-1+\cos(x))}{4}}{a}$ | 28 |
| risch | $\frac{e^{ix}}{\left(e^{ix}+1\right)^{2}a}+\frac{\ln\left(e^{ix}-1\right)}{2a}-\frac{\ln\left(e^{ix}+1\right)}{2a}$ | 46 |

input
```
int(csc(x)/(a+a*cos(x)),x,method=_RETURNVERBOSE)
```

output
```
1/4*(tan(1/2*x)^2+2*ln(tan(1/2*x)))/a
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.61

$$\int \frac{\csc(x)}{a + a\cos(x)} \, dx = -\frac{(\cos(x) + 1)\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right) - (\cos(x) + 1)\log\left(-\frac{1}{2}\cos(x) + \frac{1}{2}\right) - 2}{4\,(a\cos(x) + a)}$$

input
```
integrate(csc(x)/(a+a*cos(x)),x, algorithm="fricas")
```

output
```
-1/4*((cos(x) + 1)*log(1/2*cos(x) + 1/2) - (cos(x) + 1)*log(-1/2*cos(x) +
1/2) - 2)/(a*cos(x) + a)
```

## Sympy [F]

$$\int \frac{\csc(x)}{a + a\cos(x)} \, dx = \frac{\int \frac{\csc(x)}{\cos(x)+1} \, dx}{a}$$

input
```
integrate(csc(x)/(a+a*cos(x)),x)
```

output
```
Integral(csc(x)/(cos(x) + 1), x)/a
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int \frac{\csc(x)}{a + a\cos(x)} \, dx = -\frac{\log(\cos(x) + 1)}{4\,a} + \frac{\log(\cos(x) - 1)}{4\,a} + \frac{1}{2\,(a\cos(x) + a)}$$

input
```
integrate(csc(x)/(a+a*cos(x)),x, algorithm="maxima")
```

output
```
-1/4*log(cos(x) + 1)/a + 1/4*log(cos(x) - 1)/a + 1/2/(a*cos(x) + a)
```

## Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.48

$$\int \frac{\csc(x)}{a + a\cos(x)}\,dx = -\frac{\log(\cos(x) + 1)}{4\,a} + \frac{\log(-\cos(x) + 1)}{4\,a} + \frac{1}{2\,a(\cos(x) + 1)}$$

input
```
integrate(csc(x)/(a+a*cos(x)),x, algorithm="giac")
```

output
```
-1/4*log(cos(x) + 1)/a + 1/4*log(-cos(x) + 1)/a + 1/2/(a*(cos(x) + 1))
```

## Mupad [B] (verification not implemented)

Time = 43.86 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{\csc(x)}{a + a\cos(x)}\,dx = \frac{1}{2\,a\,(\cos(x) + 1)} - \frac{\operatorname{atanh}(\cos(x))}{2\,a}$$

input
```
int(1/(sin(x)*(a + a*cos(x))),x)
```

output
```
1/(2*a*(cos(x) + 1)) - atanh(cos(x))/(2*a)
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\csc(x)}{a + a\cos(x)}\,dx = \frac{2\log\left(\tan\left(\frac{x}{2}\right)\right) + \tan\left(\frac{x}{2}\right)^2}{4a}$$

input
```
int(csc(x)/(a+a*cos(x)),x)
```

output
```
(2*log(tan(x/2)) + tan(x/2)**2)/(4*a)
```

# 3.7 $\int \frac{\csc^2(x)}{a+a\cos(x)}\,dx$



## Optimal result

Integrand size = 13, antiderivative size = 24

$$\int \frac{\csc^2(x)}{a+a\cos(x)}\,dx = -\frac{2\cot(x)}{3a}+\frac{\csc(x)}{3(a+a\cos(x))}$$

output
```
-2/3*cot(x)/a+csc(x)/(3*a+3*a*cos(x))
```

## Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25

$$\int \frac{\csc^2(x)}{a+a\cos(x)}\,dx = -\frac{(2\cos(x)+\cos(2x))\csc\left(\frac{x}{2}\right)\sec^3\left(\frac{x}{2}\right)}{12a}$$

input
```
Integrate[Csc[x]^2/(a + a*Cos[x]),x]
```

output
```
-1/12*((2*Cos[x] + Cos[2*x])*Csc[x/2]*Sec[x/2]^3)/a
```

## Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3042, 3151, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(x)}{a\cos(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos\left(x-\frac{\pi}{2}\right)^2\left(a-a\sin\left(x-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3151}$$

$$\frac{2\int \csc^2(x)dx}{3a}+\frac{\csc(x)}{3(a\cos(x)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \csc(x)^2dx}{3a}+\frac{\csc(x)}{3(a\cos(x)+a)}$$

$$\Big\downarrow \text{4254}$$

$$\frac{\csc(x)}{3(a\cos(x)+a)}-\frac{2\int 1d\cot(x)}{3a}$$

$$\Big\downarrow \text{24}$$

$$\frac{\csc(x)}{3(a\cos(x)+a)}-\frac{2\cot(x)}{3a}$$

input
```
Int[Csc[x]^2/(a + a*Cos[x]),x]
```

output
```
(-2*Cot[x])/(3*a) + Csc[x]/(3*(a + a*Cos[x]))
```

### Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3151
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Simp[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1])   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simpli
fy[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

## Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

| method | result | size |
|---|---|---|
| parallelrisch | $\frac{\tan\left(\frac{x}{2}\right)^{3}-3\cot\left(\frac{x}{2}\right)+6\tan\left(\frac{x}{2}\right)}{12a}$ | 25 |
| default | $\frac{\frac{\tan\left(\frac{x}{2}\right)^{3}}{3}+2\tan\left(\frac{x}{2}\right)-\frac{1}{\tan\left(\frac{x}{2}\right)}}{4a}$ | 29 |
| risch | $-\frac{4i\left(1+2\,{\mathrm e}^{i x}\right)}{3\left({\mathrm e}^{i x}+1\right)^{3} a \left({\mathrm e}^{i x}-1\right)}$ | 34 |
| norman | $\frac{-\frac{1}{4a}+\frac{\tan\left(\frac{x}{2}\right)^{2}}{2a}+\frac{\tan\left(\frac{x}{2}\right)^{4}}{12a}}{\tan\left(\frac{x}{2}\right)}$ | 36 |

input
```
int(csc(x)^2/(a+a*cos(x)),x,method=_RETURNVERBOSE)
```

output
```
1/12*(tan(1/2*x)^3-3*cot(1/2*x)+6*tan(1/2*x))/a
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\csc^2(x)}{a+a\cos(x)}\,dx=-\frac{2\,\cos\left(x\right)^{2}+2\,\cos\left(x\right)-1}{3\,\left(a\cos\left(x\right)+a\right)\sin\left(x\right)}$$

input
```
integrate(csc(x)^2/(a+a*cos(x)),x, algorithm="fricas")
```

output
```
-1/3*(2*cos(x)^2 + 2*cos(x) - 1)/((a*cos(x) + a)*sin(x))
```

## Sympy [F]

$$\int \frac{\csc^2(x)}{a+a\cos(x)}\,dx=\frac{\int \frac{\csc^{2}\left(x\right)}{\cos\left(x\right)+1}\,dx}{a}$$

input
```
integrate(csc(x)**2/(a+a*cos(x)),x)
```

output
```
Integral(csc(x)**2/(cos(x) + 1), x)/a
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(20) = 40.

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.71

$$\int \frac{\csc^2(x)}{a+a\cos(x)}\,dx=\frac{\frac{6\,\sin(x)}{\cos(x)+1}+\frac{\sin(x)^{3}}{(\cos(x)+1)^{3}}}{12\,a}-\frac{\cos\left(x\right)+1}{4\,a\sin\left(x\right)}$$

input
```
integrate(csc(x)^2/(a+a*cos(x)),x, algorithm="maxima")
```

output
```
1/12*(6*sin(x)/(cos(x) + 1) + sin(x)^3/(cos(x) + 1)^3)/a - 1/4*(cos(x) + 1
)/(a*sin(x))
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{\csc^2(x)}{a+a\cos(x)}\,dx = \frac{a^2\tan\left(\frac{1}{2}\,x\right)^3+6\,a^2\tan\left(\frac{1}{2}\,x\right)}{12\,a^3} - \frac{1}{4\,a\tan\left(\frac{1}{2}\,x\right)}$$

input
```
integrate(csc(x)^2/(a+a*cos(x)),x, algorithm="giac")
```

output
```
1/12*(a^2*tan(1/2*x)^3 + 6*a^2*tan(1/2*x))/a^3 - 1/4/(a*tan(1/2*x))
```

## Mupad [B] (verification not implemented)

Time = 40.36 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \frac{\csc^2(x)}{a+a\cos(x)}\,dx = \frac{-8\cos\left(\frac{x}{2}\right)^4+4\cos\left(\frac{x}{2}\right)^2+1}{12\,a\cos\left(\frac{x}{2}\right)^3\sin\left(\frac{x}{2}\right)}$$

input
```
int(1/(sin(x)^2*(a + a*cos(x))),x)
```

output
```
(4*cos(x/2)^2 - 8*cos(x/2)^4 + 1)/(12*a*cos(x/2)^3*sin(x/2))
```

## Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.12

$$\int \frac{\csc^2(x)}{a + a\cos(x)}\,dx = \frac{\tan\left(\frac{x}{2}\right)^4 + 6\tan\left(\frac{x}{2}\right)^2 - 3}{12\tan\left(\frac{x}{2}\right)a}$$

input
```
int(csc(x)^2/(a+a*cos(x)),x)
```

output
```
(tan(x/2)**4 + 6*tan(x/2)**2 - 3)/(12*tan(x/2)*a)
```

# 3.8 $\int \frac{\csc^3(x)}{a+a\cos(x)}\,dx$



## Optimal result

Integrand size = 13, antiderivative size = 55

$$\int \frac{\csc^3(x)}{a+a\cos(x)}\,dx = -\frac{3\text{arctanh}(\cos(x))}{8a}-\frac{1}{8(a-a\cos(x))}+\frac{1}{4(a+a\cos(x))}+\frac{a^3}{8\left(a^2+a^2\cos(x)\right)^2}$$

output
```
-3/8*arctanh(cos(x))/a-1/(8*a-8*a*cos(x))+1/(4*a+4*a*cos(x))+1/8*a^3/(a^2+
a^2*cos(x))^2
```

## Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09

$$\int \frac{\csc^3(x)}{a+a\cos(x)}\,dx = \frac{4-2\cot^2\left(\frac{x}{2}\right)-12\cos^2\left(\frac{x}{2}\right)\left(\log\left(\cos\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)\right)\right)+\sec^2\left(\frac{x}{2}\right)}{16a(1+\cos(x))}$$

input
```
Integrate[Csc[x]^3/(a + a*Cos[x]),x]
```

output
```
(4 - 2*Cot[x/2]^2 - 12*Cos[x/2]^2*(Log[Cos[x/2]] - Log[Sin[x/2]]) + Sec[x/
2]^2)/(16*a*(1 + Cos[x]))
```

## Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 3146, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(x)}{a\cos(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos\left(x-\frac{\pi}{2}\right)^3\left(a-a\sin\left(x-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3146}$$

$$-a^3\int \frac{1}{(a-a\cos(x))^2(\cos(x)a+a)^3}d(a\cos(x))$$

$$\Big\downarrow \text{54}$$

$$-a^3\int \left(\frac{1}{8a^3(a-a\cos(x))^2}+\frac{1}{4a^3(\cos(x)a+a)^2}+\frac{1}{4a^2(\cos(x)a+a)^3}+\frac{3}{8a^3\left(a^2-a^2\cos^2(x)\right)}\right)d(a\cos(x))$$

$$\Big\downarrow \text{2009}$$

$$-a^3\left(\frac{3\text{arctanh}(\cos(x))}{8a^4}+\frac{1}{8a^3(a-a\cos(x))}-\frac{1}{4a^3(a\cos(x)+a)}-\frac{1}{8a^2(a\cos(x)+a)^2}\right)$$

input
```
Int[Csc[x]^3/(a + a*Cos[x]),x]
```

output
```
-(a^3*((3*ArcTanh[Cos[x]])/(8*a^4) + 1/(8*a^3*(a - a*Cos[x])) - 1/(8*a^2*(
a + a*Cos[x])^2) - 1/(4*a^3*(a + a*Cos[x]))))
```

### Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

## Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.65

| method | result | size |
|---|---|---|
| parallelrisch | $\frac{\tan\left(\frac{x}{2}\right)^{4}-2\cot\left(\frac{x}{2}\right)^{2}+6\tan\left(\frac{x}{2}\right)^{2}+12\ln\left(\tan\left(\frac{x}{2}\right)\right)}{32a}$ | 36 |
| default | $\frac{\frac{1}{8\left(\cos\left(x\right)+1\right)^{2}}+\frac{1}{4\cos\left(x\right)+4}-\frac{3\ln\left(\cos\left(x\right)+1\right)}{16}+\frac{1}{-8+8\cos\left(x\right)}+\frac{3\ln\left(-1+\cos\left(x\right)\right)}{16}}{a}$ | 44 |
| norman | $\frac{-\frac{1}{16a}+\frac{3\tan\left(\frac{x}{2}\right)^{4}}{16a}+\frac{\tan\left(\frac{x}{2}\right)^{6}}{32a}}{\tan\left(\frac{x}{2}\right)^{2}}+\frac{3\ln\left(\tan\left(\frac{x}{2}\right)\right)}{8a}$ | 47 |
| risch | $\frac{3\,{\mathrm e}^{5ix}+6\,{\mathrm e}^{4ix}-2\,{\mathrm e}^{3ix}+6\,{\mathrm e}^{2ix}+3\,{\mathrm e}^{ix}}{4\left({\mathrm e}^{ix}+1\right)^{4}a\left({\mathrm e}^{ix}-1\right)^{2}}+\frac{3\ln\left({\mathrm e}^{ix}-1\right)}{8a}-\frac{3\ln\left({\mathrm e}^{ix}+1\right)}{8a}$ | 87 |

input
```
int(csc(x)^3/(a+a*cos(x)),x,method=_RETURNVERBOSE)
```

output
```
1/32*(tan(1/2*x)^4-2*cot(1/2*x)^2+6*tan(1/2*x)^2+12*ln(tan(1/2*x)))/a
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.51

$$\int \frac{\csc^3(x)}{a + a\cos(x)}\,dx = \frac{6\cos(x)^2 - 3\left(\cos(x)^3 + \cos(x)^2 - \cos(x) - 1\right)\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right) + 3\left(\cos(x)^3 + \cos(x)^2 - \cos(x) - \right.}{16\left(a\cos(x)^3 + a\cos(x)^2 - a\cos(x) - a\right)}$$

input
```
integrate(csc(x)^3/(a+a*cos(x)),x, algorithm="fricas")
```

output
```
1/16*(6*cos(x)^2 - 3*(cos(x)^3 + cos(x)^2 - cos(x) - 1)*log(1/2*cos(x) + 1
/2) + 3*(cos(x)^3 + cos(x)^2 - cos(x) - 1)*log(-1/2*cos(x) + 1/2) + 6*cos(
x) - 4)/(a*cos(x)^3 + a*cos(x)^2 - a*cos(x) - a)
```

## Sympy [F]

$$\int \frac{\csc^3(x)}{a + a\cos(x)}\,dx = \frac{\int \frac{\csc^3(x)}{\cos(x)+1}\,dx}{a}$$

input
```
integrate(csc(x)**3/(a+a*cos(x)),x)
```

output
```
Integral(csc(x)**3/(cos(x) + 1), x)/a
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.05

$$\int \frac{\csc^3(x)}{a + a\cos(x)}\,dx = \frac{3\cos(x)^2 + 3\cos(x) - 2}{8\left(a\cos(x)^3 + a\cos(x)^2 - a\cos(x) - a\right)} - \frac{3\log(\cos(x) + 1)}{16a} + \frac{3\log(\cos(x) - 1)}{16a}$$

input
```
integrate(csc(x)^3/(a+a*cos(x)),x, algorithm="maxima")
```

output
```
1/8*(3*cos(x)^2 + 3*cos(x) - 2)/(a*cos(x)^3 + a*cos(x)^2 - a*cos(x) - a) -
 3/16*log(cos(x) + 1)/a + 3/16*log(cos(x) - 1)/a
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

$$\int \frac{\csc^3(x)}{a+a\cos(x)}\,dx = -\frac{3\,\log\left(\cos\left(x\right)+1\right)}{16\,a}+\frac{3\,\log\left(-\cos\left(x\right)+1\right)}{16\,a} + \frac{3\,\cos\left(x\right)^2+3\,\cos\left(x\right)-2}{8\,a{\left(\cos\left(x\right)+1\right)}^2\left(\cos\left(x\right)-1\right)}$$

input
```
integrate(csc(x)^3/(a+a*cos(x)),x, algorithm="giac")
```

output
```
-3/16*log(cos(x) + 1)/a + 3/16*log(-cos(x) + 1)/a + 1/8*(3*cos(x)^2 + 3*co
s(x) - 2)/(a*(cos(x) + 1)^2*(cos(x) - 1))
```

## Mupad [B] (verification not implemented)

Time = 41.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.82

$$\int \frac{\csc^3(x)}{a+a\cos(x)}\,dx = -\frac{\frac{3\cos(x)^2}{8}+\frac{3\,\cos(x)}{8}-\frac{1}{4}}{-a\cos\left(x\right)^3-a\cos\left(x\right)^2+a\,\cos\left(x\right)+a}-\frac{3\,\mathrm{atanh}(\cos\left(x\right))}{8\,a}$$

input
```
int(1/(sin(x)^3*(a + a*cos(x))),x)
```

output
```
- ((3*cos(x))/8 + (3*cos(x)^2)/8 - 1/4)/(a + a*cos(x) - a*cos(x)^2 - a*cos
(x)^3) - (3*atanh(cos(x)))/(8*a)
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.73

$$\int \frac{\csc^3(x)}{a+a\cos(x)}\,dx = \frac{12\log\left(\tan\left(\frac{x}{2}\right)\right)\tan\left(\frac{x}{2}\right)^2+\tan\left(\frac{x}{2}\right)^6+6\tan\left(\frac{x}{2}\right)^4-2}{32\tan\left(\frac{x}{2}\right)^2 a}$$

input
```
int(csc(x)^3/(a+a*cos(x)),x)
```

output
```
(12*log(tan(x/2))*tan(x/2)**2 + tan(x/2)**6 + 6*tan(x/2)**4 - 2)/(32*tan(x
/2)**2*a)
```

# 3.9 $\int \frac{\csc^4(x)}{a+a\cos(x)}\,dx$



## Optimal result

Integrand size = 13, antiderivative size = 37

$$\int \frac{\csc^4(x)}{a+a\cos(x)}\,dx = -\frac{4\cot(x)}{5a} - \frac{4\cot^3(x)}{15a} + \frac{\csc^3(x)}{5(a+a\cos(x))}$$

output
```
-4/5*cot(x)/a-4/15*cot(x)^3/a+csc(x)^3/(5*a+5*a*cos(x))
```

## Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

$$\int \frac{\csc^4(x)}{a+a\cos(x)}\,dx = \frac{(-6\cos(x) - 2\cos(2x) + 2\cos(3x) + \cos(4x))\csc^3(x)}{15a(1+\cos(x))}$$

input
```
Integrate[Csc[x]^4/(a + a*Cos[x]),x]
```

output
```
((-6*Cos[x] - 2*Cos[2*x] + 2*Cos[3*x] + Cos[4*x])*Csc[x]^3)/(15*a*(1 + Cos
[x]))
```

## Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3042, 3151, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^4(x)}{a\cos(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos\left(x-\frac{\pi}{2}\right)^4\left(a-a\sin\left(x-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3151}$$

$$\frac{4\int \csc^4(x)dx}{5a}+\frac{\csc^3(x)}{5(a\cos(x)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4\int \csc(x)^4dx}{5a}+\frac{\csc^3(x)}{5(a\cos(x)+a)}$$

$$\Big\downarrow \text{4254}$$

$$\frac{\csc^3(x)}{5(a\cos(x)+a)}-\frac{4\int \left(\cot^2(x)+1\right)d\cot(x)}{5a}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\csc^3(x)}{5(a\cos(x)+a)}-\frac{4\left(\frac{\cot^3(x)}{3}+\cot(x)\right)}{5a}$$

input
```
Int[Csc[x]^4/(a + a*Cos[x]),x]
```

output
```
(-4*(Cot[x] + Cot[x]^3/3))/(5*a) + Csc[x]^3/(5*(a + a*Cos[x]))
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3151
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Simp[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1])   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simpli
fy[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

## Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.16

| method | result | size |
|---|---|---|
| parallelrisch | $\frac{3 \tan \left(\frac{x}{2}\right)^{5}-5 \cot \left(\frac{x}{2}\right)^{3}+20 \tan \left(\frac{x}{2}\right)^{3}-60 \cot \left(\frac{x}{2}\right)+90 \tan \left(\frac{x}{2}\right)}{240 a}$ | 43 |
| default | $\frac{\frac{\tan \left(\frac{x}{2}\right)^{5}}{5}+\frac{4 \tan \left(\frac{x}{2}\right)^{3}}{3}+6 \tan \left(\frac{x}{2}\right)-\frac{1}{3 \tan \left(\frac{x}{2}\right)^{3}}-\frac{4}{\tan \left(\frac{x}{2}\right)}}{16 a}$ | 45 |
| risch | $\frac{16 i \left(6 \,{\mathrm e}^{3 i x}+2 \,{\mathrm e}^{2 i x}-2 \,{\mathrm e}^{i x}-1\right)}{15 \left({\mathrm e}^{i x}-1\right)^{3} a \left({\mathrm e}^{i x}+1\right)^{5}}$ | 48 |
| norman | $\frac{-\frac{1}{48 a}-\frac{\tan \left(\frac{x}{2}\right)^{2}}{4 a}+\frac{3 \tan \left(\frac{x}{2}\right)^{4}}{8 a}+\frac{\tan \left(\frac{x}{2}\right)^{6}}{12 a}+\frac{\tan \left(\frac{x}{2}\right)^{8}}{80 a}}{\tan \left(\frac{x}{2}\right)^{3}}$ | 58 |

input
```
int(csc(x)^4/(a+a*cos(x)),x,method=_RETURNVERBOSE)
```

output
```
1/240*(3*tan(1/2*x)^5-5*cot(1/2*x)^3+20*tan(1/2*x)^3-60*cot(1/2*x)+90*tan(
1/2*x))/a
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.43

$$\int \frac{\csc^4(x)}{a+a\cos(x)}\,dx = -\frac{8\,\cos\left(x\right)^4+8\,\cos\left(x\right)^3-12\,\cos\left(x\right)^2-12\,\cos\left(x\right)+3}{15\,\left(a\cos\left(x\right)^3+a\cos\left(x\right)^2-a\cos\left(x\right)-a\right)\sin\left(x\right)}$$

input
```
integrate(csc(x)^4/(a+a*cos(x)),x, algorithm="fricas")
```

output
```
-1/15*(8*cos(x)^4 + 8*cos(x)^3 - 12*cos(x)^2 - 12*cos(x) + 3)/((a*cos(x)^3
 + a*cos(x)^2 - a*cos(x) - a)*sin(x))
```

## Sympy [F]

$$\int \frac{\csc^4(x)}{a+a\cos(x)}\,dx = \frac{\int \frac{\csc^4\left(x\right)}{\cos\left(x\right)+1}\,dx}{a}$$

input
```
integrate(csc(x)**4/(a+a*cos(x)),x)
```

output
```
Integral(csc(x)**4/(cos(x) + 1), x)/a
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(31) = 62.

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.89

$$\int \frac{\csc^4(x)}{a+a\cos(x)}\,dx = \frac{\frac{90\sin(x)}{\cos(x)+1}+\frac{20\sin(x)^3}{(\cos(x)+1)^3}+\frac{3\sin(x)^5}{(\cos(x)+1)^5}}{240\,a} - \frac{\left(\frac{12\sin(x)^2}{(\cos(x)+1)^2}+1\right)(\cos(x)+1)^3}{48\,a\sin(x)^3}$$

input
```
integrate(csc(x)^4/(a+a*cos(x)),x, algorithm="maxima")
```

output
```
1/240*(90*sin(x)/(cos(x) + 1) + 20*sin(x)^3/(cos(x) + 1)^3 + 3*sin(x)^5/(c
os(x) + 1)^5)/a - 1/48*(12*sin(x)^2/(cos(x) + 1)^2 + 1)*(cos(x) + 1)^3/(a*
sin(x)^3)
```

## Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.59

$$\int \frac{\csc^4(x)}{a+a\cos(x)}\,dx = -\frac{12\tan\left(\frac{1}{2}x\right)^2+1}{48\,a\tan\left(\frac{1}{2}x\right)^3} + \frac{3\,a^4\tan\left(\frac{1}{2}x\right)^5+20\,a^4\tan\left(\frac{1}{2}x\right)^3+90\,a^4\tan\left(\frac{1}{2}x\right)}{240\,a^5}$$

input
```
integrate(csc(x)^4/(a+a*cos(x)),x, algorithm="giac")
```

output
```
-1/48*(12*tan(1/2*x)^2 + 1)/(a*tan(1/2*x)^3) + 1/240*(3*a^4*tan(1/2*x)^5 +
 20*a^4*tan(1/2*x)^3 + 90*a^4*tan(1/2*x))/a^5
```

## Mupad [B] (verification not implemented)

Time = 42.17 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.22

$$\int \frac{\csc^4(x)}{a+a\cos(x)}\,dx=\frac{3\tan\left(\frac{x}{2}\right)^8+20\tan\left(\frac{x}{2}\right)^6+90\tan\left(\frac{x}{2}\right)^4-60\tan\left(\frac{x}{2}\right)^2-5}{240\,a\tan\left(\frac{x}{2}\right)^3}$$

input
```
int(1/(sin(x)^4*(a + a*cos(x))),x)
```

output
```
(90*tan(x/2)^4 - 60*tan(x/2)^2 + 20*tan(x/2)^6 + 3*tan(x/2)^8 - 5)/(240*a*
tan(x/2)^3)
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.11

$$\int \frac{\csc^4(x)}{a+a\cos(x)}\,dx=\frac{-8\cos(x)\sin(x)^2-4\cos(x)+8\sin(x)^4-4\sin(x)^2-1}{15\sin(x)^3\,a(\cos(x)+1)}$$

input
```
int(csc(x)^4/(a+a*cos(x)),x)
```

output
```
( - 8*cos(x)*sin(x)**2 - 4*cos(x) + 8*sin(x)**4 - 4*sin(x)**2 - 1)/(15*sin
(x)**3*a*(cos(x) + 1))
```

# 3.10 $\int \frac{\sin(2x)}{1+\cos(2x)}\,dx$



## Optimal result

Integrand size = 13, antiderivative size = 5

$$\int \frac{\sin(2x)}{1+\cos(2x)}\,dx = -\log(\cos(x))$$

output
```
-ln(cos(x))
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{\sin(2x)}{1+\cos(2x)}\,dx = -\log(\cos(x))$$

input
```
Integrate[Sin[2*x]/(1 + Cos[2*x]),x]
```

output
```
-Log[Cos[x]]
```

## Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 11 vs. $2(5) = 10$.

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 2.20, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(2x)}{\cos(2x)+1}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos\left(2x-\frac{\pi}{2}\right)}{1-\sin\left(2x-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3146}$$

$$-\frac{1}{2}\int \frac{1}{\cos(2x)+1}d\cos(2x)$$

$$\Big\downarrow \text{16}$$

$$-\frac{1}{2}\log(\cos(2x)+1)$$

input
```
Int[Sin[2*x]/(1 + Cos[2*x]),x]
```

output
```
-1/2*Log[1 + Cos[2*x]]
```

### Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

## Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.60

| method | result | size |
|---|---|---|
| parallelrisch | $\ln\left(\sqrt{\sec\left(x\right)^{2}}\right)$ | 8 |
| derivativedivides | $-\frac{\ln(\cos(2x)+1)}{2}$ | 10 |
| default | $-\frac{\ln(\cos(2x)+1)}{2}$ | 10 |
| norman | $\frac{\ln\left(1+\tan(x)^{2}\right)}{2}$ | 10 |
| risch | $ix-\ln\left(e^{2ix}+1\right)$ | 16 |

input
```
int(sin(2*x)/(cos(2*x)+1),x,method=_RETURNVERBOSE)
```

output
```
ln((sec(x)^2)^(1/2))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11 vs. 2(5) = 10.

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 2.20

$$\int \frac{\sin(2x)}{1+\cos(2x)}\,dx=-\frac{1}{2}\,\log\left(\frac{1}{2}\,\cos\left(2\,x\right)+\frac{1}{2}\right)$$

input
```
integrate(sin(2*x)/(1+cos(2*x)),x, algorithm="fricas")
```

output
```
-1/2*log(1/2*cos(2*x) + 1/2)
```

## Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 2.00

$$\int \frac{\sin(2x)}{1+\cos(2x)}\,dx = -\frac{\log(\cos(2x)+1)}{2}$$

input
```
integrate(sin(2*x)/(1+cos(2*x)),x)
```

output
```
-log(cos(2*x) + 1)/2
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.80

$$\int \frac{\sin(2x)}{1+\cos(2x)}\,dx = -\frac{1}{2}\log(\cos(2x)+1)$$

input
```
integrate(sin(2*x)/(1+cos(2*x)),x, algorithm="maxima")
```

output
```
-1/2*log(cos(2*x) + 1)
```

## Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.80

$$\int \frac{\sin(2x)}{1+\cos(2x)}\,dx = -\frac{1}{2}\log(\cos(2x)+1)$$

input
```
integrate(sin(2*x)/(1+cos(2*x)),x, algorithm="giac")
```

output
```
-1/2*log(cos(2*x) + 1)
```

## Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.40

$$\int \frac{\sin(2x)}{1+\cos(2x)}\,dx = -\frac{\ln\left(\cos\left(x\right)^2\right)}{2}$$

input
```
int(sin(2*x)/(cos(2*x) + 1),x)
```

output
```
-log(cos(x)^2)/2
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.80

$$\int \frac{\sin(2x)}{1+\cos(2x)}\,dx = -\frac{\log(\cos\left(2x\right)+1)}{2}$$

input
```
int(sin(2*x)/(1+cos(2*x)),x)
```

output
```
( - log(cos(2*x) + 1))/2
```

# 3.11 $\int \frac{\sin(2x)}{1-\cos(2x)}\,dx$



## Optimal result

Integrand size = 15, antiderivative size = 3

$$\int \frac{\sin(2x)}{1-\cos(2x)}\,dx = \log(\sin(x))$$

output
```
ln(sin(x))
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{\sin(2x)}{1-\cos(2x)}\,dx = \log(\sin(x))$$

input
```
Integrate[Sin[2*x]/(1 - Cos[2*x]),x]
```

output
```
Log[Sin[x]]
```

## Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 13 vs. $2(3) = 6$.

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 4.33, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(2x)}{1-\cos(2x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cos\left(2x-\frac{\pi}{2}\right)}{\sin\left(2x-\frac{\pi}{2}\right)+1}dx$$
$$\Big\downarrow \text{3146}$$
$$\frac{1}{2}\int \frac{1}{1-\cos(2x)}d(-\cos(2x))$$
$$\Big\downarrow \text{16}$$
$$\frac{1}{2}\log(1-\cos(2x))$$

input
```
Int[Sin[2*x]/(1 - Cos[2*x]),x]
```

output
```
Log[1 - Cos[2*x]]/2
```

### Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 11 vs. $2(3) = 6$.

Time = 0.41 (sec) , antiderivative size = 12, normalized size of antiderivative = 4.00

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\ln(1-\cos(2x))}{2}$ | 12 |
| default | $\frac{\ln(1-\cos(2x))}{2}$ | 12 |
| parallelrisch | $\ln\left(\frac{1}{\sqrt{\sec(x)^2}}\right)+\ln(\tan(x))$ | 12 |
| norman | $-\frac{\ln\left(1+\tan(x)^2\right)}{2}+\ln(\tan(x))$ | 14 |
| risch | $-ix+\ln\left(e^{2ix}-1\right)$ | 14 |

input
```
int(sin(2*x)/(1-cos(2*x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(1-cos(2*x))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11 vs. $2(3) = 6$.

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 3.67

$$\int \frac{\sin(2x)}{1-\cos(2x)}\,dx = \frac{1}{2}\,\log\left(-\frac{1}{2}\,\cos\left(2\,x\right)+\frac{1}{2}\right)$$

input
```
integrate(sin(2*x)/(1-cos(2*x)),x, algorithm="fricas")
```

output
```
1/2*log(-1/2*cos(2*x) + 1/2)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8 vs. $2(3) = 6$.

Time = 0.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 2.67

$$\int \frac{\sin(2x)}{1-\cos(2x)}\,dx = \frac{\log\left(\cos\left(2x\right)-1\right)}{2}$$

input
```
integrate(sin(2*x)/(1-cos(2*x)),x)
```

output
```
log(cos(2*x) - 1)/2
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9 vs. $2(3) = 6$.

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 3.00

$$\int \frac{\sin(2x)}{1-\cos(2x)}\,dx = \frac{1}{2}\log\left(\cos\left(2\,x\right)-1\right)$$

input
```
integrate(sin(2*x)/(1-cos(2*x)),x, algorithm="maxima")
```

output
```
1/2*log(cos(2*x) - 1)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11 vs. $2(3) = 6$.

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 3.67

$$\int \frac{\sin(2x)}{1-\cos(2x)}\,dx = \frac{1}{2}\log\left(-\cos\left(2\,x\right)+1\right)$$

input
```
integrate(sin(2*x)/(1-cos(2*x)),x, algorithm="giac")
```

output
```
1/2*log(-cos(2*x) + 1)
```

## Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 9, normalized size of antiderivative = 3.00

$$\int \frac{\sin(2x)}{1-\cos(2x)}\,dx = \frac{\ln\left(-\sin\left(x\right)^2\right)}{2}$$

input
```
int(-sin(2*x)/(cos(2*x) - 1),x)
```

output
```
log(-sin(x)^2)/2
```

## Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 9, normalized size of antiderivative = 3.00

$$\int \frac{\sin(2x)}{1-\cos(2x)}\,dx = \frac{\log(\cos\left(2x\right)-1)}{2}$$

input
```
int(sin(2*x)/(1-cos(2*x)),x)
```

output
```
log(cos(2*x) - 1)/2
```

# 3.12 $\int \frac{\sin(x)}{(1+\cos(x))^2} \, dx$



## Optimal result

Integrand size = 9, antiderivative size = 6

$$\int \frac{\sin(x)}{(1+\cos(x))^2} \, dx = \frac{1}{1+\cos(x)}$$

output
```
1/(1+cos(x))
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 2.00

$$\int \frac{\sin(x)}{(1+\cos(x))^2} \, dx = \frac{1}{2}\sec^2\left(\frac{x}{2}\right)$$

input
```
Integrate[Sin[x]/(1 + Cos[x])^2,x]
```

output
```
Sec[x/2]^2/2
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3146, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)}{(\cos(x)+1)^2} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos\left(x-\frac{\pi}{2}\right)}{\left(1-\sin\left(x-\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3146}$$

$$-\int \frac{1}{(\cos(x)+1)^2}d\cos(x)$$

$$\Big\downarrow \text{17}$$

$$\frac{1}{\cos(x)+1}$$

input
```
Int[Sin[x]/(1 + Cos[x])^2,x]
```

output
```
(1 + Cos[x])^(-1)
```

### Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

## Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{1}{\cos(x)+1}$ | 7 |
| default | $\frac{1}{\cos(x)+1}$ | 7 |
| norman | $\frac{\tan\left(\frac{x}{2}\right)^{2}}{2}$ | 9 |
| parallelrisch | $\frac{\tan\left(\frac{x}{2}\right)^{2}}{2}$ | 9 |
| risch | $\frac{2\,\mathrm{e}^{ix}}{\left(\mathrm{e}^{ix}+1\right)^{2}}$ | 17 |

input
```
int(sin(x)/(cos(x)+1)^2,x,method=_RETURNVERBOSE)
```

output
```
1/(cos(x)+1)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{(1+\cos(x))^2}\,dx = \frac{1}{\cos{(x)}+1}$$

input
```
integrate(sin(x)/(1+cos(x))^2,x, algorithm="fricas")
```

output
```
1/(cos(x) + 1)
```

## Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int \frac{\sin(x)}{(1+\cos(x))^2}\,dx = \frac{1}{\cos(x)+1}$$

input
```
integrate(sin(x)/(1+cos(x))**2,x)
```

output
```
1/(cos(x) + 1)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{(1+\cos(x))^2}\,dx = \frac{1}{\cos(x)+1}$$

input
```
integrate(sin(x)/(1+cos(x))^2,x, algorithm="maxima")
```

output
```
1/(cos(x) + 1)
```

## Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{(1+\cos(x))^2}\,dx = \frac{1}{\cos(x)+1}$$

input
```
integrate(sin(x)/(1+cos(x))^2,x, algorithm="giac")
```

output
```
1/(cos(x) + 1)
```

## Mupad [B] (verification not implemented)

Time = 42.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{(1+\cos(x))^2}\,dx = \frac{1}{\cos(x)+1}$$

input
```
int(sin(x)/(cos(x) + 1)^2,x)
```

output
```
1/(cos(x) + 1)
```

## Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.67

$$\int \frac{\sin(x)}{(1+\cos(x))^2}\,dx = -\frac{\cos(x)}{\cos(x)+1}$$

input
```
int(sin(x)/(1+cos(x))^2,x)
```

output
```
( - cos(x))/(cos(x) + 1)
```

# 3.13 $\int \frac{\sin(x)}{(1-\cos(x))^2} dx$



## Optimal result

Integrand size = 11, antiderivative size = 10

$$\int \frac{\sin(x)}{(1-\cos(x))^2} dx = -\frac{1}{1-\cos(x)}$$

output
```
-1/(1-cos(x))
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\sin(x)}{(1-\cos(x))^2} dx = -\frac{1}{2}\csc^2\left(\frac{x}{2}\right)$$

input
```
Integrate[Sin[x]/(1 - Cos[x])^2,x]
```

output
```
-1/2*Csc[x/2]^2
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 3146, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)}{(1-\cos(x))^2} \, dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\cos\left(x-\frac{\pi}{2}\right)}{\left(\sin\left(x-\frac{\pi}{2}\right)+1\right)^2}dx$$

$$\Big\downarrow \ 3146$$

$$\int \frac{1}{(1-\cos(x))^2}d(-\cos(x))$$

$$\Big\downarrow \ 17$$

$$-\frac{1}{1-\cos(x)}$$

input
```
Int[Sin[x]/(1 - Cos[x])^2,x]
```

output
```
-(1 - Cos[x])^(-1)
```

### Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

## Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

| method | result | size |
|---|---|---|
| parallelrisch | $-\frac{\cot\left(\frac{x}{2}\right)^{2}}{2}$ | 9 |
| derivativedivides | $-\frac{1}{1-\cos(x)}$ | 11 |
| default | $-\frac{1}{1-\cos(x)}$ | 11 |
| risch | $\frac{2\,{\mathrm e}^{i x}}{\left({\mathrm e}^{i x}-1\right)^{2}}$ | 17 |
| norman | $\frac{-\frac{\tan\left(\frac{x}{2}\right)^{3}}{2}-\frac{\tan\left(\frac{x}{2}\right)}{2}}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)\tan\left(\frac{x}{2}\right)^{3}}$ | 33 |

input
```
int(sin(x)/(1-cos(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*cot(1/2*x)^2
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{\sin(x)}{(1-\cos(x))^2}\,dx = \frac{1}{\cos\left(x\right)-1}$$

input
```
integrate(sin(x)/(1-cos(x))^2,x, algorithm="fricas")
```

output
```
1/(cos(x) - 1)
```

## Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.50

$$\int \frac{\sin(x)}{(1-\cos(x))^2}\,dx = \frac{1}{\cos{(x)}-1}$$

input
```
integrate(sin(x)/(1-cos(x))**2,x)
```

output
```
1/(cos(x) - 1)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{\sin(x)}{(1-\cos(x))^2}\,dx = \frac{1}{\cos{(x)}-1}$$

input
```
integrate(sin(x)/(1-cos(x))^2,x, algorithm="maxima")
```

output
```
1/(cos(x) - 1)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{\sin(x)}{(1-\cos(x))^2}\,dx = \frac{1}{\cos{(x)}-1}$$

input
```
integrate(sin(x)/(1-cos(x))^2,x, algorithm="giac")
```

output
```
1/(cos(x) - 1)
```

## Mupad [B] (verification not implemented)

Time = 42.13 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{\sin(x)}{(1-\cos(x))^2}\,dx = \frac{1}{\cos(x)-1}$$

input
```
int(sin(x)/(cos(x) - 1)^2,x)
```

output
```
1/(cos(x) - 1)
```

## Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{\sin(x)}{(1-\cos(x))^2}\,dx = \frac{\cos(x)}{\cos(x)-1}$$

input
```
int(sin(x)/(1-cos(x))^2,x)
```

output
```
cos(x)/(cos(x) - 1)
```

# 3.14 $\int \frac{\sin^2(x)}{(1+\cos(x))^2} \, dx$



## Optimal result

Integrand size = 11, antiderivative size = 14

$$\int \frac{\sin^2(x)}{(1+\cos(x))^2} \, dx = -x + \frac{2\sin(x)}{1+\cos(x)}$$

output
```
-x+2*sin(x)/(1+cos(x))
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \frac{\sin^2(x)}{(1+\cos(x))^2} \, dx = -2\arctan\left(\tan\left(\frac{x}{2}\right)\right) + 2\tan\left(\frac{x}{2}\right)$$

input
```
Integrate[Sin[x]^2/(1 + Cos[x])^2,x]
```

output
```
-2*ArcTan[Tan[x/2]] + 2*Tan[x/2]
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 3159, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(x)}{(\cos(x)+1)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos\left(x-\frac{\pi}{2}\right)^2}{\left(1-\sin\left(x-\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3159}$$

$$\frac{2\sin(x)}{\cos(x)+1}-\int 1dx$$

$$\Big\downarrow \text{24}$$

$$\frac{2\sin(x)}{\cos(x)+1}-x$$

input
```
Int[Sin[x]^2/(1 + Cos[x])^2,x]
```

output
```
-x + (2*Sin[x])/(1 + Cos[x])
```

### Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

## Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

| method | result | size |
|---|---|---|
| parallelrisch | $-x+2\tan\left(\frac{x}{2}\right)$ | 11 |
| default | $2\tan\left(\frac{x}{2}\right)-2\arctan\left(\tan\left(\frac{x}{2}\right)\right)$ | 15 |
| risch | $-x+\frac{4i}{{\mathrm e}^{ix}+1}$ | 17 |
| norman | $\frac{-x+4\tan\left(\frac{x}{2}\right)^{3}+2\tan\left(\frac{x}{2}\right)^{5}-2\tan\left(\frac{x}{2}\right)^{2}x-\tan\left(\frac{x}{2}\right)^{4}x+2\tan\left(\frac{x}{2}\right)}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)^{2}}$ | 56 |

input
```
int(sin(x)^2/(cos(x)+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-x+2*tan(1/2*x)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \frac{\sin^2(x)}{(1+\cos(x))^2}\,dx=-\frac{x\cos\left(x\right)+x-2\,\sin\left(x\right)}{\cos\left(x\right)+1}$$

input
```
integrate(sin(x)^2/(1+cos(x))^2,x, algorithm="fricas")
```

output
```
-(x*cos(x) + x - 2*sin(x))/(cos(x) + 1)
```

## Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.50

$$\int \frac{\sin^2(x)}{(1+\cos(x))^2}\,dx = -x + 2\tan\left(\frac{x}{2}\right)$$

input
```
integrate(sin(x)**2/(1+cos(x))**2,x)
```

output
```
-x + 2*tan(x/2)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.64

$$\int \frac{\sin^2(x)}{(1+\cos(x))^2}\,dx = \frac{2\sin(x)}{\cos(x)+1} - 2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

input
```
integrate(sin(x)^2/(1+cos(x))^2,x, algorithm="maxima")
```

output
```
2*sin(x)/(cos(x) + 1) - 2*arctan(sin(x)/(cos(x) + 1))
```

## Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{\sin^2(x)}{(1+\cos(x))^2}\,dx = -x + 2\tan\left(\frac{1}{2}x\right)$$

input
```
integrate(sin(x)^2/(1+cos(x))^2,x, algorithm="giac")
```

output
```
-x + 2*tan(1/2*x)
```

## Mupad [B] (verification not implemented)

Time = 40.56 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{\sin^2(x)}{(1+\cos(x))^2}\,dx = 2\tan\left(\frac{x}{2}\right) - x$$

input
```
int(sin(x)^2/(cos(x) + 1)^2,x)
```

output
```
2*tan(x/2) - x
```

## Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{\sin^2(x)}{(1+\cos(x))^2}\,dx = 2\tan\left(\frac{x}{2}\right) - x$$

input
```
int(sin(x)^2/(1+cos(x))^2,x)
```

output
```
2*tan(x/2) - x
```

# 3.15 $\int \frac{\sin^2(x)}{(1-\cos(x))^2} dx$



## Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{\sin^2(x)}{(1-\cos(x))^2} dx = -x - \frac{2\sin(x)}{1-\cos(x)}$$

output
```
-x-2*sin(x)/(1-cos(x))
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int \frac{\sin^2(x)}{(1-\cos(x))^2} dx = -2\cot\left(\frac{x}{2}\right) \text{Hypergeometric2F1}\left(-\frac{1}{2}, 1, \frac{1}{2}, -\tan^2\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[Sin[x]^2/(1 - Cos[x])^2,x]
```

output
```
-2*Cot[x/2]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[x/2]^2]
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3042, 3159, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(x)}{(1-\cos(x))^2} \, dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\cos\left(x-\frac{\pi}{2}\right)^2}{\left(\sin\left(x-\frac{\pi}{2}\right)+1\right)^2}dx$$

$$\Big\downarrow \; 3159$$

$$-\int 1dx-\frac{2\sin(x)}{1-\cos(x)}$$

$$\Big\downarrow \; 24$$

$$-x-\frac{2\sin(x)}{1-\cos(x)}$$

input
```
Int[Sin[x]^2/(1 - Cos[x])^2,x]
```

output
```
-x - (2*Sin[x])/(1 - Cos[x])
```

### Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

## Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.69

| method | result | size |
|---|---|---|
| parallelrisch | $-x-2\cot\left(\frac{x}{2}\right)$ | 11 |
| default | $-\frac{2}{\tan\left(\frac{x}{2}\right)}-2\arctan\left(\tan\left(\frac{x}{2}\right)\right)$ | 17 |
| risch | $-x-\frac{4i}{e^{ix}-1}$ | 17 |
| norman | $\frac{-2\tan\left(\frac{x}{2}\right)^{2}-4\tan\left(\frac{x}{2}\right)^{4}-2\tan\left(\frac{x}{2}\right)^{6}-x\tan\left(\frac{x}{2}\right)^{3}-2x\tan\left(\frac{x}{2}\right)^{5}-x\tan\left(\frac{x}{2}\right)^{7}}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)^{2}\tan\left(\frac{x}{2}\right)^{3}}$ | 70 |

input
```
int(sin(x)^2/(1-cos(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-x-2*cot(1/2*x)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sin^2(x)}{(1-\cos(x))^2}\,dx=-\frac{x\sin(x)+2\cos(x)+2}{\sin(x)}$$

input
```
integrate(sin(x)^2/(1-cos(x))^2,x, algorithm="fricas")
```

output
```
-(x*sin(x) + 2*cos(x) + 2)/sin(x)
```

## Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.50

$$\int \frac{\sin^2(x)}{(1-\cos(x))^2}\,dx = -x - \frac{2}{\tan\left(\frac{x}{2}\right)}$$

input
```
integrate(sin(x)**2/(1-cos(x))**2,x)
```

output
```
-x - 2/tan(x/2)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.44

$$\int \frac{\sin^2(x)}{(1-\cos(x))^2}\,dx = -\frac{2\left(\cos\left(x\right)+1\right)}{\sin\left(x\right)} - 2\,\arctan\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}\right)$$

input
```
integrate(sin(x)^2/(1-cos(x))^2,x, algorithm="maxima")
```

output
```
-2*(cos(x) + 1)/sin(x) - 2*arctan(sin(x)/(cos(x) + 1))
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{\sin^2(x)}{(1-\cos(x))^2}\,dx = -x - \frac{2}{\tan\left(\frac{1}{2}\,x\right)}$$

input
```
integrate(sin(x)^2/(1-cos(x))^2,x, algorithm="giac")
```

output
```
-x - 2/tan(1/2*x)
```

## Mupad [B] (verification not implemented)

Time = 43.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{\sin^2(x)}{(1-\cos(x))^2}\,dx = -x-2\cot\left(\frac{x}{2}\right)$$

input
```
int(sin(x)^2/(cos(x) - 1)^2,x)
```

output
```
- x - 2*cot(x/2)
```

## Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sin^2(x)}{(1-\cos(x))^2}\,dx = \frac{-\tan\left(\frac{x}{2}\right)x-2}{\tan\left(\frac{x}{2}\right)}$$

input
```
int(sin(x)^2/(1-cos(x))^2,x)
```

output
```
( - tan(x/2)*x - 2)/tan(x/2)
```

# 3.16 $\int \frac{\sin^3(x)}{(1+\cos(x))^2} \, dx$



## Optimal result

Integrand size = 11, antiderivative size = 10

$$\int \frac{\sin^3(x)}{(1+\cos(x))^2} \, dx = \cos(x) - 2\log(1+\cos(x))$$

output
```
cos(x)-2*ln(1+cos(x))
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.30

$$\int \frac{\sin^3(x)}{(1+\cos(x))^2} \, dx = -1 + \cos(x) - 4\log\left(\cos\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[Sin[x]^3/(1 + Cos[x])^2,x]
```

output
```
-1 + Cos[x] - 4*Log[Cos[x/2]]
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 3146, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(x)}{(\cos(x)+1)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos\left(x-\frac{\pi}{2}\right)^3}{\left(1-\sin\left(x-\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3146}$$

$$-\int \frac{1-\cos(x)}{\cos(x)+1}d\cos(x)$$

$$\Big\downarrow \text{49}$$

$$-\int \left(\frac{2}{\cos(x)+1}-1\right)d\cos(x)$$

$$\Big\downarrow \text{2009}$$

$$\cos(x)-2\log(\cos(x)+1)$$

input
```
Int[Sin[x]^3/(1 + Cos[x])^2,x]
```

output
```
Cos[x] - 2*Log[1 + Cos[x]]
```

### Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

## Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

| method | result | size |
|---|---|---|
| derivativedivides | $\cos(x)-2\ln(\cos(x)+1)$ | 11 |
| default | $\cos(x)-2\ln(\cos(x)+1)$ | 11 |
| parallelrisch | $\cos(x)+1+2\ln\left(\sec\left(\frac{x}{2}\right)^2\right)$ | 14 |
| risch | $2ix+\frac{e^{ix}}{2}+\frac{e^{-ix}}{2}-4\ln(e^{ix}+1)$ | 30 |
| norman | $\frac{2\tan\left(\frac{x}{2}\right)^4+4\tan\left(\frac{x}{2}\right)^2+2}{\left(1+\tan\left(\frac{x}{2}\right)^2\right)^3}+2\ln\left(1+\tan\left(\frac{x}{2}\right)^2\right)$ | 42 |

input
```
int(sin(x)^3/(cos(x)+1)^2,x,method=_RETURNVERBOSE)
```

output
```
cos(x)-2*ln(cos(x)+1)
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\sin^3(x)}{(1+\cos(x))^2}\,dx = \cos(x) - 2\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right)$$

input
```
integrate(sin(x)^3/(1+cos(x))^2,x, algorithm="fricas")
```

output
```
cos(x) - 2*log(1/2*cos(x) + 1/2)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(10) = 20.

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 5.80

$$\int \frac{\sin^3(x)}{(1+\cos(x))^2}\,dx = -\frac{2\log(\cos(x)+1)\cos(x)}{\cos(x)+1} - \frac{2\log(\cos(x)+1)}{\cos(x)+1} + \frac{\sin^2(x)}{\cos(x)+1} + \frac{2\cos^2(x)}{\cos(x)+1} - \frac{2}{\cos(x)+1}$$

input
```
integrate(sin(x)**3/(1+cos(x))**2,x)
```

output
```
-2*log(cos(x) + 1)*cos(x)/(cos(x) + 1) - 2*log(cos(x) + 1)/(cos(x) + 1) +
sin(x)**2/(cos(x) + 1) + 2*cos(x)**2/(cos(x) + 1) - 2/(cos(x) + 1)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\sin^3(x)}{(1+\cos(x))^2}\,dx = \cos(x) - 2\log(\cos(x) + 1)$$

input
```
integrate(sin(x)^3/(1+cos(x))^2,x, algorithm="maxima")
```

output
```
cos(x) - 2*log(cos(x) + 1)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\sin^3(x)}{(1+\cos(x))^2}\,dx = \cos(x) - 2\,\log(\cos(x)+1)$$

input
```
integrate(sin(x)^3/(1+cos(x))^2,x, algorithm="giac")
```

output
```
cos(x) - 2*log(cos(x) + 1)
```

## Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\sin^3(x)}{(1+\cos(x))^2}\,dx = \cos(x) - 2\,\ln(\cos(x)+1)$$

input
```
int(sin(x)^3/(cos(x) + 1)^2,x)
```

output
```
cos(x) - 2*log(cos(x) + 1)
```

## Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 38, normalized size of antiderivative = 3.80

$$\int \frac{\sin^3(x)}{(1+\cos(x))^2}\,dx = \frac{2\cos(x)^2 - 2\cos(x)\log(\cos(x)+1) + 2\cos(x) - 2\log(\cos(x)+1) + \sin(x)^2}{\cos(x)+1}$$

input
```
int(sin(x)^3/(1+cos(x))^2,x)
```

output
```
(2*cos(x)**2 - 2*cos(x)*log(cos(x) + 1) + 2*cos(x) - 2*log(cos(x) + 1) + s
in(x)**2)/(cos(x) + 1)
```

# 3.17 $\int \frac{\sin^3(x)}{(1-\cos(x))^2}\,dx$



## Optimal result

Integrand size = 13, antiderivative size = 12

$$\int \frac{\sin^3(x)}{(1-\cos(x))^2}\,dx = \cos(x)+2\log(1-\cos(x))$$

output
```
cos(x)+2*ln(1-cos(x))
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{\sin^3(x)}{(1-\cos(x))^2}\,dx = -1+\cos(x)+4\log\left(\sin\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[Sin[x]^3/(1 - Cos[x])^2,x]
```

output
```
-1 + Cos[x] + 4*Log[Sin[x/2]]
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 3146, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(x)}{(1-\cos(x))^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos\left(x-\frac{\pi}{2}\right)^3}{\left(\sin\left(x-\frac{\pi}{2}\right)+1\right)^2}dx$$

$$\Big\downarrow \text{ 3146}$$

$$\int \frac{\cos(x)+1}{1-\cos(x)}d(-\cos(x))$$

$$\Big\downarrow \text{ 49}$$

$$\int \left(\frac{2}{1-\cos(x)}-1\right)d(-\cos(x))$$

$$\Big\downarrow \text{ 2009}$$

$$\cos(x)+2\log(1-\cos(x))$$

input
```
Int[Sin[x]^3/(1 - Cos[x])^2,x]
```

output
```
Cos[x] + 2*Log[1 - Cos[x]]
```

### Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

## Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

| method | result | size |
|---|---|---|
| derivativedivides | $\cos(x)+2\ln(-1+\cos(x))$ | 11 |
| default | $\cos(x)+2\ln(-1+\cos(x))$ | 11 |
| parallelrisch | $\cos(x)+1+4\ln\left(\tan\left(\frac{x}{2}\right)\right)-2\ln\left(\sec\left(\frac{x}{2}\right)^{2}\right)$ | 21 |
| risch | $-2ix+\frac{e^{ix}}{2}+\frac{e^{-ix}}{2}+4\ln(e^{ix}-1)$ | 30 |
| norman | $\frac{2\tan\left(\frac{x}{2}\right)^{3}+2\tan\left(\frac{x}{2}\right)^{7}+4\tan\left(\frac{x}{2}\right)^{5}}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)^{3}\tan\left(\frac{x}{2}\right)^{3}}+4\ln\left(\tan\left(\frac{x}{2}\right)\right)-2\ln\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)$ | 62 |

input
```
int(sin(x)^3/(1-cos(x))^2,x,method=_RETURNVERBOSE)
```

output
```
cos(x)+2*ln(-1+cos(x))
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sin^3(x)}{(1-\cos(x))^2}\,dx = \cos\left(x\right) + 2\,\log\left(-\frac{1}{2}\,\cos\left(x\right) + \frac{1}{2}\right)$$

input
```
integrate(sin(x)^3/(1-cos(x))^2,x, algorithm="fricas")
```

output
```
cos(x) + 2*log(-1/2*cos(x) + 1/2)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(10) = 20.

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 4.83

$$\int \frac{\sin^3(x)}{(1-\cos(x))^2}\,dx = \frac{2\log\left(\cos\left(x\right)-1\right)\cos\left(x\right)}{\cos\left(x\right)-1} - \frac{2\log\left(\cos\left(x\right)-1\right)}{\cos\left(x\right)-1} + \frac{\sin^2\left(x\right)}{\cos\left(x\right)-1} + \frac{2\cos^2\left(x\right)}{\cos\left(x\right)-1} - \frac{2}{\cos\left(x\right)-1}$$

input
```
integrate(sin(x)**3/(1-cos(x))**2,x)
```

output
```
2*log(cos(x) - 1)*cos(x)/(cos(x) - 1) - 2*log(cos(x) - 1)/(cos(x) - 1) + s
in(x)**2/(cos(x) - 1) + 2*cos(x)**2/(cos(x) - 1) - 2/(cos(x) - 1)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\sin^3(x)}{(1-\cos(x))^2}\,dx = \cos\left(x\right) + 2\,\log\left(\cos\left(x\right) - 1\right)$$

input
```
integrate(sin(x)^3/(1-cos(x))^2,x, algorithm="maxima")
```

output
```
cos(x) + 2*log(cos(x) - 1)
```

## Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sin^3(x)}{(1-\cos(x))^2}\,dx = \cos(x) + 2\log(-\cos(x) + 1)$$

input
```
integrate(sin(x)^3/(1-cos(x))^2,x, algorithm="giac")
```

output
```
cos(x) + 2*log(-cos(x) + 1)
```

## Mupad [B] (verification not implemented)

Time = 43.75 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\sin^3(x)}{(1-\cos(x))^2}\,dx = 2\ln(\cos(x) - 1) + \cos(x)$$

input
```
int(sin(x)^3/(cos(x) - 1)^2,x)
```

output
```
2*log(cos(x) - 1) + cos(x)
```

## Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 3.17

$$\int \frac{\sin^3(x)}{(1-\cos(x))^2}\,dx = \frac{2\cos(x)^2 + 2\cos(x)\log(\cos(x)-1) - 2\cos(x) - 2\log(\cos(x)-1) + \sin(x)^2}{\cos(x)-1}$$

input
```
int(sin(x)^3/(1-cos(x))^2,x)
```

output
```
(2*cos(x)**2 + 2*cos(x)*log(cos(x) - 1) - 2*cos(x) - 2*log(cos(x) - 1) + s
in(x)**2)/(cos(x) - 1)
```

# 3.18 $\int \frac{\sin(x)}{(1+\cos(x))^3} dx$



## Optimal result

Integrand size = 9, antiderivative size = 10

$$\int \frac{\sin(x)}{(1+\cos(x))^3} dx = \frac{1}{2(1+\cos(x))^2}$$

output
```
1/2/(1+cos(x))^2
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\sin(x)}{(1+\cos(x))^3} dx = \frac{1}{8}\sec^4\left(\frac{x}{2}\right)$$

input
```
Integrate[Sin[x]/(1 + Cos[x])^3,x]
```

output
```
Sec[x/2]^4/8
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3146, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)}{(\cos(x)+1)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos\left(x-\frac{\pi}{2}\right)}{\left(1-\sin\left(x-\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3146}$$

$$-\int \frac{1}{(\cos(x)+1)^3}d\cos(x)$$

$$\Big\downarrow \text{17}$$

$$\frac{1}{2(\cos(x)+1)^2}$$

input
```
Int[Sin[x]/(1 + Cos[x])^3,x]
```

output
```
1/(2*(1 + Cos[x])^2)
```

### Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

## Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{1}{2(\cos(x)+1)^2}$ | 9 |
| default | $\frac{1}{2(\cos(x)+1)^2}$ | 9 |
| risch | $\frac{2\,\mathrm{e}^{2ix}}{(\mathrm{e}^{ix}+1)^4}$ | 17 |
| norman | $\frac{\tan(\frac{x}{2})^2}{4}+\frac{\tan(\frac{x}{2})^4}{8}$ | 18 |
| parallelrisch | $\frac{\tan(\frac{x}{2})^2}{4}+\frac{\tan(\frac{x}{2})^4}{8}$ | 18 |

input
```
int(sin(x)/(cos(x)+1)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2/(cos(x)+1)^2
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.40

$$\int \frac{\sin(x)}{(1+\cos(x))^3}\,dx = \frac{1}{2\left(\cos\left(x\right)^2+2\,\cos\left(x\right)+1\right)}$$

input
```
integrate(sin(x)/(1+cos(x))^3,x, algorithm="fricas")
```

output
```
1/2/(cos(x)^2 + 2*cos(x) + 1)
```

## Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.40

$$\int \frac{\sin(x)}{(1+\cos(x))^3}\,dx = \frac{1}{2\cos^2(x) + 4\cos(x) + 2}$$

input
```
integrate(sin(x)/(1+cos(x))**3,x)
```

output
```
1/(2*cos(x)**2 + 4*cos(x) + 2)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\sin(x)}{(1+\cos(x))^3}\,dx = \frac{1}{2\,(\cos(x)+1)^2}$$

input
```
integrate(sin(x)/(1+cos(x))^3,x, algorithm="maxima")
```

output
```
1/2/(cos(x) + 1)^2
```

## Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\sin(x)}{(1+\cos(x))^3}\,dx = \frac{1}{2\,(\cos(x)+1)^2}$$

input
```
integrate(sin(x)/(1+cos(x))^3,x, algorithm="giac")
```

output
```
1/2/(cos(x) + 1)^2
```

## Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\sin(x)}{(1+\cos(x))^3}\,dx = \frac{1}{2\left(\cos\left(x\right)+1\right)^2}$$

input
```
int(sin(x)/(cos(x) + 1)^3,x)
```

output
```
1/(2*(cos(x) + 1)^2)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.40

$$\int \frac{\sin(x)}{(1+\cos(x))^3}\,dx = \frac{1}{2\cos\left(x\right)^2+4\cos\left(x\right)+2}$$

input
```
int(sin(x)/(1+cos(x))^3,x)
```

output
```
1/(2*(cos(x)**2 + 2*cos(x) + 1))
```

# 3.19 $\int \frac{\sin(x)}{(1-\cos(x))^3} dx$



## Optimal result

Integrand size = 11, antiderivative size = 12

$$\int \frac{\sin(x)}{(1-\cos(x))^3} dx = -\frac{1}{2(1-\cos(x))^2}$$

output
```
-1/2/(1-cos(x))^2
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{(1-\cos(x))^3} dx = -\frac{1}{8}\csc^4\left(\frac{x}{2}\right)$$

input
```
Integrate[Sin[x]/(1 - Cos[x])^3,x]
```

output
```
-1/8*Csc[x/2]^4
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 3146, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)}{(1-\cos(x))^3} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos\left(x-\frac{\pi}{2}\right)}{\left(\sin\left(x-\frac{\pi}{2}\right)+1\right)^3}dx$$

$$\Big\downarrow \text{3146}$$

$$\int \frac{1}{(1-\cos(x))^3}d(-\cos(x))$$

$$\Big\downarrow \text{17}$$

$$-\frac{1}{2(1-\cos(x))^2}$$

input
```
Int[Sin[x]/(1 - Cos[x])^3,x]
```

output
```
-1/2*1/(1 - Cos[x])^2
```

### Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

## Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{1}{2(1-\cos(x))^2}$ | 11 |
| default | $-\frac{1}{2(1-\cos(x))^2}$ | 11 |
| risch | $-\frac{2\,e^{2ix}}{(e^{ix}-1)^4}$ | 17 |
| parallelrisch | $-\frac{\cot(\frac{x}{2})^4}{8}-\frac{\cot(\frac{x}{2})^2}{4}$ | 18 |
| norman | $\frac{-\frac{\tan(\frac{x}{2})^5}{4}-\frac{3\tan(\frac{x}{2})^3}{8}-\frac{\tan(\frac{x}{2})}{8}}{\left(1+\tan(\frac{x}{2})^2\right)\tan(\frac{x}{2})^5}$ | 41 |

input
```
int(sin(x)/(1-cos(x))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/(1-cos(x))^2
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\sin(x)}{(1-\cos(x))^3}\,dx = -\frac{1}{2\left(\cos(x)^2-2\cos(x)+1\right)}$$

input
```
integrate(sin(x)/(1-cos(x))^3,x, algorithm="fricas")
```

output
```
-1/2/(cos(x)^2 - 2*cos(x) + 1)
```

## Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{\sin(x)}{(1-\cos(x))^3}\,dx = -\frac{1}{2\cos^2(x) - 4\cos(x) + 2}$$

input
```
integrate(sin(x)/(1-cos(x))**3,x)
```

output
```
-1/(2*cos(x)**2 - 4*cos(x) + 2)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{\sin(x)}{(1-\cos(x))^3}\,dx = -\frac{1}{2\left(\cos(x) - 1\right)^2}$$

input
```
integrate(sin(x)/(1-cos(x))^3,x, algorithm="maxima")
```

output
```
-1/2/(cos(x) - 1)^2
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{\sin(x)}{(1-\cos(x))^3}\,dx = -\frac{1}{2\left(\cos(x) - 1\right)^2}$$

input
```
integrate(sin(x)/(1-cos(x))^3,x, algorithm="giac")
```

output
```
-1/2/(cos(x) - 1)^2
```

## Mupad [B] (verification not implemented)

Time = 43.69 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{\sin(x)}{(1-\cos(x))^3}\,dx = -\frac{1}{2\left(\cos\left(x\right)-1\right)^2}$$

input
```
int(-sin(x)/(cos(x) - 1)^3,x)
```

output
```
-1/(2*(cos(x) - 1)^2)
```

## Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \frac{\sin(x)}{(1-\cos(x))^3}\,dx = -\frac{1}{2\cos\left(x\right)^2-4\cos\left(x\right)+2}$$

input
```
int(sin(x)/(1-cos(x))^3,x)
```

output
```
( - 1)/(2*(cos(x)**2 - 2*cos(x) + 1))
```

# 3.20 $\int \frac{\sin^2(x)}{(1+\cos(x))^3}\,dx$



## Optimal result

Integrand size = 11, antiderivative size = 14

$$\int \frac{\sin^2(x)}{(1+\cos(x))^3}\,dx = \frac{\sin^3(x)}{3(1+\cos(x))^3}$$

output
```
1/3*sin(x)^3/(1+cos(x))^3
```

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\sin^2(x)}{(1+\cos(x))^3}\,dx = \frac{1}{3}\tan^3\left(\frac{x}{2}\right)$$

input
```
Integrate[Sin[x]^2/(1 + Cos[x])^3,x]
```

output
```
Tan[x/2]^3/3
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3042, 3150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(x)}{(\cos(x)+1)^3} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos\left(x-\frac{\pi}{2}\right)^2}{\left(1-\sin\left(x-\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3150}$$

$$\frac{\sin^3(x)}{3(\cos(x)+1)^3}$$

input
```
Int[Sin[x]^2/(1 + Cos[x])^3,x]
```

output
```
Sin[x]^3/(3*(1 + Cos[x])^3)
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3150
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] && !ILtQ[p, 0]
```

## Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.64

| method | result | size |
|---|---|---|
| default | $\frac{\tan\left(\frac{x}{2}\right)^{3}}{3}$ | 9 |
| parallelrisch | $\frac{\tan\left(\frac{x}{2}\right)^{3}}{3}$ | 9 |
| risch | $-\frac{2i\left(3\,{\mathrm e}^{2ix}+1\right)}{3\left({\mathrm e}^{ix}+1\right)^{3}}$ | 22 |
| norman | $\frac{\frac{\tan\left(\frac{x}{2}\right)^{3}}{3}+\frac{2\tan\left(\frac{x}{2}\right)^{5}}{3}+\frac{\tan\left(\frac{x}{2}\right)^{7}}{3}}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)^{2}}$ | 37 |

input
```
int(sin(x)^2/(cos(x)+1)^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*tan(1/2*x)^3
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{\sin^2(x)}{(1+\cos(x))^3}\,dx = -\frac{(\cos(x)-1)\sin(x)}{3\left(\cos(x)^2+2\,\cos(x)+1\right)}$$

input
```
integrate(sin(x)^2/(1+cos(x))^3,x, algorithm="fricas")
```

output
```
-1/3*(cos(x) - 1)*sin(x)/(cos(x)^2 + 2*cos(x) + 1)
```

## Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.50

$$\int \frac{\sin^2(x)}{(1+\cos(x))^3}\,dx = \frac{\tan^3\left(\frac{x}{2}\right)}{3}$$

input
```
integrate(sin(x)**2/(1+cos(x))**3,x)
```

output
```
tan(x/2)**3/3
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\sin^2(x)}{(1+\cos(x))^3}\,dx = \frac{\sin\left(x\right)^3}{3\left(\cos\left(x\right)+1\right)^3}$$

input
```
integrate(sin(x)^2/(1+cos(x))^3,x, algorithm="maxima")
```

output
```
1/3*sin(x)^3/(cos(x) + 1)^3
```

## Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int \frac{\sin^2(x)}{(1+\cos(x))^3}\,dx = \frac{1}{3}\,\tan\left(\frac{1}{2}\,x\right)^3$$

input
```
integrate(sin(x)^2/(1+cos(x))^3,x, algorithm="giac")
```

output
```
1/3*tan(1/2*x)^3
```

## Mupad [B] (verification not implemented)

Time = 43.73 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int \frac{\sin^2(x)}{(1+\cos(x))^3}\,dx = \frac{\tan\left(\frac{x}{2}\right)^3}{3}$$

input
```
int(sin(x)^2/(cos(x) + 1)^3,x)
```

output
```
tan(x/2)^3/3
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int \frac{\sin^2(x)}{(1+\cos(x))^3}\,dx = \frac{\tan\left(\frac{x}{2}\right)^3}{3}$$

input
```
int(sin(x)^2/(1+cos(x))^3,x)
```

output
```
tan(x/2)**3/3
```

# 3.21 $\int \frac{\sin^2(x)}{(1-\cos(x))^3} \, dx$



## Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{\sin^2(x)}{(1-\cos(x))^3} \, dx = -\frac{\sin^3(x)}{3(1-\cos(x))^3}$$

output
```
-1/3*sin(x)^3/(1-cos(x))^3
```

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{\sin^2(x)}{(1-\cos(x))^3} \, dx = -\frac{1}{3}\cot^3\left(\frac{x}{2}\right)$$

input
```
Integrate[Sin[x]^2/(1 - Cos[x])^3,x]
```

output
```
-1/3*Cot[x/2]^3
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3042, 3150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(x)}{(1-\cos(x))^3} \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos\left(x-\frac{\pi}{2}\right)^2}{\left(\sin\left(x-\frac{\pi}{2}\right)+1\right)^3}dx$$

$$\Big\downarrow \text{ 3150}$$

$$-\frac{\sin^3(x)}{3(1-\cos(x))^3}$$

input
```
Int[Sin[x]^2/(1 - Cos[x])^3,x]
```

output
```
-1/3*Sin[x]^3/(1 - Cos[x])^3
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3150
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] && !ILtQ[p, 0]
```

## Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.56

| method | result | size |
|---|---|---|
| default | $-\frac{1}{3\tan\left(\frac{x}{2}\right)^{3}}$ | 9 |
| parallelrisch | $-\frac{\cot\left(\frac{x}{2}\right)^{3}}{3}$ | 9 |
| risch | $\frac{2i\left(3\,\mathrm{e}^{2ix}+1\right)}{3\left(\mathrm{e}^{ix}-1\right)^{3}}$ | 22 |
| norman | $\frac{-\frac{\tan\left(\frac{x}{2}\right)^{2}}{3}-\frac{2\tan\left(\frac{x}{2}\right)^{4}}{3}-\frac{\tan\left(\frac{x}{2}\right)^{6}}{3}}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)^{2}\tan\left(\frac{x}{2}\right)^{5}}$ | 43 |

input
```
int(sin(x)^2/(1-cos(x))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/3/tan(1/2*x)^3
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{\sin^2(x)}{(1-\cos(x))^3}\,dx=\frac{\cos\left(x\right)^{2}+2\,\cos\left(x\right)+1}{3\left(\cos\left(x\right)-1\right)\sin\left(x\right)}$$

input
```
integrate(sin(x)^2/(1-cos(x))^3,x, algorithm="fricas")
```

output
```
1/3*(cos(x)^2 + 2*cos(x) + 1)/((cos(x) - 1)*sin(x))
```

## Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{\sin^2(x)}{(1-\cos(x))^3}\,dx = -\frac{1}{3\tan^3\left(\frac{x}{2}\right)}$$

input
```
integrate(sin(x)**2/(1-cos(x))**3,x)
```

output
```
-1/(3*tan(x/2)**3)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{\sin^2(x)}{(1-\cos(x))^3}\,dx = -\frac{(\cos(x)+1)^3}{3\sin(x)^3}$$

input
```
integrate(sin(x)^2/(1-cos(x))^3,x, algorithm="maxima")
```

output
```
-1/3*(cos(x) + 1)^3/sin(x)^3
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.50

$$\int \frac{\sin^2(x)}{(1-\cos(x))^3}\,dx = -\frac{1}{3\tan\left(\frac{1}{2}x\right)^3}$$

input
```
integrate(sin(x)^2/(1-cos(x))^3,x, algorithm="giac")
```

output
```
-1/3/tan(1/2*x)^3
```

## Mupad [B] (verification not implemented)

Time = 44.86 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.50

$$\int \frac{\sin^2(x)}{(1-\cos(x))^3}\,dx = -\frac{\cot\left(\frac{x}{2}\right)^3}{3}$$

input
```
int(-sin(x)^2/(cos(x) - 1)^3,x)
```

output
```
-cot(x/2)^3/3
```

## Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.50

$$\int \frac{\sin^2(x)}{(1-\cos(x))^3}\,dx = -\frac{1}{3\tan\left(\frac{x}{2}\right)^3}$$

input
```
int(sin(x)^2/(1-cos(x))^3,x)
```

output
```
( - 1)/(3*tan(x/2)**3)
```

# 3.22 $\int \frac{\sin^3(x)}{(1+\cos(x))^3} dx$



## Optimal result

Integrand size = 11, antiderivative size = 14

$$\int \frac{\sin^3(x)}{(1+\cos(x))^3} dx = \frac{2}{1+\cos(x)} + \log(1+\cos(x))$$

output
```
2/(1+cos(x))+ln(1+cos(x))
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \frac{\sin^3(x)}{(1+\cos(x))^3} dx = 2\log\left(\cos\left(\frac{x}{2}\right)\right) + \sec^2\left(\frac{x}{2}\right)$$

input
```
Integrate[Sin[x]^3/(1 + Cos[x])^3,x]
```

output
```
2*Log[Cos[x/2]] + Sec[x/2]^2
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 3146, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(x)}{(\cos(x)+1)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos\left(x-\frac{\pi}{2}\right)^3}{\left(1-\sin\left(x-\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3146}$$

$$-\int \frac{1-\cos(x)}{(\cos(x)+1)^2}d\cos(x)$$

$$\Big\downarrow \text{49}$$

$$-\int \left(\frac{2}{(\cos(x)+1)^2}+\frac{1}{-\cos(x)-1}\right)d\cos(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{\cos(x)+1}+\log(\cos(x)+1)$$

input
```
Int[Sin[x]^3/(1 + Cos[x])^3,x]
```

output
```
2/(1 + Cos[x]) + Log[1 + Cos[x]]
```

### Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

## Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{2}{\cos(x)+1}+\ln\left(\cos\left(x\right)+1\right)$ | 15 |
| default | $\frac{2}{\cos(x)+1}+\ln\left(\cos\left(x\right)+1\right)$ | 15 |
| parallelrisch | $\tan\left(\frac{x}{2}\right)^{2}-\ln\left(\sec\left(\frac{x}{2}\right)^{2}\right)$ | 17 |
| risch | $-ix+\frac{4\,\mathrm{e}^{ix}}{\left(\mathrm{e}^{ix}+1\right)^{2}}+2\ln\left(\mathrm{e}^{ix}+1\right)$ | 32 |
| norman | $\frac{\tan\left(\frac{x}{2}\right)^{8}-8\tan\left(\frac{x}{2}\right)^{2}-6\tan\left(\frac{x}{2}\right)^{4}-3}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)^{3}}-\ln\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)$ | 48 |

input
```
int(sin(x)^3/(cos(x)+1)^3,x,method=_RETURNVERBOSE)
```

output
```
2/(cos(x)+1)+ln(cos(x)+1)
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.50

$$\int \frac{\sin^3(x)}{(1+\cos(x))^3}\,dx = \frac{(\cos(x)+1)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+2}{\cos(x)+1}$$

input
```
integrate(sin(x)^3/(1+cos(x))^3,x, algorithm="fricas")
```

output
```
((cos(x) + 1)*log(1/2*cos(x) + 1/2) + 2)/(cos(x) + 1)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(12) = 24.

Time = 0.25 (sec) , antiderivative size = 126, normalized size of antiderivative = 9.00

$$\int \frac{\sin^3(x)}{(1+\cos(x))^3}\,dx = \frac{2\log(\cos(x)+1)\cos^2(x)}{2\cos^2(x)+4\cos(x)+2}+\frac{4\log(\cos(x)+1)\cos(x)}{2\cos^2(x)+4\cos(x)+2}+\frac{2\log(\cos(x)+1)}{2\cos^2(x)+4\cos(x)+2}+\frac{\sin^2(x)}{2\cos^2(x)+4\cos(x)+2}+\frac{2\cos(x)}{2\cos^2(x)+4\cos(x)+2}+\frac{2}{2\cos^2(x)+4\cos(x)+2}$$

input
```
integrate(sin(x)**3/(1+cos(x))**3,x)
```

output
```
2*log(cos(x) + 1)*cos(x)**2/(2*cos(x)**2 + 4*cos(x) + 2) + 4*log(cos(x) +
1)*cos(x)/(2*cos(x)**2 + 4*cos(x) + 2) + 2*log(cos(x) + 1)/(2*cos(x)**2 +
4*cos(x) + 2) + sin(x)**2/(2*cos(x)**2 + 4*cos(x) + 2) + 2*cos(x)/(2*cos(x
)**2 + 4*cos(x) + 2) + 2/(2*cos(x)**2 + 4*cos(x) + 2)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\sin^3(x)}{(1+\cos(x))^3}\,dx = \frac{2}{\cos(x)+1} + \log(\cos(x)+1)$$

input
```
integrate(sin(x)^3/(1+cos(x))^3,x, algorithm="maxima")
```

output
```
2/(cos(x) + 1) + log(cos(x) + 1)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\sin^3(x)}{(1+\cos(x))^3}\,dx = \frac{2}{\cos(x)+1} + \log(\cos(x)+1)$$

input
```
integrate(sin(x)^3/(1+cos(x))^3,x, algorithm="giac")
```

output
```
2/(cos(x) + 1) + log(cos(x) + 1)
```

## Mupad [B] (verification not implemented)

Time = 44.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\sin^3(x)}{(1+\cos(x))^3}\,dx = \ln(\cos(x)+1) + \frac{2}{\cos(x)+1}$$

input
```
int(sin(x)^3/(cos(x) + 1)^3,x)
```

output
```
log(cos(x) + 1) + 2/(cos(x) + 1)
```

## Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 54, normalized size of antiderivative = 3.86

$$\int \frac{\sin^3(x)}{(1+\cos(x))^3}\,dx$$
$$= \frac{2\cos(x)^2\log(\cos(x)+1) - \cos(x)^2 + 4\cos(x)\log(\cos(x)+1) + 2\log(\cos(x)+1) + \sin(x)^2 + 1}{2\cos(x)^2 + 4\cos(x) + 2}$$

input
```
int(sin(x)^3/(1+cos(x))^3,x)
```

output
```
(2*cos(x)**2*log(cos(x) + 1) - cos(x)**2 + 4*cos(x)*log(cos(x) + 1) + 2*lo
g(cos(x) + 1) + sin(x)**2 + 1)/(2*(cos(x)**2 + 2*cos(x) + 1))
```

# 3.23 $\int \frac{\sin^3(x)}{(1-\cos(x))^3} dx$



## Optimal result

Integrand size = 13, antiderivative size = 20

$$\int \frac{\sin^3(x)}{(1-\cos(x))^3} dx = -\frac{2}{1-\cos(x)} - \log(1-\cos(x))$$

output
```
-2/(1-cos(x))-ln(1-cos(x))
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sin^3(x)}{(1-\cos(x))^3} dx = -\csc^2\left(\frac{x}{2}\right) - 2\log\left(\sin\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[Sin[x]^3/(1 - Cos[x])^3,x]
```

output
```
-Csc[x/2]^2 - 2*Log[Sin[x/2]]
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 3146, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(x)}{(1-\cos(x))^3}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\cos\left(x-\frac{\pi}{2}\right)^3}{\left(\sin\left(x-\frac{\pi}{2}\right)+1\right)^3}dx$$
$$\Big\downarrow \text{ 3146}$$
$$\int \frac{\cos(x)+1}{(1-\cos(x))^2}d(-\cos(x))$$
$$\Big\downarrow \text{ 49}$$
$$\int \left(\frac{2}{(1-\cos(x))^2}+\frac{1}{\cos(x)-1}\right)d(-\cos(x))$$
$$\Big\downarrow \text{ 2009}$$
$$-\frac{2}{1-\cos(x)}-\log(1-\cos(x))$$

input
```
Int[Sin[x]^3/(1 - Cos[x])^3,x]
```

output
```
-2/(1 - Cos[x]) - Log[1 - Cos[x]]
```

### Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

## Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{2}{-1+\cos\left(x\right)}-\ln\left(-1+\cos\left(x\right)\right)$ | 17 |
| default | $\frac{2}{-1+\cos\left(x\right)}-\ln\left(-1+\cos\left(x\right)\right)$ | 17 |
| parallelrisch | $-2\ln\left(\tan\left(\frac{x}{2}\right)\right)+\ln\left(\sec\left(\frac{x}{2}\right)^{2}\right)-\cot\left(\frac{x}{2}\right)^{2}$ | 24 |
| risch | $ix+\frac{4\,{\mathrm e}^{ix}}{\left({\mathrm e}^{ix}-1\right)^{2}}-2\ln\left({\mathrm e}^{ix}-1\right)$ | 32 |
| norman | $\frac{-3\tan\left(\frac{x}{2}\right)^{5}-3\tan\left(\frac{x}{2}\right)^{7}-\tan\left(\frac{x}{2}\right)^{9}-\tan\left(\frac{x}{2}\right)^{3}}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)^{3}\tan\left(\frac{x}{2}\right)^{5}}-2\ln\left(\tan\left(\frac{x}{2}\right)\right)+\ln\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)$ | 68 |

input
```
int(sin(x)^3/(1-cos(x))^3,x,method=_RETURNVERBOSE)
```

output
```
2/(-1+cos(x))-ln(-1+cos(x))
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin^3(x)}{(1-\cos(x))^3}\,dx = -\frac{(\cos(x)-1)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)-2}{\cos(x)-1}$$

input
```
integrate(sin(x)^3/(1-cos(x))^3,x, algorithm="fricas")
```

output
```
-((cos(x) - 1)*log(-1/2*cos(x) + 1/2) - 2)/(cos(x) - 1)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(14) = 28.

Time = 0.25 (sec) , antiderivative size = 126, normalized size of antiderivative = 6.30

$$\begin{aligned}\int \frac{\sin^3(x)}{(1-\cos(x))^3}\,dx = &-\frac{2\log(\cos(x)-1)\cos^2(x)}{2\cos^2(x)-4\cos(x)+2}+\frac{4\log(\cos(x)-1)\cos(x)}{2\cos^2(x)-4\cos(x)+2}\\ &-\frac{2\log(\cos(x)-1)}{2\cos^2(x)-4\cos(x)+2}-\frac{\sin^2(x)}{2\cos^2(x)-4\cos(x)+2}\\ &+\frac{2\cos(x)}{2\cos^2(x)-4\cos(x)+2}-\frac{2}{2\cos^2(x)-4\cos(x)+2}\end{aligned}$$

input
```
integrate(sin(x)**3/(1-cos(x))**3,x)
```

output
```
-2*log(cos(x) - 1)*cos(x)**2/(2*cos(x)**2 - 4*cos(x) + 2) + 4*log(cos(x) -
 1)*cos(x)/(2*cos(x)**2 - 4*cos(x) + 2) - 2*log(cos(x) - 1)/(2*cos(x)**2 -
 4*cos(x) + 2) - sin(x)**2/(2*cos(x)**2 - 4*cos(x) + 2) + 2*cos(x)/(2*cos(
x)**2 - 4*cos(x) + 2) - 2/(2*cos(x)**2 - 4*cos(x) + 2)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{\sin^3(x)}{(1-\cos(x))^3}\,dx = \frac{2}{\cos(x)-1} - \log(\cos(x)-1)$$

input
```
integrate(sin(x)^3/(1-cos(x))^3,x, algorithm="maxima")
```

output
```
2/(cos(x) - 1) - log(cos(x) - 1)
```

## Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\sin^3(x)}{(1-\cos(x))^3}\,dx = \frac{2}{\cos(x)-1} - \log(-\cos(x)+1)$$

input
```
integrate(sin(x)^3/(1-cos(x))^3,x, algorithm="giac")
```

output
```
2/(cos(x) - 1) - log(-cos(x) + 1)
```

## Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{\sin^3(x)}{(1-\cos(x))^3}\,dx = \frac{2}{\cos(x)-1} - \ln(\cos(x)-1)$$

input
```
int(-sin(x)^3/(cos(x) - 1)^3,x)
```

output
```
2/(cos(x) - 1) - log(cos(x) - 1)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.70

$$\int \frac{\sin^3(x)}{(1-\cos(x))^3}\,dx$$
$$= \frac{-2\cos(x)^2\log(\cos(x)-1)+\cos(x)^2+4\cos(x)\log(\cos(x)-1)-2\log(\cos(x)-1)-\sin(x)^2-1}{2\cos(x)^2-4\cos(x)+2}$$

input
```
int(sin(x)^3/(1-cos(x))^3,x)
```

output
```
( - 2*cos(x)**2*log(cos(x) - 1) + cos(x)**2 + 4*cos(x)*log(cos(x) - 1) - 2
*log(cos(x) - 1) - sin(x)**2 - 1)/(2*(cos(x)**2 - 2*cos(x) + 1))
```

# 3.24 $\int \frac{\sin^4(x)}{a+b\cos(x)}\,dx$



## Optimal result

Integrand size = 13, antiderivative size = 104

$$\int \frac{\sin^4(x)}{a+b\cos(x)}\,dx = -\frac{a(2a^2-3b^2)\,x}{2b^4} + \frac{2(a-b)^{3/2}(a+b)^{3/2}\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^4} + \frac{(2(a^2-b^2)-ab\cos(x))\sin(x)}{2b^3} - \frac{\sin^3(x)}{3b}$$

output
```
-1/2*a*(2*a^2-3*b^2)*x/b^4+2*(a-b)^(3/2)*(a+b)^(3/2)*arctan((a-b)^(1/2)*ta
n(1/2*x)/(a+b)^(1/2))/b^4+1/2*(2*a^2-2*b^2-a*b*cos(x))*sin(x)/b^3-1/3*sin(
x)^3/b
```

## Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.92

$$\int \frac{\sin^4(x)}{a+b\cos(x)}\,dx = \frac{-12a^3x+18ab^2x-24(-a^2+b^2)^{3/2}\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)+3b(4a^2-5b^2)\sin(x)-3ab^2\sin(2x)+b^3\sin}{12b^4}$$

input
```
Integrate[Sin[x]^4/(a + b*Cos[x]),x]
```

output
```
(-12*a^3*x + 18*a*b^2*x - 24*(-a^2 + b^2)^(3/2)*ArcTanh[((a - b)*Tan[x/2])
/Sqrt[-a^2 + b^2]] + 3*b*(4*a^2 - 5*b^2)*Sin[x] - 3*a*b^2*Sin[2*x] + b^3*S
in[3*x])/(12*b^4)
```

## Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.20, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.846$, Rules used = {3042, 3174, 25, 3042, 3344, 25, 3042, 3214, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(x)}{a+b\cos(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos\left(x-\frac{\pi}{2}\right)^4}{a-b\sin\left(x-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3174}$$

$$-\frac{\int -\frac{(b+a\cos(x))\sin^2(x)}{a+b\cos(x)}dx}{b}-\frac{\sin^3(x)}{3b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(b+a\cos(x))\sin^2(x)}{a+b\cos(x)}dx}{b}-\frac{\sin^3(x)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\cos\left(x+\frac{\pi}{2}\right)^2\left(b+a\sin\left(x+\frac{\pi}{2}\right)\right)}{a+b\sin\left(x+\frac{\pi}{2}\right)}dx}{b}-\frac{\sin^3(x)}{3b}$$

$$\Big\downarrow \text{3344}$$

$$\frac{\frac{\int -\frac{b\left(a^2-2b^2\right)+a\left(2a^2-3b^2\right)\cos(x)}{a+b\cos(x)}dx}{2b^2}+\frac{\sin(x)\left(2\left(a^2-b^2\right)-ab\cos(x)\right)}{2b^2}}{b}-\frac{\sin^3(x)}{3b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\sin(x)(2(a^2-b^2)-ab\cos(x))}{2b^2}-\frac{\int \frac{b\left(a^2-2b^2\right)+a\left(2a^2-3b^2\right)\cos(x)}{a+b\cos(x)}dx}{2b^2}}{b}-\frac{\sin^3(x)}{3b}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\sin(x)(2(a^2-b^2)-ab\cos(x))}{2b^2}-\frac{\int \frac{b\left(a^2-2b^2\right)+a\left(2a^2-3b^2\right)\sin\left(x+\frac{\pi}{2}\right)}{a+b\sin\left(x+\frac{\pi}{2}\right)}dx}{2b^2}}{b}-\frac{\sin^3(x)}{3b}$$

$\Big\downarrow$ 3214

$$\frac{\frac{\sin(x)(2(a^2-b^2)-ab\cos(x))}{2b^2}-\frac{\frac{ax\left(2a^2-3b^2\right)}{b}-\frac{2\left(a^2-b^2\right)^2\int \frac{1}{a+b\cos(x)}dx}{b}}{2b^2}}{b}-\frac{\sin^3(x)}{3b}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\sin(x)(2(a^2-b^2)-ab\cos(x))}{2b^2}-\frac{\frac{ax\left(2a^2-3b^2\right)}{b}-\frac{2\left(a^2-b^2\right)^2\int \frac{1}{a+b\sin\left(x+\frac{\pi}{2}\right)}dx}{b}}{2b^2}}{b}-\frac{\sin^3(x)}{3b}$$

$\Big\downarrow$ 3138

$$\frac{\frac{\sin(x)(2(a^2-b^2)-ab\cos(x))}{2b^2}-\frac{\frac{ax\left(2a^2-3b^2\right)}{b}-\frac{4\left(a^2-b^2\right)^2\int \frac{1}{(a-b)\tan^2\left(\frac{x}{2}\right)+a+b}d\tan\left(\frac{x}{2}\right)}{b}}{2b^2}}{b}-\frac{\sin^3(x)}{3b}$$

$\Big\downarrow$ 218

$$\frac{\frac{\sin(x)(2(a^2-b^2)-ab\cos(x))}{2b^2}-\frac{\frac{ax\left(2a^2-3b^2\right)}{b}-\frac{4\left(a^2-b^2\right)^2\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b\sqrt{a-b}\sqrt{a+b}}}{2b^2}}{b}-\frac{\sin^3(x)}{3b}$$

input
```
Int[Sin[x]^4/(a + b*Cos[x]),x]
```

output
```
-1/3*Sin[x]^3/b + (-1/2*((a*(2*a^2 - 3*b^2)*x)/b - (4*(a^2 - b^2)^2*ArcTan
[(Sqrt[a - b]*Tan[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*b*Sqrt[a + b]))/b^2 + (
(2*(a^2 - b^2) - a*b*Cos[x])*Sin[x])/(2*b^2))/b
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3174
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Simp[g^2*((p - 1)/(b*(m + p)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; F
reeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
 0] && IntegersQ[2*m, 2*p]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3344
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Simp[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Si
n[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^
2*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1
, 0] && IntegerQ[2*m]
```

## Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.46

| method | result |
|---|---|
| default | $-\frac{2\left(\frac{\left(-a^{2} b-\frac{1}{2} b^{2} a+b^{3}\right) \tan\left(\frac{x}{2}\right)^{5}+\left(-2 a^{2} b+\frac{10}{3} b^{3}\right) \tan\left(\frac{x}{2}\right)^{3}+\left(-a^{2} b+b^{3}+\frac{1}{2} b^{2} a\right) \tan\left(\frac{x}{2}\right)}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)^{3}}+\frac{a\left(2 a^{2}-3 b^{2}\right) \arctan\left(\tan\left(\frac{x}{2}\right)\right)}{2}\right)}{b^{4}}+$ $2(a+b)^2(a-$ |
| risch | $-\frac{a^{3} x}{b^{4}}+\frac{3 a x}{2 b^{2}}-\frac{i e^{i x} a^{2}}{2 b^{3}}+\frac{5 i e^{i x}}{8 b}+\frac{i e^{-i x} a^{2}}{2 b^{3}}-\frac{5 i e^{-i x}}{8 b}+\frac{\sqrt{-a^{2}+b^{2}}\, \ln\left(e^{i x}-\frac{i \sqrt{-a^{2}+b^{2}}-a}{b}\right) a^{2}}{b^{4}}-$ $\frac{\sqrt{-a^{2}+b^{2}}\, \ln\left(e^{i x}-\frac{i\sqrt{-}}{}\right.}{b^{2}}$ |

input
```
int(sin(x)^4/(a+b*cos(x)),x,method=_RETURNVERBOSE)
```

output
```
-2/b^4*(((-a^2*b-1/2*b^2*a+b^3)*tan(1/2*x)^5+(-2*a^2*b+10/3*b^3)*tan(1/2*x
)^3+(-a^2*b+b^3+1/2*b^2*a)*tan(1/2*x))/(1+tan(1/2*x)^2)^3+1/2*a*(2*a^2-3*b
^2)*arctan(tan(1/2*x)))+2*(a+b)^2*(a-b)^2/b^4/((a-b)*(a+b))^(1/2)*arctan((
a-b)*tan(1/2*x)/((a-b)*(a+b))^(1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.34

$$\int \frac{\sin^4(x)}{a+b\cos(x)}\,dx$$
$$= \left[-\frac{3\left(a^{2}-b^{2}\right)\sqrt{-a^{2}+b^{2}}\log\left(\frac{2\,ab\cos(x)+\left(2\,a^{2}-b^{2}\right)\cos(x)^{2}+2\,\sqrt{-a^{2}+b^{2}}\left(a\cos(x)+b\right)\sin(x)-a^{2}+2\,b^{2}}{b^{2}\cos(x)^{2}+2\,ab\cos(x)+a^{2}}\right)+3\left(2\,a^{3}-3\,ab^{2}\right)x}{6\,b^{4}}\right.$$

input
```
integrate(sin(x)^4/(a+b*cos(x)),x, algorithm="fricas")
```

output
```
[-1/6*(3*(a^2 - b^2)*sqrt(-a^2 + b^2)*log((2*a*b*cos(x) + (2*a^2 - b^2)*co
s(x)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(x) + b)*sin(x) - a^2 + 2*b^2)/(b^2*cos(
x)^2 + 2*a*b*cos(x) + a^2)) + 3*(2*a^3 - 3*a*b^2)*x - (2*b^3*cos(x)^2 - 3*
a*b^2*cos(x) + 6*a^2*b - 8*b^3)*sin(x))/b^4, 1/6*(6*(a^2 - b^2)^(3/2)*arct
an(-(a*cos(x) + b)/(sqrt(a^2 - b^2)*sin(x))) - 3*(2*a^3 - 3*a*b^2)*x + (2*
b^3*cos(x)^2 - 3*a*b^2*cos(x) + 6*a^2*b - 8*b^3)*sin(x))/b^4]
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^4(x)}{a + b\cos(x)}\, dx = \text{Timed out}$$

input
```
integrate(sin(x)**4/(a+b*cos(x)),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\sin^4(x)}{a + b\cos(x)}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate(sin(x)^4/(a+b*cos(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(86) = 172$.

Time = 0.11 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.87

$$\int \frac{\sin^4(x)}{a+b\cos(x)}\,dx = -\frac{(2\,a^3-3\,ab^2)x}{2\,b^4} - \frac{2\,(a^4-2\,a^2b^2+b^4)\left(\pi\left\lfloor\frac{x}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,x\right)-b\tan\left(\frac{1}{2}\,x\right)}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}b^4} + \frac{6\,a^2\tan\left(\frac{1}{2}\,x\right)^5+3\,ab\tan\left(\frac{1}{2}\,x\right)^5-6\,b^2\tan\left(\frac{1}{2}\,x\right)^5+12\,a^2\tan\left(\frac{1}{2}\,x\right)^3-20\,b^2\tan\left(\frac{1}{2}\,x\right)^3+6\,a^2\tan\left(\frac{1}{2}\,x\right)}{3\left(\tan\left(\frac{1}{2}\,x\right)^2+1\right)^3b^3}$$

input
```
integrate(sin(x)^4/(a+b*cos(x)),x, algorithm="giac")
```

output
```
-1/2*(2*a^3 - 3*a*b^2)*x/b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*(pi*floor(1/2*x/p
i + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x) - b*tan(1/2*x))/sqrt(a^2
- b^2)))/(sqrt(a^2 - b^2)*b^4) + 1/3*(6*a^2*tan(1/2*x)^5 + 3*a*b*tan(1/2*x
)^5 - 6*b^2*tan(1/2*x)^5 + 12*a^2*tan(1/2*x)^3 - 20*b^2*tan(1/2*x)^3 + 6*a
^2*tan(1/2*x) - 3*a*b*tan(1/2*x) - 6*b^2*tan(1/2*x))/((tan(1/2*x)^2 + 1)^3
*b^3)
```

## Mupad [B] (verification not implemented)

Time = 44.97 (sec) , antiderivative size = 1677, normalized size of antiderivative = 16.12

$$\int \frac{\sin^4(x)}{a+b\cos(x)}\,dx = \text{Too large to display}$$

input
```
int(sin(x)^4/(a + b*cos(x)),x)
```

output
```
((4*tan(x/2)^3*(3*a^2 - 5*b^2))/(3*b^3) - (tan(x/2)*(a*b - 2*a^2 + 2*b^2))
/b^3 + (tan(x/2)^5*(a*b + 2*a^2 - 2*b^2))/b^3)/(3*tan(x/2)^2 + 3*tan(x/2)^
4 + tan(x/2)^6 + 1) - (2*atanh((64*tan(x/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4
*b^2)^(1/2))/(128*a*b^2 + 112*a^2*b - 352*a^3 - 64*b^3 + (16*a^4)/b + (320
*a^5)/b^2 - (112*a^6)/b^3 - (96*a^7)/b^4 + (48*a^8)/b^5) + (144*a^2*tan(x/
2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(128*a*b^4 + 16*a^4*b + 320*
a^5 - 64*b^5 + 112*a^2*b^3 - 352*a^3*b^2 - (112*a^6)/b - (96*a^7)/b^2 + (4
8*a^8)/b^3) + (80*a^3*tan(x/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/
(128*a*b^5 + 320*a^5*b - 112*a^6 - 64*b^6 + 112*a^2*b^4 - 352*a^3*b^3 + 16
*a^4*b^2 - (96*a^7)/b + (48*a^8)/b^2) - (144*a^4*tan(x/2)*(b^6 - a^6 - 3*a
^2*b^4 + 3*a^4*b^2)^(1/2))/(128*a*b^6 - 112*a^6*b - 96*a^7 - 64*b^7 + 112*
a^2*b^5 - 352*a^3*b^4 + 16*a^4*b^3 + 320*a^5*b^2 + (48*a^8)/b) + (48*a^5*t
an(x/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(128*a*b^7 - 96*a^7*b +
 48*a^8 - 64*b^8 + 112*a^2*b^6 - 352*a^3*b^5 + 16*a^4*b^4 + 320*a^5*b^3 -
112*a^6*b^2) - (192*a*tan(x/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/
(128*a*b^3 - 352*a^3*b + 16*a^4 - 64*b^4 + 112*a^2*b^2 + (320*a^5)/b - (11
2*a^6)/b^2 - (96*a^7)/b^3 + (48*a^8)/b^4))*(-(a + b)^3*(a - b)^3)^(1/2))/b
^4 + (a*atan(((a*(2*a^2 - 3*b^2)*((8*tan(x/2)*(4*a*b^8 - 16*a^8*b + 8*a^9
- 4*b^9 + 7*a^2*b^7 + 11*a^3*b^6 - 39*a^4*b^5 - 3*a^5*b^4 + 48*a^6*b^3 - 1
6*a^7*b^2))/b^6 - (a*((8*(2*a*b^12 - 4*b^13 + 10*a^2*b^11 - 6*a^3*b^10 ...
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.34

$$\int \frac{\sin^4(x)}{a+b\cos(x)}\,dx$$
$$= \frac{12\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{x}{2}\right)a-\tan\left(\frac{x}{2}\right)b}{\sqrt{a^2-b^2}}\right)a^2-12\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{x}{2}\right)a-\tan\left(\frac{x}{2}\right)b}{\sqrt{a^2-b^2}}\right)b^2-3\cos(x)\sin(x)\,a\,b^2-2\sin}{6b^4}$$

input
```
int(sin(x)^4/(a+b*cos(x)),x)
```

output
```
(12*sqrt(a**2 - b**2)*atan((tan(x/2)*a - tan(x/2)*b)/sqrt(a**2 - b**2))*a*
*2 - 12*sqrt(a**2 - b**2)*atan((tan(x/2)*a - tan(x/2)*b)/sqrt(a**2 - b**2)
)*b**2 - 3*cos(x)*sin(x)*a*b**2 - 2*sin(x)**3*b**3 + 6*sin(x)*a**2*b - 6*s
in(x)*b**3 - 6*a**3*x + 9*a*b**2*x)/(6*b**4)
```

# 3.25 $\int \frac{\sin^3(x)}{a+b\cos(x)}\,dx$



## Optimal result

Integrand size = 13, antiderivative size = 40

$$\int \frac{\sin^3(x)}{a+b\cos(x)}\,dx = -\frac{a\cos(x)}{b^2}+\frac{\cos^2(x)}{2b}+\frac{(a^2-b^2)\log(a+b\cos(x))}{b^3}$$

output
```
-a*cos(x)/b^2+1/2*cos(x)^2/b+(a^2-b^2)*ln(a+b*cos(x))/b^3
```

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{\sin^3(x)}{a+b\cos(x)}\,dx = -\frac{a\cos(x)}{b^2}+\frac{\cos(2x)}{4b}+\frac{(a^2-b^2)\log(a+b\cos(x))}{b^3}$$

input
```
Integrate[Sin[x]^3/(a + b*Cos[x]),x]
```

output
```
-((a*Cos[x])/b^2) + Cos[2*x]/(4*b) + ((a^2 - b^2)*Log[a + b*Cos[x]])/b^3
```

## Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 3147, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(x)}{a+b\cos(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos\left(x-\frac{\pi}{2}\right)^3}{a-b\sin\left(x-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3147}$$

$$-\frac{\int \frac{b^2-b^2\cos^2(x)}{a+b\cos(x)}d(b\cos(x))}{b^3}$$

$$\Big\downarrow \text{476}$$

$$-\frac{\int \left(a-b\cos(x)+\frac{b^2-a^2}{a+b\cos(x)}\right)d(b\cos(x))}{b^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\left(a^2-b^2\right)\log(a+b\cos(x))+ab\cos(x)-\frac{1}{2}b^2\cos^2(x)}{b^3}$$

input
```
Int[Sin[x]^3/(a + b*Cos[x]),x]
```

output
```
-((a*b*Cos[x] - (b^2*Cos[x]^2)/2 - (a^2 - b^2)*Log[a + b*Cos[x]])/b^3)
```

### Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

## Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

| method | result |
|---|---|
| derivativedivides | $-\frac{-\frac{b\cos(x)^2}{2}+a\cos(x)}{b^2}+\frac{(a^2-b^2)\ln(a+b\cos(x))}{b^3}$ |
| default | $-\frac{-\frac{b\cos(x)^2}{2}+a\cos(x)}{b^2}+\frac{(a^2-b^2)\ln(a+b\cos(x))}{b^3}$ |
| parallelrisch | $\frac{(a^2-b^2)\ln\left(2b+\sec\left(\frac{x}{2}\right)^2(a-b)\right)+(-a^2+b^2)\ln\left(\sec\left(\frac{x}{2}\right)^2\right)-b\left(a\cos(x)-\frac{b\cos(2x)}{4}+a+\frac{b}{4}\right)}{b^3}$ |
| norman | $\frac{\frac{2a\tan\left(\frac{x}{2}\right)^4}{b^2}-\frac{2a-2b}{3b^2}+\frac{(4a+2b)\tan\left(\frac{x}{2}\right)^6}{3b^2}}{\left(1+\tan\left(\frac{x}{2}\right)^2\right)^3}+\frac{(a-b)(a+b)\ln\left(a\tan\left(\frac{x}{2}\right)^2-b\tan\left(\frac{x}{2}\right)^2+a+b\right)}{b^3}-\frac{(a-b)(a+b)\ln\left(1+\tan\left(\frac{x}{2}\right.\right.}{b^3}$ |
| risch | $-\frac{ix\,a^2}{b^3}+\frac{ix}{b}+\frac{e^{2ix}}{8b}-\frac{a\,e^{ix}}{2b^2}-\frac{a\,e^{-ix}}{2b^2}+\frac{e^{-2ix}}{8b}+\frac{\ln\left(e^{2ix}+\frac{2a\,e^{ix}}{b}+1\right)a^2}{b^3}-\frac{\ln\left(e^{2ix}+\frac{2a\,e^{ix}}{b}+1\right)}{b}$ |

input
```
int(sin(x)^3/(a+b*cos(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/b^2*(-1/2*b*cos(x)^2+a*cos(x))+(a^2-b^2)*ln(a+b*cos(x))/b^3
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int \frac{\sin^3(x)}{a+b\cos(x)}\,dx = \frac{b^2\cos{(x)}^2 - 2\,ab\cos{(x)} + 2\,(a^2-b^2)\log{(-b\cos{(x)}-a)}}{2\,b^3}$$

input
```
integrate(sin(x)^3/(a+b*cos(x)),x, algorithm="fricas")
```

output
```
1/2*(b^2*cos(x)^2 - 2*a*b*cos(x) + 2*(a^2 - b^2)*log(-b*cos(x) - a))/b^3
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1421 vs. 2(34) = 68.

Time = 153.08 (sec) , antiderivative size = 1421, normalized size of antiderivative = 35.52

$$\int \frac{\sin^3(x)}{a+b\cos(x)}\,dx = \text{Too large to display}$$

input
```
integrate(sin(x)**3/(a+b*cos(x)),x)
```

output
```
Piecewise((zoo*(-log(tan(x/2) - 1)*tan(x/2)**4/(tan(x/2)**4 + 2*tan(x/2)**
2 + 1) - 2*log(tan(x/2) - 1)*tan(x/2)**2/(tan(x/2)**4 + 2*tan(x/2)**2 + 1)
 - log(tan(x/2) - 1)/(tan(x/2)**4 + 2*tan(x/2)**2 + 1) - log(tan(x/2) + 1)
*tan(x/2)**4/(tan(x/2)**4 + 2*tan(x/2)**2 + 1) - 2*log(tan(x/2) + 1)*tan(x
/2)**2/(tan(x/2)**4 + 2*tan(x/2)**2 + 1) - log(tan(x/2) + 1)/(tan(x/2)**4
+ 2*tan(x/2)**2 + 1) + log(tan(x/2)**2 + 1)*tan(x/2)**4/(tan(x/2)**4 + 2*t
an(x/2)**2 + 1) + 2*log(tan(x/2)**2 + 1)*tan(x/2)**2/(tan(x/2)**4 + 2*tan(
x/2)**2 + 1) + log(tan(x/2)**2 + 1)/(tan(x/2)**4 + 2*tan(x/2)**2 + 1) - 2*
tan(x/2)**2/(tan(x/2)**4 + 2*tan(x/2)**2 + 1)), Eq(a, 0) & Eq(b, 0)), (-4*
tan(x/2)**2/(b*tan(x/2)**4 + 2*b*tan(x/2)**2 + b) - 2/(b*tan(x/2)**4 + 2*b
*tan(x/2)**2 + b), Eq(a, b)), ((-sin(x)**2*cos(x) - 2*cos(x)**3/3)/a, Eq(b
, 0)), (a**2*log(-sqrt(-a/(a - b) - b/(a - b)) + tan(x/2))*tan(x/2)**4/(b*
*3*tan(x/2)**4 + 2*b**3*tan(x/2)**2 + b**3) + 2*a**2*log(-sqrt(-a/(a - b)
- b/(a - b)) + tan(x/2))*tan(x/2)**2/(b**3*tan(x/2)**4 + 2*b**3*tan(x/2)**
2 + b**3) + a**2*log(-sqrt(-a/(a - b) - b/(a - b)) + tan(x/2))/(b**3*tan(x
/2)**4 + 2*b**3*tan(x/2)**2 + b**3) + a**2*log(sqrt(-a/(a - b) - b/(a - b)
) + tan(x/2))*tan(x/2)**4/(b**3*tan(x/2)**4 + 2*b**3*tan(x/2)**2 + b**3) +
 2*a**2*log(sqrt(-a/(a - b) - b/(a - b)) + tan(x/2))*tan(x/2)**2/(b**3*tan
(x/2)**4 + 2*b**3*tan(x/2)**2 + b**3) + a**2*log(sqrt(-a/(a - b) - b/(a -
b)) + tan(x/2))/(b**3*tan(x/2)**4 + 2*b**3*tan(x/2)**2 + b**3) - a**2*l...
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{\sin^3(x)}{a+b\cos(x)}\,dx = \frac{b\cos{(x)}^2-2\,a\cos{(x)}}{2\,b^2}+\frac{(a^2-b^2)\log{(b\cos{(x)}+a)}}{b^3}$$

input
```
integrate(sin(x)^3/(a+b*cos(x)),x, algorithm="maxima")
```

output
```
1/2*(b*cos(x)^2 - 2*a*cos(x))/b^2 + (a^2 - b^2)*log(b*cos(x) + a)/b^3
```

## Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{\sin^3(x)}{a+b\cos(x)}\,dx = \frac{b\cos{(x)}^2-2\,a\cos{(x)}}{2\,b^2}+\frac{(a^2-b^2)\log{(|b\cos{(x)}+a|)}}{b^3}$$

input
```
integrate(sin(x)^3/(a+b*cos(x)),x, algorithm="giac")
```

output
```
1/2*(b*cos(x)^2 - 2*a*cos(x))/b^2 + (a^2 - b^2)*log(abs(b*cos(x) + a))/b^3
```

## Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{\sin^3(x)}{a+b\cos(x)}\,dx = \frac{\cos{(x)}^2}{2\,b}+\frac{\ln{(a+b\,\cos{(x)})}\,\left(a^2-b^2\right)}{b^3}-\frac{a\,\cos{(x)}}{b^2}$$

input
```
int(sin(x)^3/(a + b*cos(x)),x)
```

output
```
cos(x)^2/(2*b) + (log(a + b*cos(x))*(a^2 - b^2))/b^3 - (a*cos(x))/b^2
```

## Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.62

$$\int \frac{\sin^3(x)}{a+b\cos(x)}\,dx$$
$$= \frac{-2\cos{(x)}\,ab-2\log\left(\tan{\left(\frac{x}{2}\right)}^2+1\right)a^2+2\log\left(\tan{\left(\frac{x}{2}\right)}^2+1\right)b^2+2\log\left(\tan{\left(\frac{x}{2}\right)}^2a-\tan{\left(\frac{x}{2}\right)}^2b+a+b}{2b^3}$$

input
```
int(sin(x)^3/(a+b*cos(x)),x)
```

output
```
( - 2*cos(x)*a*b - 2*log(tan(x/2)**2 + 1)*a**2 + 2*log(tan(x/2)**2 + 1)*b*
*2 + 2*log(tan(x/2)**2*a - tan(x/2)**2*b + a + b)*a**2 - 2*log(tan(x/2)**2
*a - tan(x/2)**2*b + a + b)*b**2 - sin(x)**2*b**2 + 2*a*b)/(2*b**3)
```

# 3.26 $\int \frac{\sin^2(x)}{a+b\cos(x)}\,dx$



## Optimal result

Integrand size = 13, antiderivative size = 59

$$\int \frac{\sin^2(x)}{a+b\cos(x)}\,dx=\frac{ax}{b^2}-\frac{2\sqrt{a-b}\sqrt{a+b}\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^2}-\frac{\sin(x)}{b}$$

output
```
a*x/b^2-2*(a-b)^(1/2)*(a+b)^(1/2)*arctan((a-b)^(1/2)*tan(1/2*x)/(a+b)^(1/2
))/b^2-sin(x)/b
```

## Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int \frac{\sin^2(x)}{a+b\cos(x)}\,dx=\frac{ax-2\sqrt{-a^2+b^2}\,\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)-b\sin(x)}{b^2}$$

input
```
Integrate[Sin[x]^2/(a + b*Cos[x]),x]
```

output
```
(a*x - 2*Sqrt[-a^2 + b^2]*ArcTanh[((a - b)*Tan[x/2])/Sqrt[-a^2 + b^2]] - b
*Sin[x])/b^2
```

## Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.24, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {3042, 3174, 25, 3042, 3214, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(x)}{a+b\cos(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos\left(x-\frac{\pi}{2}\right)^2}{a-b\sin\left(x-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3174}$$

$$-\frac{\int -\frac{b+a\cos(x)}{a+b\cos(x)}dx}{b}-\frac{\sin(x)}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{b+a\cos(x)}{a+b\cos(x)}dx}{b}-\frac{\sin(x)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{b+a\sin\left(x+\frac{\pi}{2}\right)}{a+b\sin\left(x+\frac{\pi}{2}\right)}dx}{b}-\frac{\sin(x)}{b}$$

$$\Big\downarrow \text{3214}$$

$$\frac{\frac{ax}{b}-\frac{(a^2-b^2)\int \frac{1}{a+b\cos(x)}dx}{b}}{b}-\frac{\sin(x)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{ax}{b}-\frac{(a^2-b^2)\int \frac{1}{a+b\sin\left(x+\frac{\pi}{2}\right)}dx}{b}}{b}-\frac{\sin(x)}{b}$$

$$\Big\downarrow \text{3138}$$

$$\frac{\frac{ax}{b}-\frac{2(a^2-b^2)\int \frac{1}{(a-b)\tan^2\left(\frac{x}{2}\right)+a+b}d\tan\left(\frac{x}{2}\right)}{b}}{b}-\frac{\sin(x)}{b}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{ax}{b}-\frac{2(a^2-b^2)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b\sqrt{a-b}\sqrt{a+b}}}{b}-\frac{\sin(x)}{b}$$

input
```
Int[Sin[x]^2/(a + b*Cos[x]),x]
```

output
```
((a*x)/b - (2*(a^2 - b^2)*ArcTan[(Sqrt[a - b]*Tan[x/2])/Sqrt[a + b]])/(Sqr
t[a - b]*b*Sqrt[a + b]))/b - Sin[x]/b
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3174
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Simp[g^2*((p - 1)/(b*(m + p)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; F
reeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
 0] && IntegersQ[2*m, 2*p]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

## Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.32

| method | result | size |
|---|---|---|
| default | $-\frac{2(a+b)(a-b)\arctan\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b^2\sqrt{(a-b)(a+b)}}+\frac{-\frac{2b\tan\left(\frac{x}{2}\right)}{1+\tan\left(\frac{x}{2}\right)^2}+2a\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{b^2}$ | 78 |
| risch | $\frac{ax}{b^2}+\frac{ie^{ix}}{2b}-\frac{ie^{-ix}}{2b}-\frac{\sqrt{-a^2+b^2}\,\ln\left(e^{ix}-\frac{i\sqrt{-a^2+b^2}-a}{b}\right)}{b^2}+\frac{\sqrt{-a^2+b^2}\,\ln\left(e^{ix}+\frac{i\sqrt{-a^2+b^2}+a}{b}\right)}{b^2}$ | 118 |

input
```
int(sin(x)^2/(a+b*cos(x)),x,method=_RETURNVERBOSE)
```

output
```
-2*(a+b)*(a-b)/b^2/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*x)/((a-b)*(a+b
))^(1/2))+2/b^2*(-b*tan(1/2*x)/(1+tan(1/2*x)^2)+a*arctan(tan(1/2*x)))
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.61

$$\int \frac{\sin^2(x)}{a+b\cos(x)}\,dx = \left[\frac{2\,ax-2\,b\sin\left(x\right)+\sqrt{-a^2+b^2}\log\left(\frac{2\,ab\cos(x)+\left(2\,a^2-b^2\right)\cos(x)^2+2\sqrt{-a^2+b^2}\left(a\cos(x)+b\right)\sin(x)-a^2+2\,b^2}{b^2\cos(x)^2+2\,ab\cos(x)+a^2}\right)}{2\,b^2},\ ax-b\sin\right.$$

input
```
integrate(sin(x)^2/(a+b*cos(x)),x, algorithm="fricas")
```

output
```
[1/2*(2*a*x - 2*b*sin(x) + sqrt(-a^2 + b^2)*log((2*a*b*cos(x) + (2*a^2 - b
^2)*cos(x)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(x) + b)*sin(x) - a^2 + 2*b^2)/(b^
2*cos(x)^2 + 2*a*b*cos(x) + a^2)))/b^2, (a*x - b*sin(x) - sqrt(a^2 - b^2)*
arctan(-(a*cos(x) + b)/(sqrt(a^2 - b^2)*sin(x))))/b^2]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 991 vs. $2(49) = 98$.

Time = 50.68 (sec) , antiderivative size = 991, normalized size of antiderivative = 16.80

$$\int \frac{\sin^2(x)}{a + b\cos(x)}\, dx = \text{Too large to display}$$

input
```
integrate(sin(x)**2/(a+b*cos(x)),x)
```

output
```
Piecewise((zoo*(-log(tan(x/2) - 1)*tan(x/2)**2/(tan(x/2)**2 + 1) - log(tan
(x/2) - 1)/(tan(x/2)**2 + 1) + log(tan(x/2) + 1)*tan(x/2)**2/(tan(x/2)**2
+ 1) + log(tan(x/2) + 1)/(tan(x/2)**2 + 1) - 2*tan(x/2)/(tan(x/2)**2 + 1))
, Eq(a, 0) & Eq(b, 0)), (x*tan(x/2)**2/(b*tan(x/2)**2 + b) + x/(b*tan(x/2)
**2 + b) - 2*tan(x/2)/(b*tan(x/2)**2 + b), Eq(a, b)), (-x*tan(x/2)**2/(b*t
an(x/2)**2 + b) - x/(b*tan(x/2)**2 + b) - 2*tan(x/2)/(b*tan(x/2)**2 + b),
Eq(a, -b)), ((x*sin(x)**2/2 + x*cos(x)**2/2 - sin(x)*cos(x)/2)/a, Eq(b, 0)
), (a*x*sqrt(-a/(a - b) - b/(a - b))*tan(x/2)**2/(b**2*sqrt(-a/(a - b) - b
/(a - b))*tan(x/2)**2 + b**2*sqrt(-a/(a - b) - b/(a - b))) + a*x*sqrt(-a/(
a - b) - b/(a - b))/(b**2*sqrt(-a/(a - b) - b/(a - b))*tan(x/2)**2 + b**2*
sqrt(-a/(a - b) - b/(a - b))) - a*log(-sqrt(-a/(a - b) - b/(a - b)) + tan(
x/2))*tan(x/2)**2/(b**2*sqrt(-a/(a - b) - b/(a - b))*tan(x/2)**2 + b**2*sq
rt(-a/(a - b) - b/(a - b))) - a*log(-sqrt(-a/(a - b) - b/(a - b)) + tan(x/
2))/(b**2*sqrt(-a/(a - b) - b/(a - b))*tan(x/2)**2 + b**2*sqrt(-a/(a - b)
- b/(a - b))) + a*log(sqrt(-a/(a - b) - b/(a - b)) + tan(x/2))*tan(x/2)**2
/(b**2*sqrt(-a/(a - b) - b/(a - b))*tan(x/2)**2 + b**2*sqrt(-a/(a - b) - b
/(a - b))) + a*log(sqrt(-a/(a - b) - b/(a - b)) + tan(x/2))/(b**2*sqrt(-a/
(a - b) - b/(a - b))*tan(x/2)**2 + b**2*sqrt(-a/(a - b) - b/(a - b))) - 2*
b*sqrt(-a/(a - b) - b/(a - b))*tan(x/2)/(b**2*sqrt(-a/(a - b) - b/(a - b))
*tan(x/2)**2 + b**2*sqrt(-a/(a - b) - b/(a - b))) - b*log(-sqrt(-a/(a -...
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\sin^2(x)}{a+b\cos(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sin(x)^2/(a+b*cos(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.53

$$\int \frac{\sin^2(x)}{a+b\cos(x)}\,dx = \frac{ax}{b^2} + \frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}x\right)-b\tan\left(\frac{1}{2}x\right)}{\sqrt{a^2-b^2}}\right)\right)\sqrt{a^2-b^2}}{b^2} - \frac{2\tan\left(\frac{1}{2}x\right)}{\left(\tan\left(\frac{1}{2}x\right)^2+1\right)b}$$

input
```
integrate(sin(x)^2/(a+b*cos(x)),x, algorithm="giac")
```

output
```
a*x/b^2 + 2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2
*x) - b*tan(1/2*x))/sqrt(a^2 - b^2)))*sqrt(a^2 - b^2)/b^2 - 2*tan(1/2*x)/(
(tan(1/2*x)^2 + 1)*b)
```

## Mupad [B] (verification not implemented)

Time = 44.73 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.25

$$\int \frac{\sin^2(x)}{a+b\cos(x)}\,dx = \frac{2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{x}{2}\right)\sqrt{b^2-a^2}}{a\,\cos\left(\frac{x}{2}\right)+b\,\cos\left(\frac{x}{2}\right)}\right)\,\sqrt{b^2-a^2}}{b^2} - \frac{\sin\left(x\right)}{b} + \frac{2\,a\,\mathrm{atan}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{b^2}$$

input
```
int(sin(x)^2/(a + b*cos(x)),x)
```

output
```
(2*atanh((sin(x/2)*(b^2 - a^2)^(1/2))/(a*cos(x/2) + b*cos(x/2)))*(b^2 - a^
2)^(1/2))/b^2 - sin(x)/b + (2*a*atan(sin(x/2)/cos(x/2)))/b^2
```

## Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int \frac{\sin^2(x)}{a+b\cos(x)}\,dx = \frac{-2\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{x}{2}\right)a-\tan\left(\frac{x}{2}\right)b}{\sqrt{a^2-b^2}}\right) - \sin\left(x\right)b + ax}{b^2}$$

input
```
int(sin(x)^2/(a+b*cos(x)),x)
```

output
```
( - 2*sqrt(a**2 - b**2)*atan((tan(x/2)*a - tan(x/2)*b)/sqrt(a**2 - b**2))
- sin(x)*b + a*x)/b**2
```

# 3.27 $\int \frac{\sin(x)}{a+b\cos(x)}\,dx$



## Optimal result

Integrand size = 11, antiderivative size = 12

$$\int \frac{\sin(x)}{a+b\cos(x)}\,dx = -\frac{\log(a+b\cos(x))}{b}$$

output
```
-ln(a+b*cos(x))/b
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{a+b\cos(x)}\,dx = -\frac{\log(a+b\cos(x))}{b}$$

input
```
Integrate[Sin[x]/(a + b*Cos[x]),x]
```

output
```
-(Log[a + b*Cos[x]]/b)
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 3147, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)}{a+b\cos(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos\left(x-\frac{\pi}{2}\right)}{a-b\sin\left(x-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 3147}$$

$$-\frac{\int \frac{1}{a+b\cos(x)}d(b\cos(x))}{b}$$

$$\Big\downarrow \text{ 16}$$

$$-\frac{\log(a+b\cos(x))}{b}$$

input
```
Int[Sin[x]/(a + b*Cos[x]),x]
```

output
```
-(Log[a + b*Cos[x]]/b)
```

### Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + 
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

## Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{\ln(a+b\cos(x))}{b}$ | 13 |
| default | $-\frac{\ln(a+b\cos(x))}{b}$ | 13 |
| parallelrisch | $\frac{\ln\left(\sec\left(\frac{x}{2}\right)^{2}\right)-\ln\left(2b+\sec\left(\frac{x}{2}\right)^{2}(a-b)\right)}{b}$ | 32 |
| risch | $\frac{ix}{b}-\frac{\ln\left(e^{2ix}+\frac{2a\,e^{ix}}{b}+1\right)}{b}$ | 33 |
| norman | $\frac{\ln\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)}{b}-\frac{\ln\left(a\tan\left(\frac{x}{2}\right)^{2}-b\tan\left(\frac{x}{2}\right)^{2}+a+b\right)}{b}$ | 41 |

input
```
int(sin(x)/(a+b*cos(x)),x,method=_RETURNVERBOSE)
```

output
```
-ln(a+b*cos(x))/b
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{\sin(x)}{a+b\cos(x)}\,dx=-\frac{\log\left(-b\cos\left(x\right)-a\right)}{b}$$

input
```
integrate(sin(x)/(a+b*cos(x)),x, algorithm="fricas")
```

output
```
-log(-b*cos(x) - a)/b
```

## Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \frac{\sin(x)}{a + b\cos(x)}\,dx = \begin{cases} -\frac{\log\left(\frac{a}{b}+\cos(x)\right)}{b} & \text{for } b \neq 0 \\ -\frac{\cos(x)}{a} & \text{otherwise} \end{cases}$$

input
```
integrate(sin(x)/(a+b*cos(x)),x)
```

output
```
Piecewise((-log(a/b + cos(x))/b, Ne(b, 0)), (-cos(x)/a, True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{a + b\cos(x)}\,dx = -\frac{\log\left(b\cos(x) + a\right)}{b}$$

input
```
integrate(sin(x)/(a+b*cos(x)),x, algorithm="maxima")
```

output
```
-log(b*cos(x) + a)/b
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{\sin(x)}{a + b\cos(x)}\,dx = -\frac{\log\left(|b\cos(x) + a|\right)}{b}$$

input
```
integrate(sin(x)/(a+b*cos(x)),x, algorithm="giac")
```

output
```
-log(abs(b*cos(x) + a))/b
```

## Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{a+b\cos(x)}\,dx = -\frac{\ln\left(a+b\,\cos\left(x\right)\right)}{b}$$

input
```
int(sin(x)/(a + b*cos(x)),x)
```

output
```
-log(a + b*cos(x))/b
```

## Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{a+b\cos(x)}\,dx = -\frac{\log(\cos\left(x\right)b+a)}{b}$$

input
```
int(sin(x)/(a+b*cos(x)),x)
```

output
```
( - log(cos(x)*b + a))/b
```

## 3.28 $\int \frac{1}{a+b\cos(x)}\,dx$



### Optimal result

Integrand size = 8, antiderivative size = 42

$$\int \frac{1}{a+b\cos(x)}\,dx = \frac{2\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}$$

output
```
2*arctan((a-b)^(1/2)*tan(1/2*x)/(a+b)^(1/2))/(a-b)^(1/2)/(a+b)^(1/2)
```

### Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98

$$\int \frac{1}{a+b\cos(x)}\,dx = -\frac{2\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}$$

input
```
Integrate[(a + b*Cos[x])^(-1),x]
```

output
```
(-2*ArcTanh[((a - b)*Tan[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2]
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\cos(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a+b\sin\left(x+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3138}$$

$$2\int \frac{1}{(a-b)\tan^2\left(\frac{x}{2}\right)+a+b}d\tan\left(\frac{x}{2}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{2\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}$$

input
```
Int[(a + b*Cos[x])^(-1),x]
```

output
```
(2*ArcTan[(Sqrt[a - b]*Tan[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b])
```

### Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

## Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

| method | result | size |
|---|---|---|
| default | $\frac{2\arctan\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{\sqrt{(a-b)(a+b)}}$ | 36 |
| risch | $-\frac{\ln\left(\mathrm{e}^{ix}+\frac{ia^2-ib^2+a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}+\frac{\ln\left(\mathrm{e}^{ix}-\frac{ia^2-ib^2-a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}$ | 125 |

input
```
int(1/(a+b*cos(x)),x,method=_RETURNVERBOSE)
```

output
```
2/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*x)/((a-b)*(a+b))^(1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 137, normalized size of antiderivative = 3.26

$$\int \frac{1}{a+b\cos(x)}\,dx = \left[-\frac{\sqrt{-a^2+b^2}\log\left(\frac{2\,ab\cos(x)+(2\,a^2-b^2)\cos(x)^2+2\,\sqrt{-a^2+b^2}(a\cos(x)+b)\sin(x)-a^2+2\,b^2}{b^2\cos(x)^2+2\,ab\cos(x)+a^2}\right)}{2\,(a^2-b^2)}, \frac{\arctan\left(-\frac{a\cos(x)+b}{\sqrt{a^2-b^2}\sin(x)}\right)}{\sqrt{a^2-b^2}}\right]$$

input
```
integrate(1/(a+b*cos(x)),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-a^2 + b^2)*log((2*a*b*cos(x) + (2*a^2 - b^2)*cos(x)^2 + 2*sqrt
(-a^2 + b^2)*(a*cos(x) + b)*sin(x) - a^2 + 2*b^2)/(b^2*cos(x)^2 + 2*a*b*co
s(x) + a^2))/(a^2 - b^2), arctan(-(a*cos(x) + b)/(sqrt(a^2 - b^2)*sin(x)))
/sqrt(a^2 - b^2)]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(34) = 68$.

Time = 1.70 (sec) , antiderivative size = 144, normalized size of antiderivative = 3.43

$$\int \frac{1}{a + b\cos(x)}\,dx = \begin{cases} \tilde{\infty}\left(-\log\left(\tan\left(\frac{x}{2}\right) - 1\right) + \log\left(\tan\left(\frac{x}{2}\right) + 1\right)\right) & \text{for}\ a = 0 \wedge b = 0 \\ \frac{\tan\left(\frac{x}{2}\right)}{b} & \text{for}\ a = b \\ \frac{1}{b\tan\left(\frac{x}{2}\right)} & \text{for}\ a = -b \\ \frac{\log\left(-\sqrt{-\frac{a}{a-b} - \frac{b}{a-b}} + \tan\left(\frac{x}{2}\right)\right)}{a\sqrt{-\frac{a}{a-b} - \frac{b}{a-b}} - b\sqrt{-\frac{a}{a-b} - \frac{b}{a-b}}} - \frac{\log\left(\sqrt{-\frac{a}{a-b} - \frac{b}{a-b}} + \tan\left(\frac{x}{2}\right)\right)}{a\sqrt{-\frac{a}{a-b} - \frac{b}{a-b}} - b\sqrt{-\frac{a}{a-b} - \frac{b}{a-b}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b*cos(x)),x)
```

output
```
Piecewise((zoo*(-log(tan(x/2) - 1) + log(tan(x/2) + 1)), Eq(a, 0) & Eq(b,
0)), (tan(x/2)/b, Eq(a, b)), (1/(b*tan(x/2)), Eq(a, -b)), (log(-sqrt(-a/(a
 - b) - b/(a - b)) + tan(x/2))/(a*sqrt(-a/(a - b) - b/(a - b)) - b*sqrt(-a
/(a - b) - b/(a - b))) - log(sqrt(-a/(a - b) - b/(a - b)) + tan(x/2))/(a*s
qrt(-a/(a - b) - b/(a - b)) - b*sqrt(-a/(a - b) - b/(a - b))), True))
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{a+b\cos(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*cos(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

## Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.45

$$\int \frac{1}{a+b\cos(x)}\,dx = -\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor \operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}x\right)-b\tan\left(\frac{1}{2}x\right)}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}}$$

input
```
integrate(1/(a+b*cos(x)),x, algorithm="giac")
```

output
```
-2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x) - b*t
an(1/2*x))/sqrt(a^2 - b^2)))/sqrt(a^2 - b^2)
```

## Mupad [B] (verification not implemented)

Time = 45.34 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int \frac{1}{a+b\cos(x)}\,dx = \frac{2\,\mathrm{atan}\left(\frac{\tan\left(\frac{x}{2}\right)(2a-2b)}{2\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

input
```
int(1/(a + b*cos(x)),x)
```

output
```
(2*atan((tan(x/2)*(2*a - 2*b))/(2*(a^2 - b^2)^(1/2))))/(a^2 - b^2)^(1/2)
```

## Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.21

$$\int \frac{1}{a+b\cos(x)}\,dx = \frac{2\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{x}{2}\right)a-\tan\left(\frac{x}{2}\right)b}{\sqrt{a^2-b^2}}\right)}{a^2-b^2}$$

input
```
int(1/(a+b*cos(x)),x)
```

output
```
(2*sqrt(a**2 - b**2)*atan((tan(x/2)*a - tan(x/2)*b)/sqrt(a**2 - b**2)))/(a
**2 - b**2)
```

# 3.29 $\int \frac{\csc(x)}{a+b\cos(x)}\,dx$



## Optimal result

Integrand size = 11, antiderivative size = 53

$$\int \frac{\csc(x)}{a+b\cos(x)}\,dx = \frac{\log(1-\cos(x))}{2(a+b)} - \frac{\log(1+\cos(x))}{2(a-b)} + \frac{b\log(a+b\cos(x))}{a^2-b^2}$$

output
```
ln(1-cos(x))/(2*a+2*b)-ln(1+cos(x))/(2*a-2*b)+b*ln(a+b*cos(x))/(a^2-b^2)
```

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

$$\int \frac{\csc(x)}{a+b\cos(x)}\,dx = \frac{(a-b)\log(1-\cos(x))-(a+b)\log(1+\cos(x))+2b\log(a+b\cos(x))}{2(a-b)(a+b)}$$

input
```
Integrate[Csc[x]/(a + b*Cos[x]),x]
```

output
```
((a - b)*Log[1 - Cos[x]] - (a + b)*Log[1 + Cos[x]] + 2*b*Log[a + b*Cos[x]]
)/(2*(a - b)*(a + b))
```

## Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.25, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 3147, 477, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(x)}{a+b\cos(x)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\cos\left(x-\frac{\pi}{2}\right)\left(a-b\sin\left(x-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \; 3147$$

$$-b\int \frac{1}{(a+b\cos(x))\left(b^2-b^2\cos^2(x)\right)}d(b\cos(x))$$

$$\Big\downarrow \; 477$$

$$-\frac{\int \left(-\frac{b^2}{(a^2-b^2)(a+b\cos(x))}+\frac{b}{2(a+b)(b-b\cos(x))}+\frac{b}{2(a-b)(\cos(x)b+b)}\right)d(b\cos(x))}{b}$$

$$\Big\downarrow \; 2009$$

$$-\frac{-\frac{b^2\log(a+b\cos(x))}{a^2-b^2}-\frac{b\log(b-b\cos(x))}{2(a+b)}+\frac{b\log(b\cos(x)+b)}{2(a-b)}}{b}$$

input
```
Int[Csc[x]/(a + b*Cos[x]),x]
```

output
```
-((-1/2*(b*Log[b - b*Cos[x]])/(a + b) - (b^2*Log[a + b*Cos[x]])/(a^2 - b^2
) + (b*Log[b + b*Cos[x]])/(2*(a - b)))/b)
```

### Defintions of rubi rules used

rule 477
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ExpandIntegrand[(c + d*x)^n*(1 - Rt[-b/a, 2]*x)^p*(1 + Rt[-b/a, 2
]*x)^p, x], x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[n] &
& NiceSqrtQ[-b/a] &&  !FractionalPowerFactorQ[Rt[-b/a, 2]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

## Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

| method | result | size |
|---|---|---|
| parallelrisch | $\frac{b\ln\left(2b+\sec\left(\frac{x}{2}\right)^{2}(a-b)\right)+\ln\left(\tan\left(\frac{x}{2}\right)\right)(a-b)}{a^{2}-b^{2}}$ | 44 |
| norman | $\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a+b}+\frac{b\ln\left(a\tan\left(\frac{x}{2}\right)^{2}-b\tan\left(\frac{x}{2}\right)^{2}+a+b\right)}{a^{2}-b^{2}}$ | 47 |
| default | $-\frac{\ln(\cos(x)+1)}{2a-2b}+\frac{\ln(-1+\cos(x))}{2a+2b}+\frac{b\ln(a+b\cos(x))}{(a-b)(a+b)}$ | 54 |
| risch | $\frac{ix}{a-b}-\frac{ix}{a+b}-\frac{2ixb}{a^{2}-b^{2}}-\frac{\ln(\mathrm{e}^{ix}+1)}{a-b}+\frac{\ln(\mathrm{e}^{ix}-1)}{a+b}+\frac{b\ln\left(\mathrm{e}^{2ix}+\frac{2a\,\mathrm{e}^{ix}}{b}+1\right)}{a^{2}-b^{2}}$ | 101 |

input
```
int(csc(x)/(a+b*cos(x)),x,method=_RETURNVERBOSE)
```

output
```
(b*ln(2*b+sec(1/2*x)^2*(a-b))+ln(tan(1/2*x))*(a-b))/(a^2-b^2)
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

$$\int \frac{\csc(x)}{a+b\cos(x)}\,dx = \frac{2\,b\log\left(-b\cos\left(x\right)-a\right)-\left(a+b\right)\log\left(\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)+\left(a-b\right)\log\left(-\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)}{2\left(a^2-b^2\right)}$$

input
```
integrate(csc(x)/(a+b*cos(x)),x, algorithm="fricas")
```

output
```
1/2*(2*b*log(-b*cos(x) - a) - (a + b)*log(1/2*cos(x) + 1/2) + (a - b)*log(
-1/2*cos(x) + 1/2))/(a^2 - b^2)
```

## Sympy [F]

$$\int \frac{\csc(x)}{a+b\cos(x)}\,dx = \int \frac{\csc\left(x\right)}{a+b\cos\left(x\right)}\,dx$$

input
```
integrate(csc(x)/(a+b*cos(x)),x)
```

output
```
Integral(csc(x)/(a + b*cos(x)), x)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.89

$$\int \frac{\csc(x)}{a+b\cos(x)}\,dx = \frac{b\log\left(b\cos\left(x\right)+a\right)}{a^2-b^2}-\frac{\log\left(\cos\left(x\right)+1\right)}{2\left(a-b\right)}+\frac{\log\left(\cos\left(x\right)-1\right)}{2\left(a+b\right)}$$

input
```
integrate(csc(x)/(a+b*cos(x)),x, algorithm="maxima")
```

output
```
b*log(b*cos(x) + a)/(a^2 - b^2) - 1/2*log(cos(x) + 1)/(a - b) + 1/2*log(co
s(x) - 1)/(a + b)
```

## Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int \frac{\csc(x)}{a+b\cos(x)}\,dx = \frac{b^2\log\left(|b\cos\left(x\right)+a|\right)}{a^2b-b^3} - \frac{\log\left(\cos\left(x\right)+1\right)}{2\left(a-b\right)} + \frac{\log\left(-\cos\left(x\right)+1\right)}{2\left(a+b\right)}$$

input
```
integrate(csc(x)/(a+b*cos(x)),x, algorithm="giac")
```

output
```
b^2*log(abs(b*cos(x) + a))/(a^2*b - b^3) - 1/2*log(cos(x) + 1)/(a - b) + 1
/2*log(-cos(x) + 1)/(a + b)
```

## Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

$$\int \frac{\csc(x)}{a+b\cos(x)}\,dx = \frac{\ln\left(\cos\left(x\right)-1\right)}{2\ \left(a+b\right)} - \frac{\ln\left(\cos\left(x\right)+1\right)}{2\ \left(a-b\right)} + \frac{b\ \ln\left(a+b\ \cos\left(x\right)\right)}{a^2-b^2}$$

input
```
int(1/(sin(x)*(a + b*cos(x))),x)
```

output
```
log(cos(x) - 1)/(2*(a + b)) - log(cos(x) + 1)/(2*(a - b)) + (b*log(a + b*c
os(x)))/(a^2 - b^2)
```

## Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{\csc(x)}{a+b\cos(x)}\,dx = \frac{\log\left(\tan\left(\frac{x}{2}\right)^2 a-\tan\left(\frac{x}{2}\right)^2 b+a+b\right) b+\log\left(\tan\left(\frac{x}{2}\right)\right) a-\log\left(\tan\left(\frac{x}{2}\right)\right) b}{a^2-b^2}$$

input
```
int(csc(x)/(a+b*cos(x)),x)
```

output
```
(log(tan(x/2)**2*a - tan(x/2)**2*b + a + b)*b + log(tan(x/2))*a - log(tan(
x/2))*b)/(a**2 - b**2)
```

# 3.30 $\int \frac{\csc^2(x)}{a+b\cos(x)}\,dx$



## Optimal result

Integrand size = 13, antiderivative size = 67

$$\int \frac{\csc^2(x)}{a+b\cos(x)}\,dx = -\frac{2b^2\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}+\frac{(b-a\cos(x))\csc(x)}{a^2-b^2}$$

output
```
-2*b^2*arctan((a-b)^(1/2)*tan(1/2*x)/(a+b)^(1/2))/(a-b)^(3/2)/(a+b)^(3/2)+
(b-a*cos(x))*csc(x)/(a^2-b^2)
```

## Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.99

$$\int \frac{\csc^2(x)}{a+b\cos(x)}\,dx = -\frac{2b^2\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{3/2}}+\frac{(b-a\cos(x))\csc(x)}{a^2-b^2}$$

input
```
Integrate[Csc[x]^2/(a + b*Cos[x]),x]
```

output
```
(-2*b^2*ArcTanh[((a - b)*Tan[x/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2) +
 ((b - a*Cos[x])*Csc[x])/(a^2 - b^2)
```

## Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3042, 3175, 27, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(x)}{a+b\cos(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos\left(x-\frac{\pi}{2}\right)^2\left(a-b\sin\left(x-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3175}$$

$$\frac{\csc(x)(b-a\cos(x))}{a^2-b^2}-\frac{\int \frac{b^2}{a+b\cos(x)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\csc(x)(b-a\cos(x))}{a^2-b^2}-\frac{b^2\int \frac{1}{a+b\cos(x)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\csc(x)(b-a\cos(x))}{a^2-b^2}-\frac{b^2\int \frac{1}{a+b\sin\left(x+\frac{\pi}{2}\right)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{3138}$$

$$\frac{\csc(x)(b-a\cos(x))}{a^2-b^2}-\frac{2b^2\int \frac{1}{(a-b)\tan^2\left(\frac{x}{2}\right)+a+b}d\tan\left(\frac{x}{2}\right)}{a^2-b^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{\csc(x)(b-a\cos(x))}{a^2-b^2}-\frac{2b^2\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}\,(a^2-b^2)}$$

input
```
Int[Csc[x]^2/(a + b*Cos[x]),x]
```

output
```
(-2*b^2*ArcTan[(Sqrt[a - b]*Tan[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a +
b]*(a^2 - b^2)) + ((b - a*Cos[x])*Csc[x])/(a^2 - b^2)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3175
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Simp[1/(g^2*
(a^2 - b^2)*(p + 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*
(a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; F
reeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ
[2*m, 2*p]
```

## Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.16

| method | result | size |
|---|---|---|
| default | $\frac{\tan\left(\frac{x}{2}\right)}{2a-2b}-\frac{1}{2(a+b)\tan\left(\frac{x}{2}\right)}-\frac{2b^{2}\arctan\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}$ | 78 |
| risch | $-\frac{2i\left(-e^{ix}b+a\right)}{\left(e^{2ix}-1\right)\left(a^{2}-b^{2}\right)}+\frac{b^{2}\ln\left(e^{ix}+\frac{ia^{2}-ib^{2}+a\sqrt{-a^{2}+b^{2}}}{b\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}\,(a+b)(a-b)}-\frac{b^{2}\ln\left(e^{ix}+\frac{-ia^{2}+ib^{2}+a\sqrt{-a^{2}+b^{2}}}{\sqrt{-a^{2}+b^{2}}\,b}\right)}{\sqrt{-a^{2}+b^{2}}\,(a+b)(a-b)}$ | 186 |

input
```
int(csc(x)^2/(a+b*cos(x)),x,method=_RETURNVERBOSE)
```

output
```
1/2/(a-b)*tan(1/2*x)-1/2/(a+b)/tan(1/2*x)-2/(a-b)*b^2/(a+b)/((a-b)*(a+b))^
(1/2)*arctan((a-b)*tan(1/2*x)/((a-b)*(a+b))^(1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 230, normalized size of antiderivative = 3.43

$$\int \frac{\csc^2(x)}{a+b\cos(x)}\,dx$$
$$=\left[\frac{\sqrt{-a^2+b^2}b^2\log\left(\frac{2\,ab\cos(x)+\left(2\,a^2-b^2\right)\cos(x)^2+2\,\sqrt{-a^2+b^2}(a\cos(x)+b)\sin(x)-a^2+2\,b^2}{b^2\cos(x)^2+2\,ab\cos(x)+a^2}\right)\sin(x)+2\,a^2b-2\,b^3-2\,(a^3}{2\,(a^4-2\,a^2b^2+b^4)\sin(x)}\right.$$
$$\left.-\frac{\sqrt{a^2-b^2}b^2\arctan\left(-\frac{a\cos(x)+b}{\sqrt{a^2-b^2}\sin(x)}\right)\sin(x)-a^2b+b^3+(a^3-ab^2)\cos(x)}{(a^4-2\,a^2b^2+b^4)\sin(x)}\right]$$

input
```
integrate(csc(x)^2/(a+b*cos(x)),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(-a^2 + b^2)*b^2*log((2*a*b*cos(x) + (2*a^2 - b^2)*cos(x)^2 + 2*
sqrt(-a^2 + b^2)*(a*cos(x) + b)*sin(x) - a^2 + 2*b^2)/(b^2*cos(x)^2 + 2*a*
b*cos(x) + a^2))*sin(x) + 2*a^2*b - 2*b^3 - 2*(a^3 - a*b^2)*cos(x))/((a^4
- 2*a^2*b^2 + b^4)*sin(x)), -(sqrt(a^2 - b^2)*b^2*arctan(-(a*cos(x) + b)/(
sqrt(a^2 - b^2)*sin(x)))*sin(x) - a^2*b + b^3 + (a^3 - a*b^2)*cos(x))/((a^
4 - 2*a^2*b^2 + b^4)*sin(x))]
```

## Sympy [F]

$$\int \frac{\csc^2(x)}{a+b\cos(x)}\,dx = \int \frac{\csc^2{(x)}}{a+b\cos{(x)}}\,dx$$

input
```
integrate(csc(x)**2/(a+b*cos(x)),x)
```

output
```
Integral(csc(x)**2/(a + b*cos(x)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\csc^2(x)}{a+b\cos(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(csc(x)^2/(a+b*cos(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

## Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.36

$$\int \frac{\csc^2(x)}{a+b\cos(x)}\,dx = \frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}x\right)-b\tan\left(\frac{1}{2}x\right)}{\sqrt{a^2-b^2}}\right)\right)b^2}{\left(a^2-b^2\right)^{\frac{3}{2}}} + \frac{\tan\left(\frac{1}{2}x\right)}{2(a-b)} - \frac{1}{2(a+b)\tan\left(\frac{1}{2}x\right)}$$

input
```
integrate(csc(x)^2/(a+b*cos(x)),x, algorithm="giac")
```

output
```
2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x) - b*ta
n(1/2*x))/sqrt(a^2 - b^2)))*b^2/(a^2 - b^2)^(3/2) + 1/2*tan(1/2*x)/(a - b)
 - 1/2/((a + b)*tan(1/2*x))
```

## Mupad [B] (verification not implemented)

Time = 44.81 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.28

$$\int \frac{\csc^2(x)}{a+b\cos(x)}\,dx = \frac{\tan\left(\frac{x}{2}\right)}{2a-2b} - \frac{2b^2\,\mathrm{atan}\left(\frac{\tan\left(\frac{x}{2}\right)\left(a^2-b^2\right)}{(a+b)^{3/2}\sqrt{a-b}}\right)}{(a+b)^{3/2}(a-b)^{3/2}} - \frac{a-b}{\tan\left(\frac{x}{2}\right)(a+b)(2a-2b)}$$

input
```
int(1/(sin(x)^2*(a + b*cos(x))),x)
```

output
```
tan(x/2)/(2*a - 2*b) - (2*b^2*atan((tan(x/2)*(a^2 - b^2))/((a + b)^(3/2)*(
a - b)^(1/2))))/((a + b)^(3/2)*(a - b)^(3/2)) - (a - b)/(tan(x/2)*(a + b)*
(2*a - 2*b))
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.37

$$\int \frac{\csc^2(x)}{a+b\cos(x)}\,dx$$
$$= \frac{-2\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{x}{2}\right)a-\tan\left(\frac{x}{2}\right)b}{\sqrt{a^2-b^2}}\right)\sin(x)\,b^2-\cos(x)\,a^3+\cos(x)\,a\,b^2+a^2b-b^3}{\sin(x)\,(a^4-2a^2b^2+b^4)}$$

input
```
int(csc(x)^2/(a+b*cos(x)),x)
```

output
```
( - 2*sqrt(a**2 - b**2)*atan((tan(x/2)*a - tan(x/2)*b)/sqrt(a**2 - b**2))*
sin(x)*b**2 - cos(x)*a**3 + cos(x)*a*b**2 + a**2*b - b**3)/(sin(x)*(a**4 -
 2*a**2*b**2 + b**4))
```

# 3.31 $\int \frac{\csc^3(x)}{a+b\cos(x)}\,dx$



## Optimal result

Integrand size = 13, antiderivative size = 100

$$\int \frac{\csc^3(x)}{a+b\cos(x)}\,dx = -\frac{1}{4(a+b)(1-\cos(x))}+\frac{1}{4(a-b)(1+\cos(x))}+\frac{(a+2b)\log(1-\cos(x))}{4(a+b)^2}-\frac{(a-2b)\log(1+\cos(x))}{4(a-b)^2}-\frac{b^3\log(a+b\cos(x))}{(a^2-b^2)^2}$$

output
```
-1/4/(a+b)/(1-cos(x))+1/4/(a-b)/(1+cos(x))+1/4*(a+2*b)*ln(1-cos(x))/(a+b)^
2-1/4*(a-2*b)*ln(1+cos(x))/(a-b)^2-b^3*ln(a+b*cos(x))/(a^2-b^2)^2
```

## Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.99

$$\int \frac{\csc^3(x)}{a+b\cos(x)}\,dx = \frac{1}{8}\left(-\frac{\csc^2\left(\frac{x}{2}\right)}{a+b}-\frac{4(a-2b)\log\left(\cos\left(\frac{x}{2}\right)\right)}{(a-b)^2}-\frac{8b^3\log(a+b\cos(x))}{(a^2-b^2)^2}+\frac{4(a+2b)\log\left(\sin\left(\frac{x}{2}\right)\right)}{(a+b)^2}+\frac{\sec^2\left(\frac{x}{2}\right)}{a-b}\right)$$

input
```
Integrate[Csc[x]^3/(a + b*Cos[x]),x]
```

output
```
(-(Csc[x/2]^2/(a + b)) - (4*(a - 2*b)*Log[Cos[x/2]])/(a - b)^2 - (8*b^3*Lo
g[a + b*Cos[x]])/(a^2 - b^2)^2 + (4*(a + 2*b)*Log[Sin[x/2]])/(a + b)^2 + S
ec[x/2]^2/(a - b))/8
```

## Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 3147, 477, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(x)}{a+b\cos(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos\left(x-\frac{\pi}{2}\right)^3\left(a-b\sin\left(x-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3147}$$

$$-b^3\int \frac{1}{(a+b\cos(x))\left(b^2-b^2\cos^2(x)\right)^2}d(b\cos(x))$$

$$\Big\downarrow \text{477}$$

$$-\frac{\int \left(\frac{b^4}{\left(a^2-b^2\right)^2(a+b\cos(x))}+\frac{b^2}{4(a+b)(b-b\cos(x))^2}+\frac{b^2}{4(a-b)(\cos(x)b+b)^2}+\frac{(a+2b)b}{4(a+b)^2(b-b\cos(x))}+\frac{(a-2b)b}{4(a-b)^2(\cos(x)b+b)}\right)d(b\cos(x))}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{b^4\log(a+b\cos(x))}{\left(a^2-b^2\right)^2}+\frac{b^2}{4(a+b)(b-b\cos(x))}-\frac{b^2}{4(a-b)(b\cos(x)+b)}-\frac{b(a+2b)\log(b-b\cos(x))}{4(a+b)^2}+\frac{b(a-2b)\log(b\cos(x)+b)}{4(a-b)^2}}{b}$$

input
```
Int[Csc[x]^3/(a + b*Cos[x]),x]
```

output
```
-((b^2/(4*(a + b)*(b - b*Cos[x])) - b^2/(4*(a - b)*(b + b*Cos[x])) - (b*(a
 + 2*b)*Log[b - b*Cos[x]])/(4*(a + b)^2) + (b^4*Log[a + b*Cos[x]])/(a^2 -
b^2)^2 + ((a - 2*b)*b*Log[b + b*Cos[x]])/(4*(a - b)^2))/b)
```

### Defintions of rubi rules used

rule 477
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ExpandIntegrand[(c + d*x)^n*(1 - Rt[-b/a, 2]*x)^p*(1 + Rt[-b/a, 2
]*x)^p, x], x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[n] &
& NiceSqrtQ[-b/a] &&  !FractionalPowerFactorQ[Rt[-b/a, 2]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

## Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.91

| method | result |
|---|---|
| parallelrisch | $\frac{-8b^3\ln\left(2b+\sec\left(\frac{x}{2}\right)^2(a-b)\right)-\left(-4(a+2b)(a-b)\ln\left(\tan\left(\frac{x}{2}\right)\right)+\left((a-b)\cot\left(\frac{x}{2}\right)^2-\tan\left(\frac{x}{2}\right)^2(a+b)\right)(a+b)\right)(a-b)}{8(a-b)^2(a+b)^2}$ |
| default | $\frac{1}{(4a-4b)(\cos(x)+1)}+\frac{(-a+2b)\ln(\cos(x)+1)}{4(a-b)^2}-\frac{b^3\ln(a+b\cos(x))}{(a+b)^2(a-b)^2}+\frac{1}{(4a+4b)(-1+\cos(x))}+\frac{(a+2b)\ln(-1+\cos(x))}{4(a+b)^2}$ |
| norman | $\frac{-\frac{1}{8(a+b)}+\frac{\tan\left(\frac{x}{2}\right)^4}{8a-8b}}{\tan\left(\frac{x}{2}\right)^2}-\frac{b^3\ln\left(a\tan\left(\frac{x}{2}\right)^2-b\tan\left(\frac{x}{2}\right)^2+a+b\right)}{a^4-2a^2b^2+b^4}+\frac{(a+2b)\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2a^2+4ab+2b^2}$ |
| risch | $-\frac{ixa}{2(a^2+2ab+b^2)}-\frac{ixb}{a^2+2ab+b^2}+\frac{ixa}{2a^2-4ab+2b^2}-\frac{ixb}{a^2-2ab+b^2}+\frac{2ixb^3}{a^4-2a^2b^2+b^4}-\frac{e^{3ix}a-2e^{2ix}b+ae^{ix}}{(e^{2ix}-1)^2(-a^2+b^2)}+\frac{\ln(e^{i}}{2a^2+4}$ |

input
```
int(csc(x)^3/(a+b*cos(x)),x,method=_RETURNVERBOSE)
```

output
```
1/8*(-8*b^3*ln(2*b+sec(1/2*x)^2*(a-b))-(-4*(a+2*b)*(a-b)*ln(tan(1/2*x))+((
a-b)*cot(1/2*x)^2-tan(1/2*x)^2*(a+b))*(a+b))*(a-b))/(a-b)^2/(a+b)^2
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. 2(90) = 180.

Time = 0.12 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.81

$$\int \frac{\csc^3(x)}{a+b\cos(x)}\,dx = \frac{2a^2b-2b^3-2(a^3-ab^2)\cos(x)+4(b^3\cos(x)^2-b^3)\log(-b\cos(x)-a)-(a^3-3ab^2-2b^3-(a^3-\ldots}{4(a^4-2a^2b^2+b^4\ldots}$$

input
```
integrate(csc(x)^3/(a+b*cos(x)),x, algorithm="fricas")
```

output
```
1/4*(2*a^2*b - 2*b^3 - 2*(a^3 - a*b^2)*cos(x) + 4*(b^3*cos(x)^2 - b^3)*log
(-b*cos(x) - a) - (a^3 - 3*a*b^2 - 2*b^3 - (a^3 - 3*a*b^2 - 2*b^3)*cos(x)^
2)*log(1/2*cos(x) + 1/2) + (a^3 - 3*a*b^2 + 2*b^3 - (a^3 - 3*a*b^2 + 2*b^3
)*cos(x)^2)*log(-1/2*cos(x) + 1/2))/(a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^2*
b^2 + b^4)*cos(x)^2)
```

## Sympy [F]

$$\int \frac{\csc^3(x)}{a+b\cos(x)}\,dx = \int \frac{\csc^3(x)}{a+b\cos(x)}\,dx$$

input
```
integrate(csc(x)**3/(a+b*cos(x)),x)
```

output
```
Integral(csc(x)**3/(a + b*cos(x)), x)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.15

$$\int \frac{\csc^3(x)}{a+b\cos(x)}\,dx = -\frac{b^3\log\left(b\cos\left(x\right)+a\right)}{a^4-2\,a^2b^2+b^4} - \frac{(a-2\,b)\log\left(\cos\left(x\right)+1\right)}{4\,(a^2-2\,ab+b^2)} + \frac{(a+2\,b)\log\left(\cos\left(x\right)-1\right)}{4\,(a^2+2\,ab+b^2)} + \frac{a\cos\left(x\right)-b}{2\,\left((a^2-b^2)\cos\left(x\right)^2-a^2+b^2\right)}$$

input
```
integrate(csc(x)^3/(a+b*cos(x)),x, algorithm="maxima")
```

output
```
-b^3*log(b*cos(x) + a)/(a^4 - 2*a^2*b^2 + b^4) - 1/4*(a - 2*b)*log(cos(x)
+ 1)/(a^2 - 2*a*b + b^2) + 1/4*(a + 2*b)*log(cos(x) - 1)/(a^2 + 2*a*b + b^
2) + 1/2*(a*cos(x) - b)/((a^2 - b^2)*cos(x)^2 - a^2 + b^2)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.36

$$\int \frac{\csc^3(x)}{a+b\cos(x)}\,dx = -\frac{b^4\log\left(|b\cos\left(x\right)+a|\right)}{a^4b-2\,a^2b^3+b^5} - \frac{(a-2\,b)\log\left(\cos\left(x\right)+1\right)}{4\,(a^2-2\,ab+b^2)} + \frac{(a+2\,b)\log\left(-\cos\left(x\right)+1\right)}{4\,(a^2+2\,ab+b^2)} - \frac{a^2b-b^3-(a^3-ab^2)\cos\left(x\right)}{2\,(a+b)^2(a-b)^2(\cos\left(x\right)+1)(\cos\left(x\right)-1)}$$

input
```
integrate(csc(x)^3/(a+b*cos(x)),x, algorithm="giac")
```

output
```
-b^4*log(abs(b*cos(x) + a))/(a^4*b - 2*a^2*b^3 + b^5) - 1/4*(a - 2*b)*log(
cos(x) + 1)/(a^2 - 2*a*b + b^2) + 1/4*(a + 2*b)*log(-cos(x) + 1)/(a^2 + 2*
a*b + b^2) - 1/2*(a^2*b - b^3 - (a^3 - a*b^2)*cos(x))/((a + b)^2*(a - b)^2
*(cos(x) + 1)*(cos(x) - 1))
```

## Mupad [B] (verification not implemented)

Time = 43.16 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.12

$$\int \frac{\csc^3(x)}{a+b\cos(x)}\,dx = \ln(\cos(x)-1)\left(\frac{b}{4(a+b)^2}+\frac{1}{4(a+b)}\right)+\frac{\frac{b}{2(a^2-b^2)}-\frac{a\cos(x)}{2(a^2-b^2)}}{\sin(x)^2} - \frac{b^3\ln(a+b\cos(x))}{a^4-2a^2b^2+b^4}-\frac{\ln(\cos(x)+1)(a-2b)}{4(a-b)^2}$$

input
```
int(1/(sin(x)^3*(a + b*cos(x))),x)
```

output
```
log(cos(x) - 1)*(b/(4*(a + b)^2) + 1/(4*(a + b))) + (b/(2*(a^2 - b^2)) - (
a*cos(x))/(2*(a^2 - b^2)))/sin(x)^2 - (b^3*log(a + b*cos(x)))/(a^4 + b^4 -
 2*a^2*b^2) - (log(cos(x) + 1)*(a - 2*b))/(4*(a - b)^2)
```

## Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.41

$$\int \frac{\csc^3(x)}{a+b\cos(x)}\,dx = \frac{-2\cos(x)a^3+2\cos(x)ab^2-4\log\left(\tan\left(\frac{x}{2}\right)^2a-\tan\left(\frac{x}{2}\right)^2b+a+b\right)\sin(x)^2b^3+2\log\left(\tan\left(\frac{x}{2}\right)\right)\sin(x)}{4\sin(x)^2(a^4-}$$

input
```
int(csc(x)^3/(a+b*cos(x)),x)
```

output
```
( - 2*cos(x)*a**3 + 2*cos(x)*a*b**2 - 4*log(tan(x/2)**2*a - tan(x/2)**2*b
+ a + b)*sin(x)**2*b**3 + 2*log(tan(x/2))*sin(x)**2*a**3 - 6*log(tan(x/2))
*sin(x)**2*a*b**2 + 4*log(tan(x/2))*sin(x)**2*b**3 - sin(x)**2*a**2*b + si
n(x)**2*b**3 + 2*a**2*b - 2*b**3)/(4*sin(x)**2*(a**4 - 2*a**2*b**2 + b**4)
)
```

# 3.32 $\int \frac{\csc^4(x)}{a+b\cos(x)}\,dx$



## Optimal result

Integrand size = 13, antiderivative size = 110

$$\int \frac{\csc^4(x)}{a+b\cos(x)}\,dx = \frac{2b^4\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}} - \frac{(3b^3+a(2a^2-5b^2)\cos(x))\csc(x)}{3\left(a^2-b^2\right)^2} + \frac{(b-a\cos(x))\csc^3(x)}{3\left(a^2-b^2\right)}$$

output
```
2*b^4*arctan((a-b)^(1/2)*tan(1/2*x)/(a+b)^(1/2))/(a-b)^(5/2)/(a+b)^(5/2)-1
/3*(3*b^3+a*(2*a^2-5*b^2)*cos(x))*csc(x)/(a^2-b^2)^2+(b-a*cos(x))*csc(x)^3
/(3*a^2-3*b^2)
```

## Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.02

$$\int \frac{\csc^4(x)}{a+b\cos(x)}\,dx = -\frac{2b^4\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\left(-a^2+b^2\right)^{5/2}} + \frac{\left((-6a^3+9ab^2)\cos(x)+6b^3\cos(2x)+\left(2a^2-5b^2\right)\left(2b+a\cos(3x)\right)\right)\csc^3(x)}{12(a-b)^2(a+b)^2}$$

input
```
Integrate[Csc[x]^4/(a + b*Cos[x]),x]
```

output
```
(-2*b^4*ArcTanh[((a - b)*Tan[x/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(5/2) +
 (((-6*a^3 + 9*a*b^2)*Cos[x] + 6*b^3*Cos[2*x] + (2*a^2 - 5*b^2)*(2*b + a*C
os[3*x]))*Csc[x]^3)/(12*(a - b)^2*(a + b)^2)
```

## Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.23, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {3042, 3175, 25, 3042, 3345, 27, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^4(x)}{a+b\cos(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos\left(x-\frac{\pi}{2}\right)^4\left(a-b\sin\left(x-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3175}$$

$$\frac{\csc^3(x)(b-a\cos(x))}{3\left(a^2-b^2\right)}-\frac{\int -\frac{\left(2a^2+2b\cos(x)a-3b^2\right)\csc^2(x)}{a+b\cos(x)}dx}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\left(2a^2+2b\cos(x)a-3b^2\right)\csc^2(x)}{a+b\cos(x)}dx}{3\left(a^2-b^2\right)}+\frac{\csc^3(x)(b-a\cos(x))}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{2a^2-2b\sin\left(x-\frac{\pi}{2}\right)a-3b^2}{\cos\left(x-\frac{\pi}{2}\right)^2\left(a-b\sin\left(x-\frac{\pi}{2}\right)\right)}dx}{3\left(a^2-b^2\right)}+\frac{\csc^3(x)(b-a\cos(x))}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3345}$$

$$\frac{-\frac{\int -\frac{3b^4}{a+b\cos(x)}dx}{a^2-b^2}-\frac{\csc(x)\left(a\left(2a^2-5b^2\right)\cos(x)+3b^3\right)}{a^2-b^2}}{3\left(a^2-b^2\right)}+\frac{\csc^3(x)(b-a\cos(x))}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3b^4\int \frac{1}{a+b\cos(x)}dx}{a^2-b^2}-\frac{\csc(x)\left(a\left(2a^2-5b^2\right)\cos(x)+3b^3\right)}{a^2-b^2}}{3\left(a^2-b^2\right)}+\frac{\csc^3(x)(b-a\cos(x))}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3b^4\int \frac{1}{a+b\sin\left(x+\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{\csc(x)\left(a\left(2a^2-5b^2\right)\cos(x)+3b^3\right)}{a^2-b^2}}{3\left(a^2-b^2\right)}+\frac{\csc^3(x)(b-a\cos(x))}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3138}$$

$$\frac{\frac{6b^4\int \frac{1}{(a-b)\tan^2\left(\frac{x}{2}\right)+a+b}d\tan\left(\frac{x}{2}\right)}{a^2-b^2}-\frac{\csc(x)\left(a\left(2a^2-5b^2\right)\cos(x)+3b^3\right)}{a^2-b^2}}{3\left(a^2-b^2\right)}+\frac{\csc^3(x)(b-a\cos(x))}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{6b^4\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}\left(a^2-b^2\right)}-\frac{\csc(x)\left(a\left(2a^2-5b^2\right)\cos(x)+3b^3\right)}{a^2-b^2}}{3\left(a^2-b^2\right)}+\frac{\csc^3(x)(b-a\cos(x))}{3\left(a^2-b^2\right)}$$

input
```
Int[Csc[x]^4/(a + b*Cos[x]),x]
```

output
```
((b - a*Cos[x])*Csc[x]^3)/(3*(a^2 - b^2)) + ((6*b^4*ArcTan[(Sqrt[a - b]*Ta
n[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b]*(a^2 - b^2)) - ((3*b^3 + a*
(2*a^2 - 5*b^2)*Cos[x])*Csc[x])/(a^2 - b^2))/(3*(a^2 - b^2))
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3175
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Simp[1/(g^2*
(a^2 - b^2)*(p + 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*
(a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; F
reeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ
[2*m, 2*p]
```

rule 3345
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Simp[1/(g^2*(a^2 - b^2)*(p +
 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
 2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[p, -1] && IntegerQ[2*m]
```

## Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.15

| method | result |
|---|---|
| default | $\frac{\frac{\tan\left(\frac{x}{2}\right)^{3} a}{3}-\frac{b\tan\left(\frac{x}{2}\right)^{3}}{3}+3a\tan\left(\frac{x}{2}\right)-5b\tan\left(\frac{x}{2}\right)}{8(a-b)^{2}}-\frac{1}{24(a+b)\tan\left(\frac{x}{2}\right)^{3}}-\frac{3a+5b}{8(a+b)^{2}\tan\left(\frac{x}{2}\right)}+\frac{2b^{4}\arctan\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)^{2}(a+b)^{2}\sqrt{(a-b)(a+b)}}$ |
| risch | $-\frac{2i\left(3b^{3}e^{5ix}-3a\,b^{2}e^{4ix}+4a^{2}b\,e^{3ix}-10b^{3}e^{3ix}-6a^{3}e^{2ix}+12a\,b^{2}e^{2ix}+3b^{3}e^{ix}+2a^{3}-5b^{2}a\right)}{3\left(a^{4}-2a^{2}b^{2}+b^{4}\right)\left(e^{2ix}-1\right)^{3}}-\frac{b^{4}\ln\left(e^{ix}+\frac{ia^{2}-ib^{2}+a\sqrt{-a^{2}+b^{2}}}{b\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}\,(a+b)^{2}(a-b)^{2}}+$ |

input
```
int(csc(x)^4/(a+b*cos(x)),x,method=_RETURNVERBOSE)
```

output
```
1/8/(a-b)^2*(1/3*tan(1/2*x)^3*a-1/3*b*tan(1/2*x)^3+3*a*tan(1/2*x)-5*b*tan(
1/2*x))-1/24/(a+b)/tan(1/2*x)^3-1/8*(3*a+5*b)/(a+b)^2/tan(1/2*x)+2/(a-b)^2
/(a+b)^2*b^4/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*x)/((a-b)*(a+b))^(1/
2))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(97) = 194.

Time = 0.11 (sec) , antiderivative size = 459, normalized size of antiderivative = 4.17

$$\int \frac{\csc^4(x)}{a+b\cos(x)}\,dx$$
$$=\left[\frac{2\,a^4b-10\,a^2b^3+8\,b^5+2\left(2\,a^5-7\,a^3b^2+5\,ab^4\right)\cos\left(x\right)^3+3\left(b^4\cos\left(x\right)^2-b^4\right)\sqrt{-a^2+b^2}\log\left(\frac{2\,ab\cos(x)-}{}\right.}{6\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6-(a^6\right.}\right.$$

input
```
integrate(csc(x)^4/(a+b*cos(x)),x, algorithm="fricas")
```

output
```
[1/6*(2*a^4*b - 10*a^2*b^3 + 8*b^5 + 2*(2*a^5 - 7*a^3*b^2 + 5*a*b^4)*cos(x
)^3 + 3*(b^4*cos(x)^2 - b^4)*sqrt(-a^2 + b^2)*log((2*a*b*cos(x) + (2*a^2 -
 b^2)*cos(x)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(x) + b)*sin(x) - a^2 + 2*b^2)/(
b^2*cos(x)^2 + 2*a*b*cos(x) + a^2))*sin(x) + 6*(a^2*b^3 - b^5)*cos(x)^2 -
6*(a^5 - 3*a^3*b^2 + 2*a*b^4)*cos(x))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6
- (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cos(x)^2)*sin(x)), 1/3*(a^4*b - 5*a^
2*b^3 + 4*b^5 + (2*a^5 - 7*a^3*b^2 + 5*a*b^4)*cos(x)^3 - 3*(b^4*cos(x)^2 -
 b^4)*sqrt(a^2 - b^2)*arctan(-(a*cos(x) + b)/(sqrt(a^2 - b^2)*sin(x)))*sin
(x) + 3*(a^2*b^3 - b^5)*cos(x)^2 - 3*(a^5 - 3*a^3*b^2 + 2*a*b^4)*cos(x))/(
(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 - (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*c
os(x)^2)*sin(x))]
```

## Sympy [F]

$$\int \frac{\csc^4(x)}{a+b\cos(x)}\,dx = \int \frac{\csc^4{(x)}}{a+b\cos{(x)}}\,dx$$

input
```
integrate(csc(x)**4/(a+b*cos(x)),x)
```

output
```
Integral(csc(x)**4/(a + b*cos(x)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\csc^4(x)}{a+b\cos(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(csc(x)^4/(a+b*cos(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(97) = 194$.

Time = 0.12 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.87

$$\int \frac{\csc^4(x)}{a+b\cos(x)}\,dx = -\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}x\right)-b\tan\left(\frac{1}{2}x\right)}{\sqrt{a^2-b^2}}\right)\right)b^4}{(a^4-2a^2b^2+b^4)\sqrt{a^2-b^2}} + \frac{a^2\tan\left(\frac{1}{2}x\right)^3-2ab\tan\left(\frac{1}{2}x\right)^3+b^2\tan\left(\frac{1}{2}x\right)^3+9a^2\tan\left(\frac{1}{2}x\right)-24ab\tan\left(\frac{1}{2}x\right)+15b^2\tan\left(\frac{1}{2}x\right)}{24(a^3-3a^2b+3ab^2-b^3)} - \frac{9a\tan\left(\frac{1}{2}x\right)^2+15b\tan\left(\frac{1}{2}x\right)^2+a+b}{24(a^2+2ab+b^2)\tan\left(\frac{1}{2}x\right)^3}$$

input
```
integrate(csc(x)^4/(a+b*cos(x)),x, algorithm="giac")
```

output
```
-2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x) - b*t
an(1/2*x))/sqrt(a^2 - b^2)))*b^4/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2))
 + 1/24*(a^2*tan(1/2*x)^3 - 2*a*b*tan(1/2*x)^3 + b^2*tan(1/2*x)^3 + 9*a^2*
tan(1/2*x) - 24*a*b*tan(1/2*x) + 15*b^2*tan(1/2*x))/(a^3 - 3*a^2*b + 3*a*b
^2 - b^3) - 1/24*(9*a*tan(1/2*x)^2 + 15*b*tan(1/2*x)^2 + a + b)/((a^2 + 2*
a*b + b^2)*tan(1/2*x)^3)
```

## Mupad [B] (verification not implemented)

Time = 42.89 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.67

$$\int \frac{\csc^4(x)}{a+b\cos(x)}\,dx = \tan\left(\frac{x}{2}\right)\left(\frac{4}{8a-8b}-\frac{8a+8b}{(8a-8b)^2}\right)+\frac{\tan\left(\frac{x}{2}\right)^3}{3(8a-8b)} - \frac{\frac{a^2-2ab+b^2}{3(a+b)}-\frac{\tan\left(\frac{x}{2}\right)^2(-3a^3+a^2b+7ab^2-5b^3)}{(a+b)^2}}{\tan\left(\frac{x}{2}\right)^3(8a^2-16ab+8b^2)} + \frac{2b^4\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)(a^4-2a^2b^2+b^4)}{(a+b)^{5/2}(a-b)^{3/2}}\right)}{(a+b)^{5/2}(a-b)^{5/2}}$$

input
```
int(1/(sin(x)^4*(a + b*cos(x))),x)
```

output
```
tan(x/2)*(4/(8*a - 8*b) - (8*a + 8*b)/(8*a - 8*b)^2) + tan(x/2)^3/(3*(8*a
- 8*b)) - ((a^2 - 2*a*b + b^2)/(3*(a + b)) - (tan(x/2)^2*(7*a*b^2 + a^2*b
- 3*a^3 - 5*b^3))/(a + b)^2)/(tan(x/2)^3*(8*a^2 - 16*a*b + 8*b^2)) + (2*b^
4*atan((tan(x/2)*(a^4 + b^4 - 2*a^2*b^2))/((a + b)^(5/2)*(a - b)^(3/2))))/
((a + b)^(5/2)*(a - b)^(5/2))
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.64

$$\int \frac{\csc^4(x)}{a+b\cos(x)}\,dx$$
$$= \frac{6\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{x}{2}\right)a-\tan\left(\frac{x}{2}\right)b}{\sqrt{a^2-b^2}}\right)\sin\left(x\right)^3 b^4-2\cos\left(x\right)\sin\left(x\right)^2 a^5+7\cos\left(x\right)\sin\left(x\right)^2 a^3b^2-5\cos\left(x\right)\sin\left(\right.}{3\sin\left(x\right)^3\left(a^6-3a^4b\right.}$$

input
```
int(csc(x)^4/(a+b*cos(x)),x)
```

output
```
(6*sqrt(a**2 - b**2)*atan((tan(x/2)*a - tan(x/2)*b)/sqrt(a**2 - b**2))*sin
(x)**3*b**4 - 2*cos(x)*sin(x)**2*a**5 + 7*cos(x)*sin(x)**2*a**3*b**2 - 5*c
os(x)*sin(x)**2*a*b**4 - cos(x)*a**5 + 2*cos(x)*a**3*b**2 - cos(x)*a*b**4
- 3*sin(x)**2*a**2*b**3 + 3*sin(x)**2*b**5 + a**4*b - 2*a**2*b**3 + b**5)/
(3*sin(x)**3*(a**6 - 3*a**4*b**2 + 3*a**2*b**4 - b**6))
```

# 3.33 $\int (a+b\cos(c+dx))(e\sin(c+dx))^{7/2}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 129

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{7/2}\,dx=\frac{10ae^4\operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{21d\sqrt{e\sin(c+dx)}}-\frac{10ae^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21d}-\frac{2ae\cos(c+dx)(e\sin(c+dx))^{5/2}}{7d}+\frac{2b(e\sin(c+dx))^{9/2}}{9de}$$

output
```
10/21*a*e^4*InverseJacobiAM(1/2*c-1/4*Pi+1/2*d*x,2^(1/2))*sin(d*x+c)^(1/2)
/d/(e*sin(d*x+c))^(1/2)-10/21*a*e^3*cos(d*x+c)*(e*sin(d*x+c))^(1/2)/d-2/7*
a*e*cos(d*x+c)*(e*sin(d*x+c))^(5/2)/d+2/9*b*(e*sin(d*x+c))^(9/2)/d/e
```

## Mathematica [A] (verified)

Time = 1.29 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.84

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{7/2}\,dx=\frac{e^3\left(-120a\,\text{EllipticF}\left(\frac{1}{4}(-2c+\pi-2dx),2\right)+(21b-138a\cos(c+dx)-28b\cos(2(c+dx))\right.}{252d\sqrt{\sin(c+dx)}}$$

input
```
Integrate[(a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(7/2),x]
```

output
```
(e^3*(-120*a*EllipticF[(-2*c + Pi - 2*d*x)/4, 2] + (21*b - 138*a*Cos[c + d
*x] - 28*b*Cos[2*(c + d*x)] + 18*a*Cos[3*(c + d*x)] + 7*b*Cos[4*(c + d*x)]
)*Sqrt[Sin[c + d*x]])*Sqrt[e*Sin[c + d*x]])/(252*d*Sqrt[Sin[c + d*x]])
```

## Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {3042, 3148, 3042, 3115, 3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e\sin(c+dx))^{7/2}(a+b\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{7/2}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3148}$$

$$a\int (e\sin(c+dx))^{7/2}dx+\frac{2b(e\sin(c+dx))^{9/2}}{9de}$$

$$\Big\downarrow \text{3042}$$

$$a\int (e\sin(c+dx))^{7/2}dx+\frac{2b(e\sin(c+dx))^{9/2}}{9de}$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{5}{7}e^2\int (e\sin(c+dx))^{3/2}dx-\frac{2e\cos(c+dx)(e\sin(c+dx))^{5/2}}{7d}\right)+\frac{2b(e\sin(c+dx))^{9/2}}{9de}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{5}{7}e^2\int (e\sin(c+dx))^{3/2}dx-\frac{2e\cos(c+dx)(e\sin(c+dx))^{5/2}}{7d}\right)+\frac{2b(e\sin(c+dx))^{9/2}}{9de}$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{5}{7}e^2\left(\frac{1}{3}e^2\int \frac{1}{\sqrt{e\sin(c+dx)}}dx-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)-\frac{2e\cos(c+dx)(e\sin(c+dx))^{5/2}}{7d}\right)+\frac{2b(e\sin(c+dx))^{9/2}}{9de}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{5}{7}e^2\left(\frac{1}{3}e^2\int \frac{1}{\sqrt{e\sin(c+dx)}}dx-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)-\frac{2e\cos(c+dx)(e\sin(c+dx))^{5/2}}{7d}\right)+\frac{2b(e\sin(c+dx))^{9/2}}{9de}$$

$$\Big\downarrow \text{3121}$$

$$a\left(\frac{5}{7}e^2\left(\frac{e^2\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}}dx}{3\sqrt{e\sin(c+dx)}}-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)-\frac{2e\cos(c+dx)(e\sin(c+dx))^{5/2}}{7d}\right)$$
$$\frac{2b(e\sin(c+dx))^{9/2}}{9de}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{5}{7}e^2\left(\frac{e^2\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}}dx}{3\sqrt{e\sin(c+dx)}}-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)-\frac{2e\cos(c+dx)(e\sin(c+dx))^{5/2}}{7d}\right)$$
$$\frac{2b(e\sin(c+dx))^{9/2}}{9de}$$

$$\Big\downarrow \text{3120}$$

$$a\left(\frac{5}{7}e^2\left(\frac{2e^2\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{3d\sqrt{e\sin(c+dx)}}-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)-\frac{2e\cos(c+dx)(e\sin}{7d}\right.$$

$$\frac{2b(e\sin(c+dx))^{9/2}}{9de}$$

input
```
Int[(a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(7/2),x]
```

output
```
(2*b*(e*Sin[c + d*x])^(9/2))/(9*d*e) + a*((-2*e*Cos[c + d*x]*(e*Sin[c + d*
x])^(5/2))/(7*d) + (5*e^2*((2*e^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Si
n[c + d*x]])/(3*d*Sqrt[e*Sin[c + d*x]]) - (2*e*Cos[c + d*x]*Sqrt[e*Sin[c +
 d*x]])/(3*d)))/7)
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

## Maple [A] (verified)

Time = 3.64 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.98

| method | result |
|---|---|
| default | $\frac{\frac{2b(e\sin(dx+c))^{\frac{9}{2}}}{9e}-\frac{e^4a\left(-6\sin(dx+c)^5+5\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sqrt{\sin(dx+c)}\,\operatorname{EllipticF}\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-4\sin(dx+c)^3+10\sin(dx+}{21\cos(dx+c)\sqrt{e\sin(dx+c)}}}{d}$ |
| parts | $-\frac{a\,e^4\left(-6\sin(dx+c)^5+5\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sqrt{\sin(dx+c)}\,\operatorname{EllipticF}\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-4\sin(dx+c)^3+10\sin(dx-}{21\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$ |

input
```
int((a+cos(d*x+c)*b)*(e*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
(2/9/e*b*(e*sin(d*x+c))^(9/2)-1/21*e^4*a*(-6*sin(d*x+c)^5+5*(1-sin(d*x+c))
^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1
/2),1/2*2^(1/2))-4*sin(d*x+c)^3+10*sin(d*x+c))/cos(d*x+c)/(e*sin(d*x+c))^(
1/2))/d
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.05

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{7/2}\,dx = \frac{2\left(15\,a\sqrt{-\frac{1}{2}i\,e}e^3\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\,\sin(dx+c))+15\,a\sqrt{\frac{1}{2}i\,e}e^3\text{weier}\right.}{}$$

input
```
integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
2/63*(15*a*sqrt(-1/2*I*e)*e^3*weierstrassPInverse(4, 0, cos(d*x + c) + I*s
in(d*x + c)) + 15*a*sqrt(1/2*I*e)*e^3*weierstrassPInverse(4, 0, cos(d*x +
c) - I*sin(d*x + c)) + (7*b*e^3*cos(d*x + c)^4 + 9*a*e^3*cos(d*x + c)^3 -
14*b*e^3*cos(d*x + c)^2 - 24*a*e^3*cos(d*x + c) + 7*b*e^3)*sqrt(e*sin(d*x
+ c)))/d
```

## Sympy [F(-1)]

Timed out.

$$\int (a + b\cos(c + dx))(e\sin(c + dx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))**(7/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int (a + b\cos(c + dx))(e\sin(c + dx))^{7/2}\,dx = \int (b\cos(dx + c) + a)(e\sin(dx + c))^{\frac{7}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)*(e*sin(d*x + c))^(7/2), x)
```

## Giac [F]

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{7/2}\,dx = \int (b\cos(dx+c)+a)(e\sin(dx+c))^{\frac{7}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)*(e*sin(d*x + c))^(7/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{7/2}\,dx = \int (e\,\sin(c+d\,x))^{7/2}\,(a+b\,\cos(c+d\,x))\,dx$$

input
```
int((e*sin(c + d*x))^(7/2)*(a + b*cos(c + d*x)),x)
```

output
```
int((e*sin(c + d*x))^(7/2)*(a + b*cos(c + d*x)), x)
```

## Reduce [F]

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{7/2}\,dx = \frac{\sqrt{e}\,e^3\left(2\sqrt{\sin(dx+c)}\,\sin(dx+c)^4 b+9\left(\int \sqrt{\sin(dx+c)}\,\sin(dx+c)^3\,dx\right)ad\right)}{9d}$$

input
```
int((a+b*cos(d*x+c))*(e*sin(d*x+c))^(7/2),x)
```

output
```
(sqrt(e)*e**3*(2*sqrt(sin(c + d*x))*sin(c + d*x)**4*b + 9*int(sqrt(sin(c +
 d*x))*sin(c + d*x)**3,x)*a*d))/(9*d)
```

# 3.34 $\int (a+b\cos(c+dx))(e\sin(c+dx))^{5/2}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 100

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{5/2}\,dx = \frac{6ae^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}} - \frac{2ae\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d} + \frac{2b(e\sin(c+dx))^{7/2}}{7de}$$

output
```
-6/5*a*e^2*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/
2)/d/sin(d*x+c)^(1/2)-2/5*a*e*cos(d*x+c)*(e*sin(d*x+c))^(3/2)/d+2/7*b*(e*s
in(d*x+c))^(7/2)/d/e
```

## Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.80

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{5/2}\,dx = \frac{2(e\sin(c+dx))^{5/2}\left(-21aE\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)+\sin^{\frac{3}{2}}(c+dx)\left(-7a\cos(c+dx)+5b\sin\right.\right.}{35d\sin^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[(a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(5/2),x]
```

output
```
(2*(e*Sin[c + d*x])^(5/2)*(-21*a*EllipticE[(-2*c + Pi - 2*d*x)/4, 2] + Sin
[c + d*x]^(3/2)*(-7*a*Cos[c + d*x] + 5*b*Sin[c + d*x]^2)))/(35*d*Sin[c + d
*x]^(5/2))
```

## Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 3148, 3042, 3115, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e\sin(c+dx))^{5/2}(a+b\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3148}$$

$$a\int (e\sin(c+dx))^{5/2}dx+\frac{2b(e\sin(c+dx))^{7/2}}{7de}$$

$$\Big\downarrow \text{3042}$$

$$a\int (e\sin(c+dx))^{5/2}dx+\frac{2b(e\sin(c+dx))^{7/2}}{7de}$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{3}{5}e^2\int \sqrt{e\sin(c+dx)}dx-\frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d}\right)+\frac{2b(e\sin(c+dx))^{7/2}}{7de}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{5}e^2\int \sqrt{e\sin(c+dx)}dx-\frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d}\right)+\frac{2b(e\sin(c+dx))^{7/2}}{7de}$$

$$\Big\downarrow \text{3121}$$

$$a\left(\frac{3e^2\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{5\sqrt{\sin(c+dx)}}-\frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d}\right)+\frac{2b(e\sin(c+dx))^{7/2}}{7de}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3e^2\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{5\sqrt{\sin(c+dx)}}-\frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d}\right)+\frac{2b(e\sin(c+dx))^{7/2}}{7de}$$

$$\Big\downarrow \text{3119}$$

$$a\left(\frac{6e^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}}-\frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d}\right)+\frac{2b(e\sin(c+dx))^{7/2}}{7de}$$

input
```
Int[(a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(5/2),x]
```

output
```
(2*b*(e*Sin[c + d*x])^(7/2))/(7*d*e) + a*((6*e^2*EllipticE[(c - Pi/2 + d*x
)/2, 2]*Sqrt[e*Sin[c + d*x]])/(5*d*Sqrt[Sin[c + d*x]]) - (2*e*Cos[c + d*x]
*(e*Sin[c + d*x])^(3/2))/(5*d))
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3148

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

## Maple [A] (verified)

Time = 3.10 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.71

| method | result |
|---|---|
| default | $\frac{\frac{2b(e\sin(dx+c))^{\frac{7}{2}}}{7e}-\frac{e^{3}a\left(6\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sqrt{\sin(dx+c)}\,\operatorname{EllipticE}\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-3\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sqrt{\sin(d}}{5\cos(dx+c)\sqrt{e\sin(dx+c)}}}{d}$ |
| parts | $-\frac{a\,e^{3}\left(6\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sqrt{\sin(dx+c)}\,\operatorname{EllipticE}\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-3\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sqrt{\sin(}}{5\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$ |

input

```
int((a+cos(d*x+c)*b)*(e*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output

```
(2/7/e*b*(e*sin(d*x+c))^(7/2)-1/5*e^3*a*(6*(1-sin(d*x+c))^(1/2)*(2+2*sin(d
*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-
3*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticF((
1-sin(d*x+c))^(1/2),1/2*2^(1/2))-2*sin(d*x+c)^4+2*sin(d*x+c)^2)/cos(d*x+c)
/(e*sin(d*x+c))^(1/2))/d
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.20

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{5/2}\,dx =$$
$$-\frac{2\left(-21i\,a\sqrt{-\frac{1}{2}i\,e}e^2\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(dx+c))+i\,\sin(dx+c)))+21i\,a\right.}{}$$

input
```
integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-2/35*(-21*I*a*sqrt(-1/2*I*e)*e^2*weierstrassZeta(4, 0, weierstrassPInvers
e(4, 0, cos(d*x + c) + I*sin(d*x + c))) + 21*I*a*sqrt(1/2*I*e)*e^2*weierst
rassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c))) +
 (5*b*e^2*cos(d*x + c)^2 + 7*a*e^2*cos(d*x + c) - 5*b*e^2)*sqrt(e*sin(d*x
+ c))*sin(d*x + c))/d
```

## Sympy [F]

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{5/2}\,dx = \int (e\sin(c+dx))^{\frac{5}{2}}\,(a+b\cos(c+dx))\,dx$$

input
```
integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))**(5/2),x)
```

output
```
Integral((e*sin(c + d*x))**(5/2)*(a + b*cos(c + d*x)), x)
```

## Maxima [F]

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{5/2}\,dx = \int (b\cos{(dx+c)}+a)(e\sin{(dx+c)})^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)*(e*sin(d*x + c))^(5/2), x)
```

## Giac [F]

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{5/2}\,dx = \int (b\cos{(dx+c)}+a)(e\sin{(dx+c)})^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)*(e*sin(d*x + c))^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{5/2}\,dx = \int {\left(e\,\sin\left(c+d\,x\right)\right)}^{5/2}\,\left(a+b\,\cos\left(c+d\,x\right)\right)\,dx$$

input
```
int((e*sin(c + d*x))^(5/2)*(a + b*cos(c + d*x)),x)
```

output
```
int((e*sin(c + d*x))^(5/2)*(a + b*cos(c + d*x)), x)
```

## Reduce [F]

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{5/2}\,dx = \frac{\sqrt{e}\,e^2\left(2\sqrt{\sin(dx+c)}\,\sin(dx+c)^3 b+7\left(\int\sqrt{\sin(dx+c)}\,\sin(dx+c)^2\,dx\right)ad\right)}{7d}$$

input
```
int((a+b*cos(d*x+c))*(e*sin(d*x+c))^(5/2),x)
```

output
```
(sqrt(e)*e**2*(2*sqrt(sin(c + d*x))*sin(c + d*x)**3*b + 7*int(sqrt(sin(c +
 d*x))*sin(c + d*x)**2,x)*a*d))/(7*d)
```

# 3.35 $\int (a+b\cos(c+dx))(e\sin(c+dx))^{3/2}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 100

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{3/2}\,dx = \frac{2ae^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3d\sqrt{e\sin(c+dx)}} - \frac{2ae\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d} + \frac{2b(e\sin(c+dx))^{5/2}}{5de}$$

output
```
2/3*a*e^2*InverseJacobiAM(1/2*c-1/4*Pi+1/2*d*x,2^(1/2))*sin(d*x+c)^(1/2)/d
/(e*sin(d*x+c))^(1/2)-2/3*a*e*cos(d*x+c)*(e*sin(d*x+c))^(1/2)/d+2/5*b*(e*s
in(d*x+c))^(5/2)/d/e
```

## Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.80

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{3/2}\,dx = \frac{2(e\sin(c+dx))^{3/2}\left(-5a\,\text{EllipticF}\left(\frac{1}{4}(-2c+\pi-2dx),2\right)+\sqrt{\sin(c+dx)}(-5a\cos(c+dx)\right.}{15d\sin^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[(a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2),x]
```

output
```
(2*(e*Sin[c + d*x])^(3/2)*(-5*a*EllipticF[(-2*c + Pi - 2*d*x)/4, 2] + Sqrt
[Sin[c + d*x]]*(-5*a*Cos[c + d*x] + 3*b*Sin[c + d*x]^2)))/(15*d*Sin[c + d*
x]^(3/2))
```

## Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3042, 3148, 3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e\sin(c+dx))^{3/2}(a+b\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3148}$$

$$a\int (e\sin(c+dx))^{3/2}dx+\frac{2b(e\sin(c+dx))^{5/2}}{5de}$$

$$\Big\downarrow \text{3042}$$

$$a\int (e\sin(c+dx))^{3/2}dx+\frac{2b(e\sin(c+dx))^{5/2}}{5de}$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{1}{3}e^2\int \frac{1}{\sqrt{e\sin(c+dx)}}dx-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)+\frac{2b(e\sin(c+dx))^{5/2}}{5de}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{1}{3}e^2\int \frac{1}{\sqrt{e\sin(c+dx)}}dx-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)+\frac{2b(e\sin(c+dx))^{5/2}}{5de}$$

$$\Big\downarrow \text{3121}$$

$$a\left(\frac{e^2\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{3\sqrt{e\sin(c+dx)}}-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)+\frac{2b(e\sin(c+dx))^{5/2}}{5de}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{e^2\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{3\sqrt{e\sin(c+dx)}}-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)+\frac{2b(e\sin(c+dx))^{5/2}}{5de}$$

$$\Big\downarrow \text{3120}$$

$$a\left(\frac{2e^2\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{3d\sqrt{e\sin(c+dx)}}-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)+\frac{2b(e\sin(c+dx))^{5/2}}{5de}$$

input
```
Int[(a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2),x]
```

output
```
(2*b*(e*Sin[c + d*x])^(5/2))/(5*d*e) + a*((2*e^2*EllipticF[(c - Pi/2 + d*x
)/2, 2]*Sqrt[Sin[c + d*x]])/(3*d*Sqrt[e*Sin[c + d*x]]) - (2*e*Cos[c + d*x]
*Sqrt[e*Sin[c + d*x]])/(3*d))
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

## Maple [A] (verified)

Time = 1.85 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.16

| method | result |
|---|---|
| default | $\frac{\frac{2b(e\sin(dx+c))^{\frac{5}{2}}}{5e}-\frac{a\,e^{2}\left(\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sqrt{\sin(dx+c)}\,\operatorname{EllipticF}\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-2\sin(dx+c)^{3}+2\sin(dx+c)\right)}{3\cos(dx+c)\sqrt{e\sin(dx+c)}}}{d}$ |
| parts | $-\frac{a\,e^{2}\left(\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sqrt{\sin(dx+c)}\,\operatorname{EllipticF}\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-2\sin(dx+c)^{3}+2\sin(dx+c)\right)}{3\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}+\frac{2b(e\sin(d\ldots}{5de\ldots}$ |

input
```
int((a+cos(d*x+c)*b)*(e*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(2/5/e*b*(e*sin(d*x+c))^(5/2)-1/3*a*e^2*((1-sin(d*x+c))^(1/2)*(2+2*sin(d*x
+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-2*
sin(d*x+c)^3+2*sin(d*x+c))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.99

$$\int(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}\,dx=\frac{2\left(5\,a\sqrt{-\frac{1}{2}i\,e}e\operatorname{weierstrassPInverse}(4,0,\cos(dx+c)+i\,\sin(dx+c))+5\,a\sqrt{\frac{1}{2}i\,e}e\operatorname{weierstras\ldots}\right.}{\ldots}$$

input
```
integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
2/15*(5*a*sqrt(-1/2*I*e)*e*weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(
d*x + c)) + 5*a*sqrt(1/2*I*e)*e*weierstrassPInverse(4, 0, cos(d*x + c) - I
*sin(d*x + c)) - (3*b*e*cos(d*x + c)^2 + 5*a*e*cos(d*x + c) - 3*b*e)*sqrt(
e*sin(d*x + c)))/d
```

## Sympy [F]

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{3/2}\,dx = \int (e\sin(c+dx))^{\frac{3}{2}}\,(a+b\cos(c+dx))\;dx$$

input
```
integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))**(3/2),x)
```

output
```
Integral((e*sin(c + d*x))**(3/2)*(a + b*cos(c + d*x)), x)
```

## Maxima [F]

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{3/2}\,dx = \int (b\cos(dx+c)+a)(e\sin(dx+c))^{\frac{3}{2}}\;dx$$

input
```
integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)*(e*sin(d*x + c))^(3/2), x)
```

## Giac [F]

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{3/2}\,dx=\int (b\cos(dx+c)+a)(e\sin(dx+c))^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)*(e*sin(d*x + c))^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{3/2}\,dx=\int (e\,\sin(c+d\,x))^{3/2}\,(a+b\,\cos(c+d\,x))\,dx$$

input
```
int((e*sin(c + d*x))^(3/2)*(a + b*cos(c + d*x)),x)
```

output
```
int((e*sin(c + d*x))^(3/2)*(a + b*cos(c + d*x)), x)
```

## Reduce [F]

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{3/2}\,dx=\frac{\sqrt{e}\,e\left(2\sqrt{\sin(dx+c)}\,\sin(dx+c)^2 b+5\left(\int \sqrt{\sin(dx+c)}\,\sin(dx+c)\,dx\right)ad\right)}{5d}$$

input
```
int((a+b*cos(d*x+c))*(e*sin(d*x+c))^(3/2),x)
```

output
```
(sqrt(e)*e*(2*sqrt(sin(c + d*x))*sin(c + d*x)**2*b + 5*int(sqrt(sin(c + d*
x))*sin(c + d*x),x)*a*d))/(5*d)
```

# 3.36 $\int (a+b\cos(c+dx))\sqrt{e\sin(c+dx)}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 68

$$\int (a+b\cos(c+dx))\sqrt{e\sin(c+dx)}\,dx=\frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}+\frac{2b(e\sin(c+dx))^{3/2}}{3de}$$

output
```
-2*a*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/d/s
in(d*x+c)^(1/2)+2/3*b*(e*sin(d*x+c))^(3/2)/d/e
```

## Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.88

$$\int (a+b\cos(c+dx))\sqrt{e\sin(c+dx)}\,dx=\frac{2\sqrt{e\sin(c+dx)}\left(-3aE\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)+b\sin^{\frac{3}{2}}(c+dx)\right)}{3d\sqrt{\sin(c+dx)}}$$

input
```
Integrate[(a + b*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]],x]
```

output
```
(2*Sqrt[e*Sin[c + d*x]]*(-3*a*EllipticE[(-2*c + Pi - 2*d*x)/4, 2] + b*Sin[
c + d*x]^(3/2)))/(3*d*Sqrt[Sin[c + d*x]])
```

## Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 3148, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{e\sin(c+dx)}(a+b\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3148}$$

$$a\int \sqrt{e\sin(c+dx)}dx+\frac{2b(e\sin(c+dx))^{3/2}}{3de}$$

$$\Big\downarrow \text{3042}$$

$$a\int \sqrt{e\sin(c+dx)}dx+\frac{2b(e\sin(c+dx))^{3/2}}{3de}$$

$$\Big\downarrow \text{3121}$$

$$\frac{a\sqrt{e\sin(c+dx)}\int \sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}+\frac{2b(e\sin(c+dx))^{3/2}}{3de}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\sqrt{e\sin(c+dx)}\int \sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}+\frac{2b(e\sin(c+dx))^{3/2}}{3de}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}+\frac{2b(e\sin(c+dx))^{3/2}}{3de}$$

input
```
Int[(a + b*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]],x]
```

output
```
(2*a*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(d*Sqrt[Sin[c
+ d*x]]) + (2*b*(e*Sin[c + d*x])^(3/2))/(3*d*e)
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

## Maple [A] (verified)

Time = 2.14 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.72

| method | result |
|---|---|
| default | $\frac{\frac{2b(e\sin(dx+c))^{\frac{3}{2}}}{3e}-\frac{ae\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sqrt{\sin(dx+c)}\left(2\,\mathrm{EllipticE}\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-\mathrm{EllipticF}\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)\right)}{\cos(dx+c)\sqrt{e\sin(dx+c)}}}{d}$ |
| parts | $-\frac{ae\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sqrt{\sin(dx+c)}\left(2\,\mathrm{EllipticE}\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-\mathrm{EllipticF}\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)\right)}{\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}+\frac{2b(e\,s}{}$ |

input
```
int((a+cos(d*x+c)*b)*(e*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(2/3*b/e*(e*sin(d*x+c))^(3/2)-a*e*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1
/2)*sin(d*x+c)^(1/2)*(2*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-Ellipt
icF((1-sin(d*x+c))^(1/2),1/2*2^(1/2)))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.21

$$\int (a + b\cos(c + dx))\sqrt{e\sin(c + dx)}\,dx$$
$$= \frac{2\left(\sqrt{e\sin(dx + c)}b\sin(dx + c) + 3i\,a\sqrt{-\frac{1}{2}i\,e}\text{weierstrassZeta}(4, 0, \text{weierstrassPInverse}(4, 0, \cos(dx + c$$

input
```
integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
2/3*(sqrt(e*sin(d*x + c))*b*sin(d*x + c) + 3*I*a*sqrt(-1/2*I*e)*weierstras
sZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*
I*a*sqrt(1/2*I*e)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x
+ c) - I*sin(d*x + c))))/d
```

## Sympy [F]

$$\int (a + b\cos(c + dx))\sqrt{e\sin(c + dx)}\,dx = \int \sqrt{e\sin(c + dx)}(a + b\cos(c + dx))\,dx$$

input
```
integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(e*sin(c + d*x))*(a + b*cos(c + d*x)), x)
```

## Maxima [F]

$$\int (a+b\cos(c+dx))\sqrt{e\sin(c+dx)}\,dx = \int (b\cos(dx+c)+a)\sqrt{e\sin(dx+c)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)*sqrt(e*sin(d*x + c)), x)
```

## Giac [F]

$$\int (a+b\cos(c+dx))\sqrt{e\sin(c+dx)}\,dx = \int (b\cos(dx+c)+a)\sqrt{e\sin(dx+c)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)*sqrt(e*sin(d*x + c)), x)
```

## Mupad [B] (verification not implemented)

Time = 42.66 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.88

$$\int (a+b\cos(c+dx))\sqrt{e\sin(c+dx)}\,dx = \frac{2\,b\,\sin(c+d\,x)\,\sqrt{e\,\sin(c+d\,x)}}{3\,d} + \frac{2\,a\,\sqrt{e\,\sin(c+d\,x)}\,\mathrm{E}\left(\frac{c}{2}-\frac{\pi}{4}+\frac{d\,x}{2}\middle|2\right)}{d\,\sqrt{\sin(c+d\,x)}}$$

input
```
int((e*sin(c + d*x))^(1/2)*(a + b*cos(c + d*x)),x)
```

output
```
(2*b*sin(c + d*x)*(e*sin(c + d*x))^(1/2))/(3*d) + (2*a*(e*sin(c + d*x))^(1
/2)*ellipticE(c/2 - pi/4 + (d*x)/2, 2))/(d*sin(c + d*x)^(1/2))
```

## Reduce [F]

$$\int (a+b\cos(c+dx))\sqrt{e\sin(c+dx)}\,dx$$
$$=\frac{\sqrt{e}\left(2\sqrt{\sin(dx+c)}\,\sin(dx+c)\,b+3\left(\int\sqrt{\sin(dx+c)}dx\right)ad\right)}{3d}$$

input
```
int((a+b*cos(d*x+c))*(e*sin(d*x+c))^(1/2),x)
```

output
```
(sqrt(e)*(2*sqrt(sin(c + d*x))*sin(c + d*x)*b + 3*int(sqrt(sin(c + d*x)),x
)*a*d))/(3*d)
```

# 3.37 $\int \frac{a+b\cos(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 66

$$\int \frac{a+b\cos(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx = \frac{2a\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{d\sqrt{e\sin(c+dx)}} + \frac{2b\sqrt{e\sin(c+dx)}}{de}$$

output
```
2*a*InverseJacobiAM(1/2*c-1/4*Pi+1/2*d*x,2^(1/2))*sin(d*x+c)^(1/2)/d/(e*si
n(d*x+c))^(1/2)+2*b*(e*sin(d*x+c))^(1/2)/d/e
```

## Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.82

$$\int \frac{a+b\cos(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx = \frac{2\left(-a\,\text{EllipticF}\left(\frac{1}{4}(-2c+\pi-2dx),2\right)\sqrt{\sin(c+dx)}+b\sin(c+dx)\right)}{d\sqrt{e\sin(c+dx)}}$$

input
```
Integrate[(a + b*Cos[c + d*x])/Sqrt[e*Sin[c + d*x]],x]
```

output
```
(2*(-(a*EllipticF[(-2*c + Pi - 2*d*x)/4, 2]*Sqrt[Sin[c + d*x]]) + b*Sin[c
+ d*x]))/(d*Sqrt[e*Sin[c + d*x]])
```

## Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 3148, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\cos(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3148}$$

$$a\int \frac{1}{\sqrt{e\sin(c+dx)}}dx+\frac{2b\sqrt{e\sin(c+dx)}}{de}$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\sqrt{e\sin(c+dx)}}dx+\frac{2b\sqrt{e\sin(c+dx)}}{de}$$

$$\Big\downarrow \text{3121}$$

$$\frac{a\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}}dx}{\sqrt{e\sin(c+dx)}}+\frac{2b\sqrt{e\sin(c+dx)}}{de}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}}dx}{\sqrt{e\sin(c+dx)}}+\frac{2b\sqrt{e\sin(c+dx)}}{de}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}+\frac{2b\sqrt{e\sin(c+dx)}}{de}$$

input
```
Int[(a + b*Cos[c + d*x])/Sqrt[e*Sin[c + d*x]],x]
```

output
```
(2*a*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(d*Sqrt[e*Sin[c
+ d*x]]) + (2*b*Sqrt[e*Sin[c + d*x]])/(d*e)
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

## Maple [A] (verified)

Time = 1.92 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.39

| method | result |
|---|---|
| default | $-\frac{a\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sqrt{\sin(dx+c)}\,\operatorname{EllipticF}\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-2\sin(dx+c)\cos(dx+c)b}{\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$ |
| parts | $-\frac{a\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sqrt{\sin(dx+c)}\,\operatorname{EllipticF}\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)}{\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}+\frac{2b\sqrt{e\sin(dx+c)}}{de}$ |
| risch | $-\frac{ib\left(e^{2i(dx+c)}-1\right)\sqrt{2}\,e^{-i(dx+c)}}{d\sqrt{-ie\left(e^{2i(dx+c)}-1\right)e^{-i(dx+c)}}}-\frac{ia\sqrt{e^{i(dx+c)}+1}\,\sqrt{-2\,e^{i(dx+c)}+2}\,\sqrt{-e^{i(dx+c)}}\,\operatorname{EllipticF}\left(\sqrt{e^{i(dx+c)}+1},\frac{\sqrt{2}}{2}\right)\sqrt{2}\,\sqrt{-ie\left(e^{2i(c\right.}}}{d\sqrt{-ie\,e^{3i(dx+c)}+ie^{i(dx+c)}e}\,\sqrt{-ie\left(e^{2i(dx+c)}-1\right)e^{-i(dx+c)}}}$ |

input
```
int((a+cos(d*x+c)*b)/(e*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*(a*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c)
)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-2*sin
(d*x+c)*cos(d*x+c)*b)/d
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.08

$$\int \frac{a+b\cos(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx = \frac{2\left(a\sqrt{-\frac{1}{2}i\,e}\,\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)+i\,\sin(dx+c))+a\sqrt{\frac{1}{2}i\,e}\,\mathrm{weierstrassPInverse}(4,0,\mathrm{co}\right.}{de}$$

input
```
integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
2*(a*sqrt(-1/2*I*e)*weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c
)) + a*sqrt(1/2*I*e)*weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x +
c)) + sqrt(e*sin(d*x + c))*b)/(d*e)
```

## Sympy [F]

$$\int \frac{a+b\cos(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx=\int \frac{a+b\cos{(c+dx)}}{\sqrt{e\sin{(c+dx)}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))**(1/2),x)
```

output
```
Integral((a + b*cos(c + d*x))/sqrt(e*sin(c + d*x)), x)
```

## Maxima [F]

$$\int \frac{a+b\cos(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx=\int \frac{b\cos{(dx+c)}+a}{\sqrt{e\sin{(dx+c)}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)/sqrt(e*sin(d*x + c)), x)
```

## Giac [F]

$$\int \frac{a+b\cos(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx=\int \frac{b\cos{(dx+c)}+a}{\sqrt{e\sin{(dx+c)}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)/sqrt(e*sin(d*x + c)), x)
```

## Mupad [B] (verification not implemented)

Time = 43.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.76

$$\int \frac{a+b\cos(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx = -\frac{2\sqrt{\sin(c+d\,x)}\,\left(a\,\mathrm{F}\left(\frac{\pi}{4}-\frac{c}{2}-\frac{d\,x}{2}\middle|2\right)-b\,\sqrt{\sin(c+d\,x)}\right)}{d\,\sqrt{e\,\sin(c+d\,x)}}$$

input
```
int((a + b*cos(c + d*x))/(e*sin(c + d*x))^(1/2),x)
```

output
```
-(2*sin(c + d*x)^(1/2)*(a*ellipticF(pi/4 - c/2 - (d*x)/2, 2) - b*sin(c + d
*x)^(1/2)))/(d*(e*sin(c + d*x))^(1/2))
```

## Reduce [F]

$$\int \frac{a+b\cos(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx = \frac{\sqrt{e}\left(2\sqrt{\sin(dx+c)}\,b+\left(\int\frac{\sqrt{\sin(dx+c)}}{\sin(dx+c)}dx\right)ad\right)}{de}$$

input
```
int((a+b*cos(d*x+c))/(e*sin(d*x+c))^(1/2),x)
```

output
```
(sqrt(e)*(2*sqrt(sin(c + d*x))*b + int(sqrt(sin(c + d*x))/sin(c + d*x),x)*
a*d))/(d*e)
```

# 3.38 $\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 96

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx = -\frac{2b}{de\sqrt{e\sin(c+dx)}} - \frac{2a\cos(c+dx)}{de\sqrt{e\sin(c+dx)}} - \frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{de^2\sqrt{\sin(c+dx)}}$$

output
```
-2*b/d/e/(e*sin(d*x+c))^(1/2)-2*a*cos(d*x+c)/d/e/(e*sin(d*x+c))^(1/2)+2*a*
EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/d/e^2/si
n(d*x+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.60

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx = -\frac{2\left(b+a\cos(c+dx)-aE\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)\sqrt{\sin(c+dx)}\right)}{de\sqrt{e\sin(c+dx)}}$$

input
```
Integrate[(a + b*Cos[c + d*x])/(e*Sin[c + d*x])^(3/2),x]
```

output
```
(-2*(b + a*Cos[c + d*x] - a*EllipticE[(-2*c + Pi - 2*d*x)/4, 2]*Sqrt[Sin[c
 + d*x]]))/(d*e*Sqrt[e*Sin[c + d*x]])
```

## Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3042, 3148, 3042, 3116, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3148}$$

$$a\int \frac{1}{(e\sin(c+dx))^{3/2}}dx-\frac{2b}{de\sqrt{e\sin(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{(e\sin(c+dx))^{3/2}}dx-\frac{2b}{de\sqrt{e\sin(c+dx)}}$$

$$\Big\downarrow \text{3116}$$

$$a\left(-\frac{\int \sqrt{e\sin(c+dx)}dx}{e^2}-\frac{2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}\right)-\frac{2b}{de\sqrt{e\sin(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$a\left(-\frac{\int \sqrt{e\sin(c+dx)}dx}{e^2}-\frac{2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}\right)-\frac{2b}{de\sqrt{e\sin(c+dx)}}$$

$$\Big\downarrow \text{3121}$$

$$a\left(-\frac{\sqrt{e\sin(c+dx)}\int \sqrt{\sin(c+dx)}dx}{e^2\sqrt{\sin(c+dx)}}-\frac{2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}\right)-\frac{2b}{de\sqrt{e\sin(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$a\left(-\frac{\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{e^2\sqrt{\sin(c+dx)}}-\frac{2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}\right)-\frac{2b}{de\sqrt{e\sin(c+dx)}}$$

$$\Big\downarrow \text{3119}$$

$$a\left(-\frac{2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{de^2\sqrt{\sin(c+dx)}}-\frac{2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}\right)-\frac{2b}{de\sqrt{e\sin(c+dx)}}$$

input
```
Int[(a + b*Cos[c + d*x])/(e*Sin[c + d*x])^(3/2),x]
```

output
```
(-2*b)/(d*e*Sqrt[e*Sin[c + d*x]]) + a*((-2*Cos[c + d*x])/(d*e*Sqrt[e*Sin[c
 + d*x]]) - (2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(d*e
^2*Sqrt[Sin[c + d*x]]))
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

## Maple [A] (verified)

Time = 2.22 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.59

| method | result |
|---|---|
| default | $\frac{2\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sqrt{\sin(dx+c)}\,\mathrm{EllipticE}\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)a-a\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sqrt{\sin(dx+c)}}{e\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$ |
| parts | $\frac{a\left(2\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sqrt{\sin(dx+c)}\,\mathrm{EllipticE}\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sqrt{\sin(dx+c)}\right.}{e\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$ |

input
```
int((a+cos(d*x+c)*b)/(e*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(2*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticE(
(1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a-a*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c)
)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-2*a*c
os(d*x+c)^2-2*cos(d*x+c)*b)/e/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/d
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.12

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx =$$
$$-\frac{2\left(i\,a\sqrt{-\frac{1}{2}i\,e}\sin\left(dx+c\right)\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)))\right.-}{}$$

input
```
integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-2*(I*a*sqrt(-1/2*I*e)*sin(d*x + c)*weierstrassZeta(4, 0, weierstrassPInve
rse(4, 0, cos(d*x + c) + I*sin(d*x + c))) - I*a*sqrt(1/2*I*e)*sin(d*x + c)
*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x
+ c))) + (a*cos(d*x + c) + b)*sqrt(e*sin(d*x + c)))/(d*e^2*sin(d*x + c))
```

## Sympy [F]

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx = \int \frac{a+b\cos\left(c+dx\right)}{\left(e\sin\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))**(3/2),x)
```

output
```
Integral((a + b*cos(c + d*x))/(e*sin(c + d*x))**(3/2), x)
```

## Maxima [F]

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx = \int \frac{b\cos\left(dx+c\right)+a}{\left(e\sin\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)/(e*sin(d*x + c))^(3/2), x)
```

## Giac [F]

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx = \int \frac{b\cos(dx+c)+a}{(e\sin(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)/(e*sin(d*x + c))^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx = \int \frac{a+b\,\cos(c+d\,x)}{(e\,\sin(c+d\,x))^{3/2}}\,dx$$

input
```
int((a + b*cos(c + d*x))/(e*sin(c + d*x))^(3/2),x)
```

output
```
int((a + b*cos(c + d*x))/(e*sin(c + d*x))^(3/2), x)
```

## Reduce [F]

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx = \frac{\sqrt{e}\left(-2\sqrt{\sin(dx+c)}\,b+\left(\int \frac{\sqrt{\sin(dx+c)}}{\sin(dx+c)^2}dx\right)\sin(dx+c)\,ad\right)}{\sin(dx+c)\,d\,e^2}$$

input
```
int((a+b*cos(d*x+c))/(e*sin(d*x+c))^(3/2),x)
```

output
```
(sqrt(e)*( - 2*sqrt(sin(c + d*x))*b + int(sqrt(sin(c + d*x))/sin(c + d*x)*
*2,x)*sin(c + d*x)*a*d))/(sin(c + d*x)*d*e**2)
```

# 3.39 $\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 102

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx=-\frac{2b}{3de(e\sin(c+dx))^{3/2}}-\frac{2a\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}}+\frac{2a\,\mathrm{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3de^2\sqrt{e\sin(c+dx)}}$$

output
```
-2/3*b/d/e/(e*sin(d*x+c))^(3/2)-2/3*a*cos(d*x+c)/d/e/(e*sin(d*x+c))^(3/2)+
2/3*a*InverseJacobiAM(1/2*c-1/4*Pi+1/2*d*x,2^(1/2))*sin(d*x+c)^(1/2)/d/e^2
/(e*sin(d*x+c))^(1/2)
```

## Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.58

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx=-\frac{2\left(b+a\cos(c+dx)+a\,\mathrm{EllipticF}\left(\frac{1}{4}(-2c+\pi-2dx),2\right)\sin^{\frac{3}{2}}(c+dx)\right)}{3de(e\sin(c+dx))^{3/2}}$$

input
```
Integrate[(a + b*Cos[c + d*x])/(e*Sin[c + d*x])^(5/2),x]
```

output
```
(-2*(b + a*Cos[c + d*x] + a*EllipticF[(-2*c + Pi - 2*d*x)/4, 2]*Sin[c + d*
x]^(3/2)))/(3*d*e*(e*Sin[c + d*x])^(3/2))
```

## Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 3148, 3042, 3116, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3148}$$

$$a\int \frac{1}{(e\sin(c+dx))^{5/2}}dx-\frac{2b}{3de(e\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{(e\sin(c+dx))^{5/2}}dx-\frac{2b}{3de(e\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3116}$$

$$a\left(\frac{\int \frac{1}{\sqrt{e\sin(c+dx)}}dx}{3e^2}-\frac{2\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}}\right)-\frac{2b}{3de(e\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{\int \frac{1}{\sqrt{e\sin(c+dx)}}dx}{3e^2}-\frac{2\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}}\right)-\frac{2b}{3de(e\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3121}$$

$$a\left(\frac{\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{3e^2\sqrt{e\sin(c+dx)}}-\frac{2\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}}\right)-\frac{2b}{3de(e\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{3e^2\sqrt{e\sin(c+dx)}}-\frac{2\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}}\right)-\frac{2b}{3de(e\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3120}$$

$$a\left(\frac{2\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{3de^2\sqrt{e\sin(c+dx)}}-\frac{2\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}}\right)-\frac{2b}{3de(e\sin(c+dx))^{3/2}}$$

input
```
Int[(a + b*Cos[c + d*x])/(e*Sin[c + d*x])^(5/2),x]
```

output
```
(-2*b)/(3*d*e*(e*Sin[c + d*x])^(3/2)) + a*((-2*Cos[c + d*x])/(3*d*e*(e*Sin
[c + d*x])^(3/2)) + (2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]]
)/(3*d*e^2*Sqrt[e*Sin[c + d*x]]))
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

## Maple [A] (verified)

Time = 2.42 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.22

| method | result |
|---|---|
| default | $\frac{-\frac{2b}{3e(e\sin(dx+c))^{\frac{3}{2}}}-\frac{a\left(\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sin(dx+c)^{\frac{5}{2}}\operatorname{EllipticF}\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-2\sin(dx+c)^{3}+2\sin(dx+c)\right)}{3e^{2}\sin(dx+c)^{2}\cos(dx+c)\sqrt{e\sin(dx+c)}}}{d}$ |
| parts | $-\frac{a\left(\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sin(dx+c)^{\frac{5}{2}}\operatorname{EllipticF}\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-2\sin(dx+c)^{3}+2\sin(dx+c)\right)}{3e^{2}\sin(dx+c)^{2}\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}-\frac{2b}{3de(e\sin(dx-$ |

input
```
int((a+cos(d*x+c)*b)/(e*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(-2/3*b/e/(e*sin(d*x+c))^(3/2)-1/3*a/e^2*((1-sin(d*x+c))^(1/2)*(2+2*sin(d*
x+c))^(1/2)*sin(d*x+c)^(5/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-2
*sin(d*x+c)^3+2*sin(d*x+c))/sin(d*x+c)^2/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/
d
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.20

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx = \frac{2\left(\left(a\cos\left(dx+c\right)^{2}-a\right)\sqrt{-\frac{1}{2}i\,e}\text{weierstrassPInverse}(4,0,\cos\left(dx+c\right)+i\,\sin\left(dx\right.\right.}{}$$

input
```
integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
2/3*((a*cos(d*x + c)^2 - a)*sqrt(-1/2*I*e)*weierstrassPInverse(4, 0, cos(d
*x + c) + I*sin(d*x + c)) + (a*cos(d*x + c)^2 - a)*sqrt(1/2*I*e)*weierstra
ssPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)) + (a*cos(d*x + c) + b)*sqr
t(e*sin(d*x + c)))/(d*e^3*cos(d*x + c)^2 - d*e^3)
```

## Sympy [F]

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx = \int \frac{a+b\cos{(c+dx)}}{(e\sin{(c+dx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))**(5/2),x)
```

output
```
Integral((a + b*cos(c + d*x))/(e*sin(c + d*x))**(5/2), x)
```

## Maxima [F]

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx = \int \frac{b\cos{(dx+c)}+a}{(e\sin{(dx+c)})^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)/(e*sin(d*x + c))^(5/2), x)
```

## Giac [F]

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx=\int \frac{b\cos(dx+c)+a}{(e\sin(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)/(e*sin(d*x + c))^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx=\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx$$

input
```
int((a + b*cos(c + d*x))/(e*sin(c + d*x))^(5/2),x)
```

output
```
int((a + b*cos(c + d*x))/(e*sin(c + d*x))^(5/2), x)
```

## Reduce [F]

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx=\frac{\sqrt{e}\left(-2\sqrt{\sin(dx+c)}\,b+3\left(\int \frac{\sqrt{\sin(dx+c)}}{\sin(dx+c)^3}dx\right)\sin(dx+c)^2 ad\right)}{3\sin(dx+c)^2 d\,e^3}$$

input
```
int((a+b*cos(d*x+c))/(e*sin(d*x+c))^(5/2),x)
```

output
```
(sqrt(e)*( - 2*sqrt(sin(c + d*x))*b + 3*int(sqrt(sin(c + d*x))/sin(c + d*x
)**3,x)*sin(c + d*x)**2*a*d))/(3*sin(c + d*x)**2*d*e**3)
```

# 3.40 $\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{7/2}}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 131

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{7/2}}\,dx = -\frac{2b}{5de(e\sin(c+dx))^{5/2}} - \frac{2a\cos(c+dx)}{5de(e\sin(c+dx))^{5/2}} - \frac{6a\cos(c+dx)}{5de^3\sqrt{e\sin(c+dx)}} - \frac{6aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5de^4\sqrt{\sin(c+dx)}}$$

output
```
-2/5*b/d/e/(e*sin(d*x+c))^(5/2)-2/5*a*cos(d*x+c)/d/e/(e*sin(d*x+c))^(5/2)-
6/5*a*cos(d*x+c)/d/e^3/(e*sin(d*x+c))^(1/2)+6/5*a*EllipticE(cos(1/2*c+1/4*
Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/d/e^4/sin(d*x+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.56

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{7/2}}\,dx = \frac{-4b-7a\cos(c+dx)+3a\cos(3(c+dx))+12aE\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)\sin^{\frac{5}{2}}(c}{10de(e\sin(c+dx))^{5/2}}$$

input
```
Integrate[(a + b*Cos[c + d*x])/(e*Sin[c + d*x])^(7/2),x]
```

output
```
(-4*b - 7*a*Cos[c + d*x] + 3*a*Cos[3*(c + d*x)] + 12*a*EllipticE[(-2*c + P
i - 2*d*x)/4, 2]*Sin[c + d*x]^(5/2))/(10*d*e*(e*Sin[c + d*x])^(5/2))
```

## Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 3148, 3042, 3116, 3042, 3116, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{3148}$$

$$a\int \frac{1}{(e\sin(c+dx))^{7/2}}dx-\frac{2b}{5de(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{(e\sin(c+dx))^{7/2}}dx-\frac{2b}{5de(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3116}$$

$$a\left(\frac{3\int \frac{1}{(e\sin(c+dx))^{3/2}}dx}{5e^2}-\frac{2\cos(c+dx)}{5de(e\sin(c+dx))^{5/2}}\right)-\frac{2b}{5de(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3\int \frac{1}{(e\sin(c+dx))^{3/2}}dx}{5e^2}-\frac{2\cos(c+dx)}{5de(e\sin(c+dx))^{5/2}}\right)-\frac{2b}{5de(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3116}$$

$$a\left(\frac{3\left(-\frac{\int\sqrt{e\sin(c+dx)}dx}{e^2}-\frac{2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}\right)}{5e^2}-\frac{2\cos(c+dx)}{5de(e\sin(c+dx))^{5/2}}\right)-\frac{2b}{5de(e\sin(c+dx))^{5/2}}$$

$\downarrow$ 3042

$$a\left(\frac{3\left(-\frac{\int\sqrt{e\sin(c+dx)}dx}{e^2}-\frac{2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}\right)}{5e^2}-\frac{2\cos(c+dx)}{5de(e\sin(c+dx))^{5/2}}\right)-\frac{2b}{5de(e\sin(c+dx))^{5/2}}$$

$\downarrow$ 3121

$$a\left(\frac{3\left(-\frac{\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{e^2\sqrt{\sin(c+dx)}}-\frac{2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}\right)}{5e^2}-\frac{2\cos(c+dx)}{5de(e\sin(c+dx))^{5/2}}\right)-\frac{2b}{5de(e\sin(c+dx))^{5/2}}$$

$\downarrow$ 3042

$$a\left(\frac{3\left(-\frac{\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{e^2\sqrt{\sin(c+dx)}}-\frac{2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}\right)}{5e^2}-\frac{2\cos(c+dx)}{5de(e\sin(c+dx))^{5/2}}\right)-\frac{2b}{5de(e\sin(c+dx))^{5/2}}$$

$\downarrow$ 3119

$$a\left(\frac{3\left(-\frac{2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)\sqrt{e\sin(c+dx)}}{de^2\sqrt{\sin(c+dx)}}-\frac{2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}\right)}{5e^2}-\frac{2\cos(c+dx)}{5de(e\sin(c+dx))^{5/2}}\right)-\frac{2b}{5de(e\sin(c+dx))^{5/2}}$$

input

```
Int[(a + b*Cos[c + d*x])/(e*Sin[c + d*x])^(7/2),x]
```

output

```
(-2*b)/(5*d*e*(e*Sin[c + d*x])^(5/2)) + a*((-2*Cos[c + d*x])/(5*d*e*(e*Sin
[c + d*x])^(5/2)) + (3*((-2*Cos[c + d*x])/(d*e*Sqrt[e*Sin[c + d*x]]) - (2*
EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(d*e^2*Sqrt[Sin[c +
 d*x]])))/(5*e^2))
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

## Maple [A] (verified)

Time = 2.25 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.43

| method | result |
|---|---|
| default | $\frac{-\frac{2b}{5e(e\sin(dx+c))^{\frac{5}{2}}}+\frac{a\left(6\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sin(dx+c)^{\frac{7}{2}}\,\text{EllipticE}\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-3\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sin(dx+}{5e^3\sin(dx+c)^3\cos(dx+c)\sqrt{e\sin(dx+c)}}}{d}$ |
| parts | $\frac{a\left(6\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sin(dx+c)^{\frac{7}{2}}\,\text{EllipticE}\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-3\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sin(dx+c)^{\frac{7}{2}}}{5e^3\sin(dx+c)^3\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$ |

input
```
int((a+cos(d*x+c)*b)/(e*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
(-2/5*b/e/(e*sin(d*x+c))^(5/2)+1/5*a/e^3*(6*(1-sin(d*x+c))^(1/2)*(2+2*sin(
d*x+c))^(1/2)*sin(d*x+c)^(7/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))
-3*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(7/2)*EllipticF(
(1-sin(d*x+c))^(1/2),1/2*2^(1/2))+6*sin(d*x+c)^5-4*sin(d*x+c)^3-2*sin(d*x+
c))/sin(d*x+c)^3/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.27

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{7/2}}\,dx =$$
$$-\frac{2\left(3\left(i\,a\cos\left(dx+c\right)^2-i\,a\right)\sqrt{-\frac{1}{2}i\,e}\sin\left(dx+c\right)\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos\left(dx+\right.\right.\right.}{}$$

input
```
integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
-2/5*(3*(I*a*cos(d*x + c)^2 - I*a)*sqrt(-1/2*I*e)*sin(d*x + c)*weierstrass
Zeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*(
-I*a*cos(d*x + c)^2 + I*a)*sqrt(1/2*I*e)*sin(d*x + c)*weierstrassZeta(4, 0
, weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c))) + (3*a*cos(d*x
 + c)^3 - 4*a*cos(d*x + c) - b)*sqrt(e*sin(d*x + c)))/((d*e^4*cos(d*x + c)
^2 - d*e^4)*sin(d*x + c))
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))**(7/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{7/2}}\,dx = \int \frac{b\cos(dx+c)+a}{(e\sin(dx+c))^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)/(e*sin(d*x + c))^(7/2), x)
```

## Giac [F]

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{7/2}}\,dx = \int \frac{b\cos(dx+c)+a}{(e\sin(dx+c))^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)/(e*sin(d*x + c))^(7/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{7/2}}\,dx = \int \frac{a+b\,\cos(c+d\,x)}{(e\,\sin(c+d\,x))^{7/2}}\,dx$$

input
```
int((a + b*cos(c + d*x))/(e*sin(c + d*x))^(7/2),x)
```

output
```
int((a + b*cos(c + d*x))/(e*sin(c + d*x))^(7/2), x)
```

## Reduce [F]

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{7/2}}\,dx = \frac{\sqrt{e}\left(-2\sqrt{\sin\left(dx+c\right)}\,b+5\left(\int \frac{\sqrt{\sin(dx+c)}}{\sin(dx+c)^4}dx\right)\sin\left(dx+c\right)^3 ad\right)}{5\sin\left(dx+c\right)^3 d\,e^4}$$

input
```
int((a+b*cos(d*x+c))/(e*sin(d*x+c))^(7/2),x)
```

output
```
(sqrt(e)*( - 2*sqrt(sin(c + d*x))*b + 5*int(sqrt(sin(c + d*x))/sin(c + d*x
)**4,x)*sin(c + d*x)**3*a*d))/(5*sin(c + d*x)**3*d*e**4)
```

# 3.41 $\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 193

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}\,dx = \frac{10(11a^2+2b^2)\,e^4\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{231d\sqrt{e\sin(c+dx)}} - \frac{10(11a^2+2b^2)\,e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{231d} - \frac{2(11a^2+2b^2)\,e\cos(c+dx)(e\sin(c+dx))^{5/2}}{77d} + \frac{26ab(e\sin(c+dx))^{9/2}}{99de} + \frac{2b(a+b\cos(c+dx))(e\sin(c+dx))^{9/2}}{11de}$$

output
```
10/231*(11*a^2+2*b^2)*e^4*InverseJacobiAM(1/2*c-1/4*Pi+1/2*d*x,2^(1/2))*si
n(d*x+c)^(1/2)/d/(e*sin(d*x+c))^(1/2)-10/231*(11*a^2+2*b^2)*e^3*cos(d*x+c)
*(e*sin(d*x+c))^(1/2)/d-2/77*(11*a^2+2*b^2)*e*cos(d*x+c)*(e*sin(d*x+c))^(5
/2)/d+26/99*a*b*(e*sin(d*x+c))^(9/2)/d/e+2/11*b*(a+b*cos(d*x+c))*(e*sin(d*
x+c))^(9/2)/d/e
```

## Mathematica [A] (verified)

Time = 2.78 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.81

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}\,dx = \frac{\left(\frac{1}{6}(924ab-6(506a^2+71b^2)\cos(c+dx)-1232ab\cos(2(c+dx))+396a^2\cos(3(c+dx))-\right.}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(7/2),x]
```

output
```
((((924*a*b - 6*(506*a^2 + 71*b^2)*Cos[c + d*x] - 1232*a*b*Cos[2*(c + d*x)
] + 396*a^2*Cos[3*(c + d*x)] - 117*b^2*Cos[3*(c + d*x)] + 308*a*b*Cos[4*(c
 + d*x)] + 63*b^2*Cos[5*(c + d*x)])*Csc[c + d*x]^3)/6 - (40*(11*a^2 + 2*b^
2)*EllipticF[(-2*c + Pi - 2*d*x)/4, 2])/Sin[c + d*x]^(7/2))*(e*Sin[c + d*x
])^(7/2))/(924*d)
```

## Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.95, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 3171, 27, 3042, 3148, 3042, 3115, 3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e\sin(c+dx))^{7/2}(a+b\cos(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{7/2}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{3171}$$

$$\frac{2}{11}\int \frac{1}{2}\left(11a^2+13b\cos(c+dx)a+2b^2\right)(e\sin(c+dx))^{7/2}dx+\frac{2b(e\sin(c+dx))^{9/2}(a+b\cos(c+dx))}{11de}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{11}\int \left(11a^2+13b\cos(c+dx)a+2b^2\right)(e\sin(c+dx))^{7/2}dx+\frac{2b(e\sin(c+dx))^{9/2}(a+b\cos(c+dx))}{11de}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{11}\int \left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{7/2}\left(11a^2-13b\sin\left(c+dx-\frac{\pi}{2}\right)a+2b^2\right)dx+\frac{2b(e\sin(c+dx))^{9/2}(a+b\cos(c+dx))}{11de}$$

$$\Big\downarrow \; 3148$$

$$\frac{1}{11}\left(\left(11a^2+2b^2\right)\int (e\sin(c+dx))^{7/2}dx+\frac{26ab(e\sin(c+dx))^{9/2}}{9de}\right)+\frac{2b(e\sin(c+dx))^{9/2}(a+b\cos(c+dx))}{11de}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{11}\left(\left(11a^2+2b^2\right)\int (e\sin(c+dx))^{7/2}dx+\frac{26ab(e\sin(c+dx))^{9/2}}{9de}\right)+\frac{2b(e\sin(c+dx))^{9/2}(a+b\cos(c+dx))}{11de}$$

$$\Big\downarrow \; 3115$$

$$\frac{1}{11}\left(\left(11a^2+2b^2\right)\left(\frac{5}{7}e^2\int (e\sin(c+dx))^{3/2}dx-\frac{2e\cos(c+dx)(e\sin(c+dx))^{5/2}}{7d}\right)+\frac{26ab(e\sin(c+dx))^{9/2}}{9de}\right)+\frac{2b(e\sin(c+dx))^{9/2}(a+b\cos(c+dx))}{11de}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{11}\left(\left(11a^2+2b^2\right)\left(\frac{5}{7}e^2\int (e\sin(c+dx))^{3/2}dx-\frac{2e\cos(c+dx)(e\sin(c+dx))^{5/2}}{7d}\right)+\frac{26ab(e\sin(c+dx))^{9/2}}{9de}\right)+\frac{2b(e\sin(c+dx))^{9/2}(a+b\cos(c+dx))}{11de}$$

$$\Big\downarrow \; 3115$$

$$\frac{1}{11}\left(\left(11a^2+2b^2\right)\left(\frac{5}{7}e^2\left(\frac{1}{3}e^2\int\frac{1}{\sqrt{e\sin(c+dx)}}dx-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)-\frac{2e\cos(c+dx)(e\sin(c+}{7d}\right.\right.$$

$$\frac{2b(e\sin(c+dx))^{9/2}(a+b\cos(c+dx))}{11de}$$

$\big\downarrow$ 3042

$$\frac{1}{11}\left(\left(11a^2+2b^2\right)\left(\frac{5}{7}e^2\left(\frac{1}{3}e^2\int\frac{1}{\sqrt{e\sin(c+dx)}}dx-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)-\frac{2e\cos(c+dx)(e\sin(c+}{7d}\right.\right.$$

$$\frac{2b(e\sin(c+dx))^{9/2}(a+b\cos(c+dx))}{11de}$$

$\big\downarrow$ 3121

$$\frac{1}{11}\left(\left(11a^2+2b^2\right)\left(\frac{5}{7}e^2\left(\frac{e^2\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{3\sqrt{e\sin(c+dx)}}-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)-\frac{2e\cos(c+dx)(e\,s}{7d}\right.\right.$$

$$\frac{2b(e\sin(c+dx))^{9/2}(a+b\cos(c+dx))}{11de}$$

$\big\downarrow$ 3042

$$\frac{1}{11}\left(\left(11a^2+2b^2\right)\left(\frac{5}{7}e^2\left(\frac{e^2\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{3\sqrt{e\sin(c+dx)}}-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)-\frac{2e\cos(c+dx)(e\,s}{7d}\right.\right.$$

$$\frac{2b(e\sin(c+dx))^{9/2}(a+b\cos(c+dx))}{11de}$$

$\big\downarrow$ 3120

$$\frac{1}{11}\left(\left(11a^2+2b^2\right)\left(\frac{5}{7}e^2\left(\frac{2e^2\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{3d\sqrt{e\sin(c+dx)}}-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)-\frac{2e}{}\right.\right.$$

$$\frac{2b(e\sin(c+dx))^{9/2}(a+b\cos(c+dx))}{11de}$$

input
```
Int[(a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(7/2),x]
```

output
```
(2*b*(a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(9/2))/(11*d*e) + ((26*a*b*(e*S
in[c + d*x])^(9/2))/(9*d*e) + (11*a^2 + 2*b^2)*((-2*e*Cos[c + d*x]*(e*Sin[
c + d*x])^(5/2))/(7*d) + (5*e^2*((2*e^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*S
qrt[Sin[c + d*x]])/(3*d*Sqrt[e*Sin[c + d*x]]) - (2*e*Cos[c + d*x]*Sqrt[e*S
in[c + d*x]])/(3*d)))/7))/11
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3171
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[1/(m + p)   Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
 && GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 11.24 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.31

| method | result |
|---|---|
| default | $\frac{4ab(e\sin(dx+c))^{\frac{9}{2}}}{9e}-\frac{e^4\left(-42b^2\cos(dx+c)^6\sin(dx+c)-66a^2\cos(dx+c)^4\sin(dx+c)+72b^2\cos(dx+c)^4\sin(dx+c)+55\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\right.}{}$ |
| parts | $-\frac{a^2e^4\left(-6\sin(dx+c)^5+5\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sqrt{\sin(dx+c)}\,\operatorname{EllipticF}\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-4\sin(dx+c)^3+10\sin(dx\right.}{21\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$ |

input
```
int((a+cos(d*x+c)*b)^2*(e*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
(4/9/e*a*b*(e*sin(d*x+c))^(9/2)-1/231*e^4*(-42*b^2*cos(d*x+c)^6*sin(d*x+c)
-66*a^2*cos(d*x+c)^4*sin(d*x+c)+72*b^2*cos(d*x+c)^4*sin(d*x+c)+55*(1-sin(d
*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+
c))^(1/2),1/2*2^(1/2))*a^2+10*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*
sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^2+176*a^2*c
os(d*x+c)^2*sin(d*x+c)-10*b^2*cos(d*x+c)^2*sin(d*x+c))/cos(d*x+c)/(e*sin(d
*x+c))^(1/2))/d
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.01

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}\,dx=\frac{2\left(15\left(11a^2+2b^2\right)\sqrt{-\frac{1}{2}i\,e}\,e^3\,\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c))+15\,(11\right.}{}$$

input
```
integrate((a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
2/693*(15*(11*a^2 + 2*b^2)*sqrt(-1/2*I*e)*e^3*weierstrassPInverse(4, 0, co
s(d*x + c) + I*sin(d*x + c)) + 15*(11*a^2 + 2*b^2)*sqrt(1/2*I*e)*e^3*weier
strassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)) + (63*b^2*e^3*cos(d*x
+ c)^5 + 154*a*b*e^3*cos(d*x + c)^4 - 308*a*b*e^3*cos(d*x + c)^2 + 9*(11*a
^2 - 12*b^2)*e^3*cos(d*x + c)^3 + 154*a*b*e^3 - 3*(88*a^2 - 5*b^2)*e^3*cos
(d*x + c))*sqrt(e*sin(d*x + c)))/d
```

## Sympy [F(-1)]

Timed out.

$$\int (a + b\cos(c + dx))^2 (e\sin(c + dx))^{7/2} \, dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**2*(e*sin(d*x+c))**(7/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int (a + b\cos(c + dx))^2 (e\sin(c + dx))^{7/2} \, dx = \int (b\cos(dx + c) + a)^2 (e\sin(dx + c))^{\frac{7}{2}} \, dx$$

input
```
integrate((a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^2*(e*sin(d*x + c))^(7/2), x)
```

## Giac [F]

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}\,dx = \int (b\cos(dx+c)+a)^2(e\sin(dx+c))^{\frac{7}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^2*(e*sin(d*x + c))^(7/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}\,dx = \int (e\sin(c+dx))^{7/2}\,(a+b\cos(c+dx))^2\,dx$$

input
```
int((e*sin(c + d*x))^(7/2)*(a + b*cos(c + d*x))^2,x)
```

output
```
int((e*sin(c + d*x))^(7/2)*(a + b*cos(c + d*x))^2, x)
```

## Reduce [F]

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}\,dx = \frac{\sqrt{e}\,e^3\left(4\sqrt{\sin(dx+c)}\,\sin(dx+c)^4\,ab+9\left(\int\sqrt{\sin(dx+c)}\,\cos(dx+c)^2\sin(dx+c)^3\,dx\right)b\right.}{9d}$$

input
```
int((a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(7/2),x)
```

output
```
(sqrt(e)*e**3*(4*sqrt(sin(c + d*x))*sin(c + d*x)**4*a*b + 9*int(sqrt(sin(c
 + d*x))*cos(c + d*x)**2*sin(c + d*x)**3,x)*b**2*d + 9*int(sqrt(sin(c + d*
x))*sin(c + d*x)**3,x)*a**2*d))/(9*d)
```

# 3.42 $\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 154

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}\,dx = \frac{2(9a^2+2b^2)\,e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{15d\sqrt{\sin(c+dx)}} - \frac{2(9a^2+2b^2)\,e\cos(c+dx)(e\sin(c+dx))^{3/2}}{45d} + \frac{22ab(e\sin(c+dx))^{7/2}}{63de} + \frac{2b(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}{9de}$$

output
```
-2/15*(9*a^2+2*b^2)*e^2*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*si
n(d*x+c))^(1/2)/d/sin(d*x+c)^(1/2)-2/45*(9*a^2+2*b^2)*e*cos(d*x+c)*(e*sin(
d*x+c))^(3/2)/d+22/63*a*b*(e*sin(d*x+c))^(7/2)/d/e+2/9*b*(a+b*cos(d*x+c))*
(e*sin(d*x+c))^(7/2)/d/e
```

## Mathematica [A] (verified)

Time = 1.43 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.75

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}\,dx =$$
$$-\frac{(e\sin(c+dx))^{5/2}\left(84(9a^2+2b^2)\,E\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\,2\right)+(21(12a^2+b^2)\cos(c+dx)+5b(-36a+36\right.}{630d\sin^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[(a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(5/2),x]
```

output
```
-1/630*((e*Sin[c + d*x])^(5/2)*(84*(9*a^2 + 2*b^2)*EllipticE[(-2*c + Pi -
2*d*x)/4, 2] + (21*(12*a^2 + b^2)*Cos[c + d*x] + 5*b*(-36*a + 36*a*Cos[2*(
c + d*x)] + 7*b*Cos[3*(c + d*x)]))*Sin[c + d*x]^(3/2)))/(d*Sin[c + d*x]^(5
/2))
```

## Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.97, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 3171, 27, 3042, 3148, 3042, 3115, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{3171}$$

$$\frac{2}{9}\int \frac{1}{2}\left(9a^2+11b\cos(c+dx)a+2b^2\right)(e\sin(c+dx))^{5/2}dx+\frac{2b(e\sin(c+dx))^{7/2}(a+b\cos(c+dx))}{9de}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{9}\int \left(9a^2+11b\cos(c+dx)a+2b^2\right)(e\sin(c+dx))^{5/2}dx+\frac{2b(e\sin(c+dx))^{7/2}(a+b\cos(c+dx))}{9de}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{9}\int \left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2}\left(9a^2-11b\sin\left(c+dx-\frac{\pi}{2}\right)a+2b^2\right)dx+\frac{2b(e\sin(c+dx))^{7/2}(a+b\cos(c+dx))}{9de}$$

$$\Big\downarrow \ 3148$$

$$\frac{1}{9}\left(\left(9a^2+2b^2\right)\int (e\sin(c+dx))^{5/2}dx+\frac{22ab(e\sin(c+dx))^{7/2}}{7de}\right)+\frac{2b(e\sin(c+dx))^{7/2}(a+b\cos(c+dx))}{9de}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{9}\left(\left(9a^2+2b^2\right)\int (e\sin(c+dx))^{5/2}dx+\frac{22ab(e\sin(c+dx))^{7/2}}{7de}\right)+\frac{2b(e\sin(c+dx))^{7/2}(a+b\cos(c+dx))}{9de}$$

$$\Big\downarrow \ 3115$$

$$\frac{1}{9}\left(\left(9a^2+2b^2\right)\left(\frac{3}{5}e^2\int \sqrt{e\sin(c+dx)}dx-\frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d}\right)+\frac{22ab(e\sin(c+dx))^{7/2}}{7de}\right)+\frac{2b(e\sin(c+dx))^{7/2}(a+b\cos(c+dx))}{9de}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{9}\left(\left(9a^2+2b^2\right)\left(\frac{3}{5}e^2\int \sqrt{e\sin(c+dx)}dx-\frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d}\right)+\frac{22ab(e\sin(c+dx))^{7/2}}{7de}\right)+\frac{2b(e\sin(c+dx))^{7/2}(a+b\cos(c+dx))}{9de}$$

$$\Big\downarrow \ 3121$$

$$\frac{1}{9}\left((9a^2+2b^2)\left(\frac{3e^2\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{5\sqrt{\sin(c+dx)}}-\frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d}\right)+\frac{22ab(e\sin(c+dx}{7de}\right.$$
$$\frac{2b(e\sin(c+dx))^{7/2}(a+b\cos(c+dx))}{9de}$$

$\Big\downarrow$ 3042

$$\frac{1}{9}\left((9a^2+2b^2)\left(\frac{3e^2\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{5\sqrt{\sin(c+dx)}}-\frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d}\right)+\frac{22ab(e\sin(c+dx}{7de}\right.$$
$$\frac{2b(e\sin(c+dx))^{7/2}(a+b\cos(c+dx))}{9de}$$

$\Big\downarrow$ 3119

$$\frac{1}{9}\left((9a^2+2b^2)\left(\frac{6e^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}}-\frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d}\right)+\frac{22ab(e\sin(c+}{7de}\right.$$
$$\frac{2b(e\sin(c+dx))^{7/2}(a+b\cos(c+dx))}{9de}$$

input
```
Int[(a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(5/2),x]
```

output
```
(2*b*(a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(7/2))/(9*d*e) + ((22*a*b*(e*Si
n[c + d*x])^(7/2))/(7*d*e) + (9*a^2 + 2*b^2)*((6*e^2*EllipticE[(c - Pi/2 +
 d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(5*d*Sqrt[Sin[c + d*x]]) - (2*e*Cos[c +
d*x]*(e*Sin[c + d*x])^(3/2))/(5*d)))/9
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3171
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[1/(m + p)   Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
 && GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 331 vs. $2(136) = 272$.

Time = 10.17 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.16

| method | result |
|---|---|
| default | $\frac{\frac{4ab(e\sin(dx+c))^{\frac{7}{2}}}{7e} - \frac{e^3\left(10\sin(dx+c)^6 b^2+54\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sqrt{\sin(dx+c)}\,\operatorname{EllipticE}\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)a^2+12\sqrt{1-\sin(dx+c)}\,\sqrt{}\right.}{}}{}$ |
| parts | $-\frac{a^2e^3\left(6\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sqrt{\sin(dx+c)}\,\operatorname{EllipticE}\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-3\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sqrt{\sin(}\right.}{5\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$ |

input
```
int((a+cos(d*x+c)*b)^2*(e*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(4/7/e*a*b*(e*sin(d*x+c))^(7/2)-1/45*e^3*(10*sin(d*x+c)^6*b^2+54*(1-sin(d*
x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticE((1-sin(d*x+c
))^(1/2),1/2*2^(1/2))*a^2+12*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*s
in(d*x+c)^(1/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^2-27*(1-sin(
d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x
+c))^(1/2),1/2*2^(1/2))*a^2-6*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*
sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^2-18*a^2*si
n(d*x+c)^4-14*sin(d*x+c)^4*b^2+18*a^2*sin(d*x+c)^2+4*b^2*sin(d*x+c)^2)/cos
(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.09

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}\,dx =$$

$$-\frac{2\left(-21i\left(9a^2+2b^2\right)\sqrt{-\frac{1}{2}i\,e}e^2\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right.\right.\right.}{}$$

input
```
integrate((a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-2/315*(-21*I*(9*a^2 + 2*b^2)*sqrt(-1/2*I*e)*e^2*weierstrassZeta(4, 0, wei
erstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) + 21*I*(9*a^2 + 2*b
^2)*sqrt(1/2*I*e)*e^2*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(
d*x + c) - I*sin(d*x + c))) + (35*b^2*e^2*cos(d*x + c)^3 + 90*a*b*e^2*cos(
d*x + c)^2 - 90*a*b*e^2 + 21*(3*a^2 - b^2)*e^2*cos(d*x + c))*sqrt(e*sin(d*
x + c))*sin(d*x + c))/d
```

## Sympy [F(-1)]

Timed out.

$$\int (a + b\cos(c + dx))^2 (e\sin(c + dx))^{5/2}\, dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**2*(e*sin(d*x+c))**(5/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int (a + b\cos(c + dx))^2 (e\sin(c + dx))^{5/2}\, dx = \int (b\cos(dx + c) + a)^2 (e\sin(dx + c))^{\frac{5}{2}}\, dx$$

input
```
integrate((a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^2*(e*sin(d*x + c))^(5/2), x)
```

## Giac [F]

$$\int (a + b\cos(c + dx))^2 (e\sin(c + dx))^{5/2}\, dx = \int (b\cos(dx + c) + a)^2 (e\sin(dx + c))^{\frac{5}{2}}\, dx$$

input
```
integrate((a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^2*(e*sin(d*x + c))^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}\,dx = \int (e\,\sin(c+d\,x))^{5/2}\,(a+b\,\cos(c+d\,x))^2\,dx$$

input
```
int((e*sin(c + d*x))^(5/2)*(a + b*cos(c + d*x))^2,x)
```

output
```
int((e*sin(c + d*x))^(5/2)*(a + b*cos(c + d*x))^2, x)
```

## Reduce [F]

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}\,dx = \frac{\sqrt{e}\,e^2\left(4\sqrt{\sin(dx+c)}\,\sin(dx+c)^3\,ab+7\left(\int\sqrt{\sin(dx+c)}\,\cos(dx+c)^2\sin(dx+c)^2\,dx\right)b\right.}{7d}$$

input
```
int((a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(5/2),x)
```

output
```
(sqrt(e)*e**2*(4*sqrt(sin(c + d*x))*sin(c + d*x)**3*a*b + 7*int(sqrt(sin(c
 + d*x))*cos(c + d*x)**2*sin(c + d*x)**2,x)*b**2*d + 7*int(sqrt(sin(c + d*
x))*sin(c + d*x)**2,x)*a**2*d))/(7*d)
```

# 3.43 $\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 154

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}\,dx = \frac{2(7a^2+2b^2)\,e^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{21d\sqrt{e\sin(c+dx)}} - \frac{2(7a^2+2b^2)\,e\cos(c+dx)\sqrt{e\sin(c+dx)}}{21d} + \frac{18ab(e\sin(c+dx))^{5/2}}{35de} + \frac{2b(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}{7de}$$

output
```
2/21*(7*a^2+2*b^2)*e^2*InverseJacobiAM(1/2*c-1/4*Pi+1/2*d*x,2^(1/2))*sin(d
*x+c)^(1/2)/d/(e*sin(d*x+c))^(1/2)-2/21*(7*a^2+2*b^2)*e*cos(d*x+c)*(e*sin(
d*x+c))^(1/2)/d+18/35*a*b*(e*sin(d*x+c))^(5/2)/d/e+2/7*b*(a+b*cos(d*x+c))*
(e*sin(d*x+c))^(5/2)/d/e
```

## Mathematica [A] (verified)

Time = 1.52 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.76

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}\,dx = \frac{\left(-\frac{1}{2}(5(28a^2+5b^2)\cos(c+dx)+3b(-28a+28a\cos(2(c+dx))+5b\cos(3(c+dx))))\csc(c\right.}{105d}$$

input
```
Integrate[(a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(3/2),x]
```

output
```
((-1/2*((5*(28*a^2 + 5*b^2)*Cos[c + d*x] + 3*b*(-28*a + 28*a*Cos[2*(c + d*
x)] + 5*b*Cos[3*(c + d*x)]))*Csc[c + d*x]) - (10*(7*a^2 + 2*b^2)*EllipticF
[(-2*c + Pi - 2*d*x)/4, 2])/Sin[c + d*x]^(3/2))*(e*Sin[c + d*x])^(3/2))/(1
05*d)
```

## Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.97, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 3171, 27, 3042, 3148, 3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{3171}$$

$$\frac{2}{7}\int \frac{1}{2}\left(7a^2+9b\cos(c+dx)a+2b^2\right)(e\sin(c+dx))^{3/2}dx+\frac{2b(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}{7de}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\int \left(7a^2+9b\cos(c+dx)a+2b^2\right)(e\sin(c+dx))^{3/2}dx+\frac{2b(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}{7de}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\int \left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}\left(7a^2-9b\sin\left(c+dx-\frac{\pi}{2}\right)a+2b^2\right)dx+\frac{2b(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}{7de}$$

$$\Big\downarrow \text{3148}$$

$$\frac{1}{7}\left(\left(7a^2+2b^2\right)\int (e\sin(c+dx))^{3/2}dx+\frac{18ab(e\sin(c+dx))^{5/2}}{5de}\right)+\frac{2b(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}{7de}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\left(7a^2+2b^2\right)\int (e\sin(c+dx))^{3/2}dx+\frac{18ab(e\sin(c+dx))^{5/2}}{5de}\right)+\frac{2b(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}{7de}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{7}\left(\left(7a^2+2b^2\right)\left(\frac{1}{3}e^2\int \frac{1}{\sqrt{e\sin(c+dx)}}dx-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)+\frac{18ab(e\sin(c+dx))^{5/2}}{5de}\right)+\frac{2b(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}{7de}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\left(7a^2+2b^2\right)\left(\frac{1}{3}e^2\int \frac{1}{\sqrt{e\sin(c+dx)}}dx-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)+\frac{18ab(e\sin(c+dx))^{5/2}}{5de}\right)+\frac{2b(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}{7de}$$

$$\Big\downarrow \text{3121}$$

$$\frac{1}{7}\left(\left(7a^2+2b^2\right)\left(\frac{e^2\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}}dx}{3\sqrt{e\sin(c+dx)}}-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)+\frac{18ab(e\sin(c+dx))^{5/2}}{5de}\right)+$$
$$\frac{2b(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}{7de}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\left(7a^2+2b^2\right)\left(\frac{e^2\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}}dx}{3\sqrt{e\sin(c+dx)}}-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)+\frac{18ab(e\sin(c+dx))^{5/2}}{5de}\right)+$$
$$\frac{2b(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}{7de}$$

$$\Big\downarrow \text{3120}$$

$$\frac{1}{7}\left(\left(7a^2+2b^2\right)\left(\frac{2e^2\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{3d\sqrt{e\sin(c+dx)}}-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)+\frac{18ab(e\sin(}{5}\right.$$
$$\frac{2b(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}{7de}$$

input
```
Int[(a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(3/2),x]
```

output
```
(2*b*(a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(5/2))/(7*d*e) + ((18*a*b*(e*Si
n[c + d*x])^(5/2))/(5*d*e) + (7*a^2 + 2*b^2)*((2*e^2*EllipticF[(c - Pi/2 +
 d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*d*Sqrt[e*Sin[c + d*x]]) - (2*e*Cos[c +
d*x]*Sqrt[e*Sin[c + d*x]])/(3*d)))/7
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3171
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[1/(m + p)   Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
 && GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 2.83 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.49

| method | result |
|---|---|
| default | $-\frac{e^{2}\left(30 b^{2} \cos \left(d x+c\right)^{4} \sin \left(d x+c\right)+35 \sqrt{1-\sin \left(d x+c\right)}\, \sqrt{2+2 \sin \left(d x+c\right)}\, \sqrt{\sin \left(d x+c\right)}\, \operatorname{EllipticF}\left(\sqrt{1-\sin \left(d x+c\right)}, \frac{\sqrt{2}}{2}\right) a^{2}+10 \sqrt{1-\sin}\right.}{}$ |
| parts | $-\frac{a^{2} e^{2}\left(\sqrt{1-\sin \left(d x+c\right)}\, \sqrt{2+2 \sin \left(d x+c\right)}\, \sqrt{\sin \left(d x+c\right)}\, \operatorname{EllipticF}\left(\sqrt{1-\sin \left(d x+c\right)}, \frac{\sqrt{2}}{2}\right)-2 \sin \left(d x+c\right)^{3}+2 \sin \left(d x+c\right)\right)}{3 \cos \left(d x+c\right) \sqrt{e \sin \left(d x+c\right)}\, d}-\frac{2 b^{2} e^{2}\left(3 \mathrm{s}\right.}{}$ |

input
```
int((a+cos(d*x+c)*b)^2*(e*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/105/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*e^2*(30*b^2*cos(d*x+c)^4*sin(d*x+c)
+35*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticF
((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2+10*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*
x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b
^2+84*a*b*cos(d*x+c)^3*sin(d*x+c)+70*a^2*cos(d*x+c)^2*sin(d*x+c)-10*b^2*co
s(d*x+c)^2*sin(d*x+c)-84*a*b*cos(d*x+c)*sin(d*x+c))/d
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.94

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}\,dx = \frac{2\left(5\left(7a^2+2b^2\right)\sqrt{-\frac{1}{2}i\,e}e\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c))+5\left(7a^2+\right.\right.}{}$$

input
```
integrate((a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
2/105*(5*(7*a^2 + 2*b^2)*sqrt(-1/2*I*e)*e*weierstrassPInverse(4, 0, cos(d*
x + c) + I*sin(d*x + c)) + 5*(7*a^2 + 2*b^2)*sqrt(1/2*I*e)*e*weierstrassPI
nverse(4, 0, cos(d*x + c) - I*sin(d*x + c)) - (15*b^2*e*cos(d*x + c)^3 + 4
2*a*b*e*cos(d*x + c)^2 - 42*a*b*e + 5*(7*a^2 - b^2)*e*cos(d*x + c))*sqrt(e
*sin(d*x + c)))/d
```

## Sympy [F]

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}\,dx = \int (e\sin(c+dx))^{\frac{3}{2}}\,(a+b\cos(c+dx))^2\,dx$$

input
```
integrate((a+b*cos(d*x+c))**2*(e*sin(d*x+c))**(3/2),x)
```

output
```
Integral((e*sin(c + d*x))**(3/2)*(a + b*cos(c + d*x))**2, x)
```

## Maxima [F]

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}\,dx = \int (b\cos(dx+c)+a)^2(e\sin(dx+c))^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^2*(e*sin(d*x + c))^(3/2), x)
```

## Giac [F]

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}\,dx = \int (b\cos(dx+c)+a)^2(e\sin(dx+c))^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^2*(e*sin(d*x + c))^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}\,dx = \int (e\,\sin(c+d\,x))^{3/2}\,(a+b\,\cos(c+d\,x))^2\,dx$$

input
```
int((e*sin(c + d*x))^(3/2)*(a + b*cos(c + d*x))^2,x)
```

output
```
int((e*sin(c + d*x))^(3/2)*(a + b*cos(c + d*x))^2, x)
```

## Reduce [F]

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}\,dx = \frac{\sqrt{e}\,e\left(4\sqrt{\sin(dx+c)}\,\sin(dx+c)^2\,ab+5\left(\int\sqrt{\sin(dx+c)}\,\cos(dx+c)^2\sin(dx+c)\,dx\right)b^2 d\right.}{5d}$$

input
```
int((a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(3/2),x)
```

output
```
(sqrt(e)*e*(4*sqrt(sin(c + d*x))*sin(c + d*x)**2*a*b + 5*int(sqrt(sin(c +
d*x))*cos(c + d*x)**2*sin(c + d*x),x)*b**2*d + 5*int(sqrt(sin(c + d*x))*si
n(c + d*x),x)*a**2*d))/(5*d)
```

# 3.44 $\int (a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 114

$$\int (a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}\,dx = \frac{2(5a^2+2b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}} + \frac{14ab(e\sin(c+dx))^{3/2}}{15de} + \frac{2b(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}{5de}$$

output
```
-2/5*(5*a^2+2*b^2)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x
+c))^(1/2)/d/sin(d*x+c)^(1/2)+14/15*a*b*(e*sin(d*x+c))^(3/2)/d/e+2/5*b*(a+
b*cos(d*x+c))*(e*sin(d*x+c))^(3/2)/d/e
```

## Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.73

$$\int (a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}\,dx = \frac{2\sqrt{e\sin(c+dx)}\left(-3(5a^2+2b^2)\,E\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)+b(10a+3b\cos(c+dx))\sin^{\frac{3}{2}}(c+dx)\right)}{15d\sqrt{\sin(c+dx)}}$$

input
```
Integrate[(a + b*Cos[c + d*x])^2*Sqrt[e*Sin[c + d*x]],x]
```

output
```
(2*Sqrt[e*Sin[c + d*x]]*(-3*(5*a^2 + 2*b^2)*EllipticE[(-2*c + Pi - 2*d*x)/
4, 2] + b*(10*a + 3*b*Cos[c + d*x])*Sin[c + d*x]^(3/2)))/(15*d*Sqrt[Sin[c
+ d*x]])
```

## Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 3171, 27, 3042, 3148, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{3171}$$

$$\frac{2}{5}\int \frac{1}{2}\left(5a^2+7b\cos(c+dx)a+2b^2\right)\sqrt{e\sin(c+dx)}dx+\frac{2b(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}{5de}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\int \left(5a^2+7b\cos(c+dx)a+2b^2\right)\sqrt{e\sin(c+dx)}dx+\frac{2b(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}{5de}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\int \sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(5a^2-7b\sin\left(c+dx-\frac{\pi}{2}\right)a+2b^2\right)dx+\frac{2b(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}{5de}$$

$$\Big\downarrow \text{3148}$$

$$\frac{1}{5}\left(\left(5a^2+2b^2\right)\int \sqrt{e\sin(c+dx)}dx+\frac{14ab(e\sin(c+dx))^{3/2}}{3de}\right)+\frac{2b(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}{5de}$$

$\big\downarrow$ 3042

$$\frac{1}{5}\left(\left(5a^2+2b^2\right)\int \sqrt{e\sin(c+dx)}dx+\frac{14ab(e\sin(c+dx))^{3/2}}{3de}\right)+\frac{2b(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}{5de}$$

$\big\downarrow$ 3121

$$\frac{1}{5}\left(\frac{\left(5a^2+2b^2\right)\sqrt{e\sin(c+dx)}\int \sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}+\frac{14ab(e\sin(c+dx))^{3/2}}{3de}\right)+\frac{2b(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}{5de}$$

$\big\downarrow$ 3042

$$\frac{1}{5}\left(\frac{\left(5a^2+2b^2\right)\sqrt{e\sin(c+dx)}\int \sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}+\frac{14ab(e\sin(c+dx))^{3/2}}{3de}\right)+\frac{2b(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}{5de}$$

$\big\downarrow$ 3119

$$\frac{1}{5}\left(\frac{2\left(5a^2+2b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}+\frac{14ab(e\sin(c+dx))^{3/2}}{3de}\right)+\frac{2b(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}{5de}$$

input
```
Int[(a + b*Cos[c + d*x])^2*Sqrt[e*Sin[c + d*x]],x]
```

output
```
(2*b*(a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2))/(5*d*e) + ((2*(5*a^2 + 2
*b^2)*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(d*Sqrt[Sin[c
 + d*x]]) + (14*a*b*(e*Sin[c + d*x])^(3/2))/(3*d*e))/5
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3171
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[1/(m + p)   Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
 && GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(100) = 200$.

Time = 3.21 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.39

| method | result |
|---|---|
| parts | $-\frac{a^{2} e \sqrt{1-\sin(dx+c)}\, \sqrt{2+2\sin(dx+c)}\, \sqrt{\sin(dx+c)}\, \left(2\,\mathrm{EllipticE}\left(\sqrt{1-\sin(dx+c)}, \frac{\sqrt{2}}{2}\right)-\mathrm{EllipticF}\left(\sqrt{1-\sin(dx+c)}, \frac{\sqrt{2}}{2}\right)\right)}{\cos(dx+c)\sqrt{e\sin(dx+c)}\, d} - \frac{2b^{2}e}{}$ |
| default | $-\frac{e\left(30\sqrt{1-\sin(dx+c)}\, \sqrt{2+2\sin(dx+c)}\, \sqrt{\sin(dx+c)}\, \mathrm{EllipticE}\left(\sqrt{1-\sin(dx+c)}, \frac{\sqrt{2}}{2}\right)a^{2}+12\sqrt{1-\sin(dx+c)}\, \sqrt{2+2\sin(dx+c)}\, \sqrt{\sin}\right.}{}$ |

input
```
int((a+cos(d*x+c)*b)^2*(e*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-a^2*e*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*(2*Ell
ipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-EllipticF((1-sin(d*x+c))^(1/2),1/
2*2^(1/2)))/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/d-2/5*b^2*e*(2*(1-sin(d*x+c))^
(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticE((1-sin(d*x+c))^(1/
2),1/2*2^(1/2))-(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/
2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))+cos(d*x+c)^4-cos(d*x+c)^2)/
cos(d*x+c)/(e*sin(d*x+c))^(1/2)/d+4/3*a*b*(e*sin(d*x+c))^(3/2)/d/e
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.03

$$\int (a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}\,dx$$
$$= \frac{2\left(\left(3\,b^{2}\cos\left(dx+c\right)+10\,ab\right)\sqrt{e\sin\left(dx+c\right)}\sin\left(dx+c\right)-3\left(-5i\,a^{2}-2i\,b^{2}\right)\sqrt{-\frac{1}{2}i\,e}\,\mathrm{weierstrassZeta}(4,0\right.}{}$$

input
```
integrate((a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
2/15*((3*b^2*cos(d*x + c) + 10*a*b)*sqrt(e*sin(d*x + c))*sin(d*x + c) - 3*
(-5*I*a^2 - 2*I*b^2)*sqrt(-1/2*I*e)*weierstrassZeta(4, 0, weierstrassPInve
rse(4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*(5*I*a^2 + 2*I*b^2)*sqrt(1/2
*I*e)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*sin
(d*x + c))))/d
```

## Sympy [F]

$$\int (a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}\,dx=\int \sqrt{e\sin{(c+dx)}}(a+b\cos{(c+dx)})^2\,dx$$

input
```
integrate((a+b*cos(d*x+c))**2*(e*sin(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(e*sin(c + d*x))*(a + b*cos(c + d*x))**2, x)
```

## Maxima [F]

$$\int (a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}\,dx=\int (b\cos{(dx+c)}+a)^2\sqrt{e\sin{(dx+c)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^2*sqrt(e*sin(d*x + c)), x)
```

## Giac [F]

$$\int (a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}\,dx = \int (b\cos(dx+c)+a)^2\sqrt{e\sin(dx+c)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^2*sqrt(e*sin(d*x + c)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}\,dx = \int \sqrt{e\,\sin(c+d\,x)}\,(a+b\,\cos(c+d\,x))^2\,dx$$

input
```
int((e*sin(c + d*x))^(1/2)*(a + b*cos(c + d*x))^2,x)
```

output
```
int((e*sin(c + d*x))^(1/2)*(a + b*cos(c + d*x))^2, x)
```

## Reduce [F]

$$\int (a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}\,dx = \frac{\sqrt{e}\left(4\sqrt{\sin(dx+c)}\,\sin(dx+c)\,ab+3\left(\int\sqrt{\sin(dx+c)}dx\right)a^2d+3\left(\int\sqrt{\sin(dx+c)}\,\cos(dx+c)^2\,dx\right.\right.}{3d}$$

input
```
int((a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(1/2),x)
```

output
```
(sqrt(e)*(4*sqrt(sin(c + d*x))*sin(c + d*x)*a*b + 3*int(sqrt(sin(c + d*x))
,x)*a**2*d + 3*int(sqrt(sin(c + d*x))*cos(c + d*x)**2,x)*b**2*d))/(3*d)
```

# 3.45 $\int \frac{(a+b\cos(c+dx))^2}{\sqrt{e\sin(c+dx)}}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 114

$$\int \frac{(a+b\cos(c+dx))^2}{\sqrt{e\sin(c+dx)}}\,dx = \frac{2(3a^2+2b^2)\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3d\sqrt{e\sin(c+dx)}} + \frac{10ab\sqrt{e\sin(c+dx)}}{3de} + \frac{2b(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}{3de}$$

output
```
2/3*(3*a^2+2*b^2)*InverseJacobiAM(1/2*c-1/4*Pi+1/2*d*x,2^(1/2))*sin(d*x+c)
^(1/2)/d/(e*sin(d*x+c))^(1/2)+10/3*a*b*(e*sin(d*x+c))^(1/2)/d/e+2/3*b*(a+b
*cos(d*x+c))*(e*sin(d*x+c))^(1/2)/d/e
```

## Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.69

$$\int \frac{(a+b\cos(c+dx))^2}{\sqrt{e\sin(c+dx)}}\,dx = \frac{-2(3a^2+2b^2)\,\text{EllipticF}\left(\frac{1}{4}(-2c+\pi-2dx),2\right)\sqrt{\sin(c+dx)}+2b(6a+b\cos(c+dx))\sin(c+dx)}{3d\sqrt{e\sin(c+dx)}}$$

input
```
Integrate[(a + b*Cos[c + d*x])^2/Sqrt[e*Sin[c + d*x]],x]
```

output
```
(-2*(3*a^2 + 2*b^2)*EllipticF[(-2*c + Pi - 2*d*x)/4, 2]*Sqrt[Sin[c + d*x]]
 + 2*b*(6*a + b*Cos[c + d*x])*Sin[c + d*x])/(3*d*Sqrt[e*Sin[c + d*x]])
```

## Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 3171, 27, 3042, 3148, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^2}{\sqrt{e\sin(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^2}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3171}$$

$$\frac{2}{3}\int \frac{3a^2+5b\cos(c+dx)a+2b^2}{2\sqrt{e\sin(c+dx)}}dx+\frac{2b\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}{3de}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{3a^2+5b\cos(c+dx)a+2b^2}{\sqrt{e\sin(c+dx)}}dx+\frac{2b\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}{3de}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{3a^2-5b\sin\left(c+dx-\frac{\pi}{2}\right)a+2b^2}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}dx+\frac{2b\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}{3de}$$

$$\Big\downarrow \text{3148}$$

$$\frac{1}{3}\left(\left(3a^2+2b^2\right)\int \frac{1}{\sqrt{e\sin(c+dx)}}dx+\frac{10ab\sqrt{e\sin(c+dx)}}{de}\right)+\frac{2b\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}{3de}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(\left(3a^2+2b^2\right)\int \frac{1}{\sqrt{e\sin(c+dx)}}dx+\frac{10ab\sqrt{e\sin(c+dx)}}{de}\right)+\frac{2b\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}{3de}$$

$\Big\downarrow$ 3121

$$\frac{1}{3}\left(\frac{\left(3a^2+2b^2\right)\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}}dx}{\sqrt{e\sin(c+dx)}}+\frac{10ab\sqrt{e\sin(c+dx)}}{de}\right)+\frac{2b\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}{3de}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(\frac{\left(3a^2+2b^2\right)\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}}dx}{\sqrt{e\sin(c+dx)}}+\frac{10ab\sqrt{e\sin(c+dx)}}{de}\right)+\frac{2b\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}{3de}$$

$\Big\downarrow$ 3120

$$\frac{1}{3}\left(\frac{2\left(3a^2+2b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}+\frac{10ab\sqrt{e\sin(c+dx)}}{de}\right)+\frac{2b\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}{3de}$$

input
```
Int[(a + b*Cos[c + d*x])^2/Sqrt[e*Sin[c + d*x]],x]
```

output
```
(2*b*(a + b*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]])/(3*d*e) + ((2*(3*a^2 + 2*b
^2)*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(d*Sqrt[e*Sin[c +
 d*x]]) + (10*a*b*Sqrt[e*Sin[c + d*x]])/(d*e))/3
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3171
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[1/(m + p)   Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
 && GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 2.74 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.49

| method | result |
|---|---|
| default | $-\frac{3\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sqrt{\sin(dx+c)}\,\mathrm{EllipticF}\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)a^2+2\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sqrt{\sin(dx-}}{3\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$ |
| parts | $-\frac{a^2\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sqrt{\sin(dx+c)}\,\mathrm{EllipticF}\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)}{\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}+\frac{b^2\left(-\frac{2\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sqrt{\sin(d}}{3}\right.}{\cos(}$ |

input
```
int((a+cos(d*x+c)*b)^2/(e*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*(3*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+
c))^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2
+2*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticF(
(1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^2-2*b^2*cos(d*x+c)^2*sin(d*x+c)-12*a*b
*cos(d*x+c)*sin(d*x+c))/d
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\cos(c+dx))^2}{\sqrt{e\sin(c+dx)}}\,dx = \frac{2\left((3a^2+2b^2)\sqrt{-\frac{1}{2}ie}\,\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c))+(3a^2+2b^2)\sqrt{\frac{1}{2}ie}\,\mathrm{weier}\right.}{3de}$$

input
```
integrate((a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
2/3*((3*a^2 + 2*b^2)*sqrt(-1/2*I*e)*weierstrassPInverse(4, 0, cos(d*x + c)
 + I*sin(d*x + c)) + (3*a^2 + 2*b^2)*sqrt(1/2*I*e)*weierstrassPInverse(4,
0, cos(d*x + c) - I*sin(d*x + c)) + (b^2*cos(d*x + c) + 6*a*b)*sqrt(e*sin(
d*x + c)))/(d*e)
```

## Sympy [F]

$$\int \frac{(a+b\cos(c+dx))^2}{\sqrt{e\sin(c+dx)}}\,dx = \int \frac{(a+b\cos{(c+dx)})^2}{\sqrt{e\sin{(c+dx)}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**2/(e*sin(d*x+c))**(1/2),x)
```

output
```
Integral((a + b*cos(c + d*x))**2/sqrt(e*sin(c + d*x)), x)
```

## Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^2}{\sqrt{e\sin(c+dx)}}\,dx = \int \frac{(b\cos{(dx+c)}+a)^2}{\sqrt{e\sin{(dx+c)}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^2/sqrt(e*sin(d*x + c)), x)
```

## Giac [F]

$$\int \frac{(a+b\cos(c+dx))^2}{\sqrt{e\sin(c+dx)}}\,dx = \int \frac{(b\cos{(dx+c)}+a)^2}{\sqrt{e\sin{(dx+c)}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^2/sqrt(e*sin(d*x + c)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^2}{\sqrt{e\sin(c+dx)}}\,dx = \int \frac{\left(a+b\,\cos\left(c+d\,x\right)\right)^2}{\sqrt{e\,\sin\left(c+d\,x\right)}}\,dx$$

input
```
int((a + b*cos(c + d*x))^2/(e*sin(c + d*x))^(1/2),x)
```

output
```
int((a + b*cos(c + d*x))^2/(e*sin(c + d*x))^(1/2), x)
```

## Reduce [F]

$$\int \frac{(a+b\cos(c+dx))^2}{\sqrt{e\sin(c+dx)}}\,dx = \frac{\sqrt{e}\left(4\sqrt{\sin(dx+c)}\,ab+\left(\int \frac{\sqrt{\sin(dx+c)}}{\sin(dx+c)}dx\right)a^2d+\left(\int \frac{\sqrt{\sin(dx+c)}\cos(dx+c)^2}{\sin(dx+c)}dx\right)b^2d\right)}{de}$$

input
```
int((a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(1/2),x)
```

output
```
(sqrt(e)*(4*sqrt(sin(c + d*x))*a*b + int(sqrt(sin(c + d*x))/sin(c + d*x),x
)*a**2*d + int((sqrt(sin(c + d*x))*cos(c + d*x)**2)/sin(c + d*x),x)*b**2*d
))/(d*e)
```

# 3.46 $\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{3/2}}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 118

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{3/2}}\,dx = -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))}{de\sqrt{e\sin(c+dx)}} - \frac{2(a^2+2b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{e\sin(c+dx)}}{de^2\sqrt{\sin(c+dx)}} - \frac{2ab(e\sin(c+dx))^{3/2}}{de^3}$$

output
```
-2*(b+a*cos(d*x+c))*(a+b*cos(d*x+c))/d/e/(e*sin(d*x+c))^(1/2)+2*(a^2+2*b^2
)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/d/e^2/
sin(d*x+c)^(1/2)-2*a*b*(e*sin(d*x+c))^(3/2)/d/e^3
```

## Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.64

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{3/2}}\,dx = \frac{-4ab-2(a^2+b^2)\cos(c+dx)+2(a^2+2b^2)\,E\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\,2\right)\sqrt{\sin(c}}{de\sqrt{e\sin(c+dx)}}$$

input
```
Integrate[(a + b*Cos[c + d*x])^2/(e*Sin[c + d*x])^(3/2),x]
```

output
```
(-4*a*b - 2*(a^2 + b^2)*Cos[c + d*x] + 2*(a^2 + 2*b^2)*EllipticE[(-2*c + P
i - 2*d*x)/4, 2]*Sqrt[Sin[c + d*x]])/(d*e*Sqrt[e*Sin[c + d*x]])
```

## Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 3170, 27, 3042, 3148, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^2}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3170}$$

$$-\frac{2\int \frac{1}{2}\left(a^2+3b\cos(c+dx)a+2b^2\right)\sqrt{e\sin(c+dx)}dx}{e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))}{de\sqrt{e\sin(c+dx)}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int \left(a^2+3b\cos(c+dx)a+2b^2\right)\sqrt{e\sin(c+dx)}dx}{e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))}{de\sqrt{e\sin(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\int \sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a^2-3b\sin\left(c+dx-\frac{\pi}{2}\right)a+2b^2\right)dx}{e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))}{de\sqrt{e\sin(c+dx)}}$$

$$\Big\downarrow \text{ 3148}$$

$$-\frac{\left(a^2+2b^2\right)\int \sqrt{e\sin(c+dx)}dx+\frac{2ab(e\sin(c+dx))^{3/2}}{de}}{e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))}{de\sqrt{e\sin(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(a^2+2b^2)\int\sqrt{e\sin(c+dx)}dx+\frac{2ab(e\sin(c+dx))^{3/2}}{de}}{e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))}{de\sqrt{e\sin(c+dx)}}$$

$$\Big\downarrow \text{3121}$$

$$-\frac{\frac{(a^2+2b^2)\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}+\frac{2ab(e\sin(c+dx))^{3/2}}{de}}{e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))}{de\sqrt{e\sin(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{(a^2+2b^2)\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}+\frac{2ab(e\sin(c+dx))^{3/2}}{de}}{e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))}{de\sqrt{e\sin(c+dx)}}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{\frac{2(a^2+2b^2)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}+\frac{2ab(e\sin(c+dx))^{3/2}}{de}}{e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))}{de\sqrt{e\sin(c+dx)}}$$

input
```
Int[(a + b*Cos[c + d*x])^2/(e*Sin[c + d*x])^(3/2),x]
```

output
```
(-2*(b + a*Cos[c + d*x])*(a + b*Cos[c + d*x]))/(d*e*Sqrt[e*Sin[c + d*x]])
- ((2*(a^2 + 2*b^2)*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])
/(d*Sqrt[Sin[c + d*x]]) + (2*a*b*(e*Sin[c + d*x])^(3/2))/(d*e))/e^2
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3170
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Simp[1/(g^2*(p + 1))
  Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) +
a^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g
}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*
p] || IntegerQ[m])
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(110) = 220$.

Time = 3.36 (sec) , antiderivative size = 283, normalized size of antiderivative = 2.40

| method | result |
|---|---|
| default | $\frac{2\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sqrt{\sin(dx+c)}\,\operatorname{EllipticE}\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)a^2+4\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sqrt{\sin(dx+c)}}{}$ |
| parts | $\frac{a^2\left(2\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sqrt{\sin(dx+c)}\,\operatorname{EllipticE}\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sqrt{\sin(dx+c)}\right.}{e\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$ |

input
```
int((a+cos(d*x+c)*b)^2/(e*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/e/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*(2*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c
))^(1/2)*sin(d*x+c)^(1/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2+
4*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticE((
1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^2-(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))
^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2-2*
(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-
sin(d*x+c))^(1/2),1/2*2^(1/2))*b^2-2*a^2*cos(d*x+c)^2-2*cos(d*x+c)^2*b^2-4
*a*b*cos(d*x+c))/d
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.14

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{3/2}}\,dx =$$
$$-\frac{2\left(\left(i\,a^2+2i\,b^2\right)\sqrt{-\frac{1}{2}i\,e}\sin\left(dx+c\right)\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos\left(dx+c\right)+i\,\sin\left(d\right.\right.\right.}{}$$

input
```
integrate((a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-2*((I*a^2 + 2*I*b^2)*sqrt(-1/2*I*e)*sin(d*x + c)*weierstrassZeta(4, 0, we
ierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) + (-I*a^2 - 2*I*b^
2)*sqrt(1/2*I*e)*sin(d*x + c)*weierstrassZeta(4, 0, weierstrassPInverse(4,
 0, cos(d*x + c) - I*sin(d*x + c))) + (2*a*b + (a^2 + b^2)*cos(d*x + c))*s
qrt(e*sin(d*x + c)))/(d*e^2*sin(d*x + c))
```

## Sympy [F]

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{3/2}}\,dx = \int \frac{(a+b\cos{(c+dx)})^2}{(e\sin{(c+dx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**2/(e*sin(d*x+c))**(3/2),x)
```

output
```
Integral((a + b*cos(c + d*x))**2/(e*sin(c + d*x))**(3/2), x)
```

## Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{3/2}}\,dx = \int \frac{(b\cos{(dx+c)}+a)^2}{(e\sin{(dx+c)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^2/(e*sin(d*x + c))^(3/2), x)
```

## Giac [F]

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{3/2}}\,dx = \int \frac{(b\cos{(dx+c)}+a)^2}{(e\sin{(dx+c)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^2/(e*sin(d*x + c))^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{3/2}}\,dx = \int \frac{\left(a+b\,\cos\left(c+d\,x\right)\right)^2}{\left(e\,\sin\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int((a + b*cos(c + d*x))^2/(e*sin(c + d*x))^(3/2),x)
```

output
```
int((a + b*cos(c + d*x))^2/(e*sin(c + d*x))^(3/2), x)
```

## Reduce [F]

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{3/2}}\,dx = \frac{\sqrt{e}\left(-4\sqrt{\sin(dx+c)}\,ab+\left(\int\frac{\sqrt{\sin(dx+c)}}{\sin(dx+c)^2}dx\right)\sin(dx+c)\,a^2d+\left(\int\frac{\sqrt{\sin(dx+c)}\,c}{\sin(dx-}\right.}{\sin(dx+c)\,d\,e^2}$$

input
```
int((a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(3/2),x)
```

output
```
(sqrt(e)*( - 4*sqrt(sin(c + d*x))*a*b + int(sqrt(sin(c + d*x))/sin(c + d*x
)**2,x)*sin(c + d*x)*a**2*d + int((sqrt(sin(c + d*x))*cos(c + d*x)**2)/sin
(c + d*x)**2,x)*sin(c + d*x)*b**2*d))/(sin(c + d*x)*d*e**2)
```

# 3.47 $\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{5/2}}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 124

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{5/2}}\,dx = -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))}{3de(e\sin(c+dx))^{3/2}} + \frac{2(a^2-2b^2)\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3de^2\sqrt{e\sin(c+dx)}} - \frac{2ab\sqrt{e\sin(c+dx)}}{3de^3}$$

output

```
-2/3*(b+a*cos(d*x+c))*(a+b*cos(d*x+c))/d/e/(e*sin(d*x+c))^(3/2)+2/3*(a^2-2
*b^2)*InverseJacobiAM(1/2*c-1/4*Pi+1/2*d*x,2^(1/2))*sin(d*x+c)^(1/2)/d/e^2
/(e*sin(d*x+c))^(1/2)-2/3*a*b*(e*sin(d*x+c))^(1/2)/d/e^3
```

## Mathematica [A] (verified)

Time = 1.23 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.61

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{5/2}}\,dx = -\frac{2\left(2ab+(a^2+b^2)\cos(c+dx)+(a^2-2b^2)\,\text{EllipticF}\left(\frac{1}{4}(-2c+\pi-2dx),2\right)\sin^{\frac{3}{2}}(c+dx)\right)}{3de(e\sin(c+dx))^{3/2}}$$

input

```
Integrate[(a + b*Cos[c + d*x])^2/(e*Sin[c + d*x])^(5/2),x]
```

output
```
(-2*(2*a*b + (a^2 + b^2)*Cos[c + d*x] + (a^2 - 2*b^2)*EllipticF[(-2*c + Pi
 - 2*d*x)/4, 2]*Sin[c + d*x]^(3/2)))/(3*d*e*(e*Sin[c + d*x])^(3/2))
```

## Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 3170, 27, 3042, 3148, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{5/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^2}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \; 3170$$

$$-\frac{2\int -\frac{a^2-b\cos(c+dx)a-2b^2}{2\sqrt{e\sin(c+dx)}}dx}{3e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))}{3de(e\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{a^2-b\cos(c+dx)a-2b^2}{\sqrt{e\sin(c+dx)}}dx}{3e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))}{3de(e\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{a^2+b\sin\left(c+dx-\frac{\pi}{2}\right)a-2b^2}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}dx}{3e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))}{3de(e\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3148$$

$$\frac{(a^2-2b^2)\int \frac{1}{\sqrt{e\sin(c+dx)}}dx-\frac{2ab\sqrt{e\sin(c+dx)}}{de}}{3e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))}{3de(e\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\left(a^2-2b^2\right)\int \frac{1}{\sqrt{e\sin(c+dx)}}dx-\frac{2ab\sqrt{e\sin(c+dx)}}{de}}{3e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))}{3de(e\sin(c+dx))^{3/2}}$$

$\Big\downarrow$ 3121

$$\frac{\frac{\left(a^2-2b^2\right)\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}}dx}{\sqrt{e\sin(c+dx)}}-\frac{2ab\sqrt{e\sin(c+dx)}}{de}}{3e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))}{3de(e\sin(c+dx))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\left(a^2-2b^2\right)\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}}dx}{\sqrt{e\sin(c+dx)}}-\frac{2ab\sqrt{e\sin(c+dx)}}{de}}{3e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))}{3de(e\sin(c+dx))^{3/2}}$$

$\Big\downarrow$ 3120

$$\frac{\frac{2\left(a^2-2b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-\frac{2ab\sqrt{e\sin(c+dx)}}{de}}{3e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))}{3de(e\sin(c+dx))^{3/2}}$$

input
```
Int[(a + b*Cos[c + d*x])^2/(e*Sin[c + d*x])^(5/2),x]
```

output
```
(-2*(b + a*Cos[c + d*x])*(a + b*Cos[c + d*x]))/(3*d*e*(e*Sin[c + d*x])^(3/
2)) + ((2*(a^2 - 2*b^2)*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]
])/(d*Sqrt[e*Sin[c + d*x]]) - (2*a*b*Sqrt[e*Sin[c + d*x]])/(d*e))/(3*e^2)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3170
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Simp[1/(g^2*(p + 1))
  Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) +
a^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g
}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*
p] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 3.22 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.63

| method | result |
|---|---|
| default | $\frac{-\frac{4ab}{3e(e\sin(dx+c))^{\frac{3}{2}}}-\frac{\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sin(dx+c)^{\frac{5}{2}}\,\text{EllipticF}\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)a^{2}-2b^{2}\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sin(dx}{3e^{2}\sin(dx+c)^{2}\cos(dx+c)\sqrt{e\sin(dx}}}{d}$ |
| parts | $-\frac{a^{2}\left(\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sin(dx+c)^{\frac{5}{2}}\,\text{EllipticF}\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-2\sin(dx+c)^{3}+2\sin(dx+c)\right)}{3e^{2}\sin(dx+c)^{2}\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}+\frac{2b^{2}\left(\sqrt{1-\sin}\right.}{}$ |

input
```
int((a+cos(d*x+c)*b)^2/(e*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(-4/3*a*b/e/(e*sin(d*x+c))^(3/2)-1/3/e^2*((1-sin(d*x+c))^(1/2)*(2+2*sin(d*
x+c))^(1/2)*sin(d*x+c)^(5/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a
^2-2*b^2*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(5/2)*Elli
pticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))+2*a^2*cos(d*x+c)^2*sin(d*x+c)+2*b^
2*cos(d*x+c)^2*sin(d*x+c))/sin(d*x+c)^2/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.30

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{5/2}}\,dx = \frac{2\left(\left((a^2-2\,b^2)\cos{(dx+c)}^2-a^2+2\,b^2\right)\sqrt{-\frac{1}{2}i\,e}\,\mathrm{weierstrassPInverse}(4,0,\cos\right.}{}$$

input
```
integrate((a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
2/3*(((a^2 - 2*b^2)*cos(d*x + c)^2 - a^2 + 2*b^2)*sqrt(-1/2*I*e)*weierstra
ssPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c)) + ((a^2 - 2*b^2)*cos(d*x +
 c)^2 - a^2 + 2*b^2)*sqrt(1/2*I*e)*weierstrassPInverse(4, 0, cos(d*x + c)
- I*sin(d*x + c)) + (2*a*b + (a^2 + b^2)*cos(d*x + c))*sqrt(e*sin(d*x + c)
))/(d*e^3*cos(d*x + c)^2 - d*e^3)
```

## Sympy [F]

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{5/2}}\,dx = \int \frac{(a+b\cos{(c+dx)})^2}{(e\sin{(c+dx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**2/(e*sin(d*x+c))**(5/2),x)
```

output
```
Integral((a + b*cos(c + d*x))**2/(e*sin(c + d*x))**(5/2), x)
```

## Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{5/2}}\,dx = \int \frac{(b\cos(dx+c)+a)^2}{(e\sin(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^2/(e*sin(d*x + c))^(5/2), x)
```

## Giac [F]

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{5/2}}\,dx = \int \frac{(b\cos(dx+c)+a)^2}{(e\sin(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^2/(e*sin(d*x + c))^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{5/2}}\,dx = \int \frac{(a+b\,\cos(c+d\,x))^2}{(e\,\sin(c+d\,x))^{5/2}}\,dx$$

input
```
int((a + b*cos(c + d*x))^2/(e*sin(c + d*x))^(5/2),x)
```

output
```
int((a + b*cos(c + d*x))^2/(e*sin(c + d*x))^(5/2), x)
```

## Reduce [F]

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{5/2}}\,dx = \frac{\sqrt{e}\left(-4\sqrt{\sin(dx+c)}\,ab+3\left(\int\frac{\sqrt{\sin(dx+c)}}{\sin(dx+c)^3}dx\right)\sin(dx+c)^2a^2d+3\right.}{3\sin(dx+c)^2\,d\,e^3}$$

input
```
int((a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(5/2),x)
```

output
```
(sqrt(e)*( - 4*sqrt(sin(c + d*x))*a*b + 3*int(sqrt(sin(c + d*x))/sin(c + d
*x)**3,x)*sin(c + d*x)**2*a**2*d + 3*int((sqrt(sin(c + d*x))*cos(c + d*x)*
*2)/sin(c + d*x)**3,x)*sin(c + d*x)**2*b**2*d))/(3*sin(c + d*x)**2*d*e**3)
```

# 3.48 $\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{7/2}}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 165

$$\begin{aligned}\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{7/2}}\,dx =& -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))}{5de(e\sin(c+dx))^{5/2}} \\ &-\frac{2ab}{5de^3\sqrt{e\sin(c+dx)}}-\frac{2(3a^2-2b^2)\cos(c+dx)}{5de^3\sqrt{e\sin(c+dx)}} \\ &-\frac{2(3a^2-2b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5de^4\sqrt{\sin(c+dx)}}\end{aligned}$$

output
```
-2/5*(b+a*cos(d*x+c))*(a+b*cos(d*x+c))/d/e/(e*sin(d*x+c))^(5/2)-2/5*a*b/d/
e^3/(e*sin(d*x+c))^(1/2)-2/5*(3*a^2-2*b^2)*cos(d*x+c)/d/e^3/(e*sin(d*x+c))
^(1/2)+2/5*(3*a^2-2*b^2)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*s
in(d*x+c))^(1/2)/d/e^4/sin(d*x+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 1.54 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.66

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{7/2}}\,dx = -\frac{8ab+(7a^2+2b^2)\cos(c+dx)-3a^2\cos(3(c+dx))+2b^2\cos(3(c+dx))-4(3a^2-2b^2)\,E\left(\frac{1}{4}(-2c+\pi-\right.}{10de(e\sin(c+dx))^{5/2}}$$

input
```
Integrate[(a + b*Cos[c + d*x])^2/(e*Sin[c + d*x])^(7/2),x]
```

output
```
-1/10*(8*a*b + (7*a^2 + 2*b^2)*Cos[c + d*x] - 3*a^2*Cos[3*(c + d*x)] + 2*b
^2*Cos[3*(c + d*x)] - 4*(3*a^2 - 2*b^2)*EllipticE[(-2*c + Pi - 2*d*x)/4, 2
]*Sin[c + d*x]^(5/2))/(d*e*(e*Sin[c + d*x])^(5/2))
```

## Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.96, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 3170, 27, 3042, 3148, 3042, 3116, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^2}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{ 3170}$$

$$-\frac{2\int -\frac{3a^2+b\cos(c+dx)a-2b^2}{2(e\sin(c+dx))^{3/2}}dx}{5e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))}{5de(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{3a^2+b\cos(c+dx)a-2b^2}{(e\sin(c+dx))^{3/2}}dx}{5e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))}{5de(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{3a^2-b\sin\left(c+dx-\frac{\pi}{2}\right)a-2b^2}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}}dx}{5e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))}{5de(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3148}$$

$$\frac{\left(3a^2-2b^2\right)\int \frac{1}{(e\sin(c+dx))^{3/2}}dx-\frac{2ab}{de\sqrt{e\sin(c+dx)}}}{5e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))}{5de(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(3a^2-2b^2\right)\int \frac{1}{(e\sin(c+dx))^{3/2}}dx-\frac{2ab}{de\sqrt{e\sin(c+dx)}}}{5e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))}{5de(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3116}$$

$$\frac{\left(3a^2-2b^2\right)\left(-\frac{\int \sqrt{e\sin(c+dx)}dx}{e^2}-\frac{2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}\right)-\frac{2ab}{de\sqrt{e\sin(c+dx)}}}{5e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))}{5de(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(3a^2-2b^2\right)\left(-\frac{\int \sqrt{e\sin(c+dx)}dx}{e^2}-\frac{2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}\right)-\frac{2ab}{de\sqrt{e\sin(c+dx)}}}{5e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))}{5de(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3121}$$

$$\frac{\left(3a^2-2b^2\right)\left(-\frac{\sqrt{e\sin(c+dx)}\int \sqrt{\sin(c+dx)}dx}{e^2\sqrt{\sin(c+dx)}}-\frac{2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}\right)-\frac{2ab}{de\sqrt{e\sin(c+dx)}}}{5e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))}{5de(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(3a^2-2b^2\right)\left(-\frac{\sqrt{e\sin(c+dx)}\int \sqrt{\sin(c+dx)}dx}{e^2\sqrt{\sin(c+dx)}}-\frac{2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}\right)-\frac{2ab}{de\sqrt{e\sin(c+dx)}}}{5e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))}{5de(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{(3a^2-2b^2)\left(-\frac{2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)\sqrt{e\sin(c+dx)}}{de^2\sqrt{\sin(c+dx)}}-\frac{2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}\right)-\frac{2ab}{de\sqrt{e\sin(c+dx)}}}{5e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))}{5de(e\sin(c+dx))^{5/2}}$$

input
```
Int[(a + b*Cos[c + d*x])^2/(e*Sin[c + d*x])^(7/2),x]
```

output
```
(-2*(b + a*Cos[c + d*x])*(a + b*Cos[c + d*x]))/(5*d*e*(e*Sin[c + d*x])^(5/
2)) + ((-2*a*b)/(d*e*Sqrt[e*Sin[c + d*x]]) + (3*a^2 - 2*b^2)*((-2*Cos[c +
d*x])/(d*e*Sqrt[e*Sin[c + d*x]]) - (2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqr
t[e*Sin[c + d*x]])/(d*e^2*Sqrt[Sin[c + d*x]])))/(5*e^2)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3170
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Simp[1/(g^2*(p + 1))
  Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) +
a^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g
}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*
p] || IntegerQ[m])
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(147) = 294$.

Time = 3.34 (sec) , antiderivative size = 351, normalized size of antiderivative = 2.13

| method | result |
|---|---|
| default | $-\frac{4ab}{5e(e\sin(dx+c))^{\frac{5}{2}}}+\frac{6\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sin(dx+c)^{\frac{7}{2}}\,\text{EllipticE}\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)a^2-4\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sin(dx+\ldots}{\ldots}$ |
| parts | $\frac{a^2\left(6\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sin(dx+c)^{\frac{7}{2}}\,\text{EllipticE}\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-3\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sin(dx+c)^{\ldots}\ldots\right)}{5e^3\sin(dx+c)^3\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$ |

input
```
int((a+cos(d*x+c)*b)^2/(e*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
(-4/5*a*b/e/(e*sin(d*x+c))^(5/2)+1/5/e^3*(6*(1-sin(d*x+c))^(1/2)*(2+2*sin(
d*x+c))^(1/2)*sin(d*x+c)^(7/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))
*a^2-4*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(7/2)*Ellipt
icE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^2-3*(1-sin(d*x+c))^(1/2)*(2+2*sin(
d*x+c))^(1/2)*sin(d*x+c)^(7/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))
*a^2+2*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(7/2)*Ellipt
icF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^2+6*a^2*cos(d*x+c)^4*sin(d*x+c)-4*
b^2*cos(d*x+c)^4*sin(d*x+c)-8*a^2*cos(d*x+c)^2*sin(d*x+c)+2*b^2*cos(d*x+c)
^2*sin(d*x+c))/sin(d*x+c)^3/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.31

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{7/2}}\,dx =$$
$$-\frac{2\left(\left((3i\,a^2-2i\,b^2)\cos(dx+c)^2-3i\,a^2+2i\,b^2\right)\sqrt{-\frac{1}{2}i\,e}\sin(dx+c)\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInv}\right.}{}$$

input
```
integrate((a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
-2/5*(((3*I*a^2 - 2*I*b^2)*cos(d*x + c)^2 - 3*I*a^2 + 2*I*b^2)*sqrt(-1/2*I
*e)*sin(d*x + c)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x +
 c) + I*sin(d*x + c))) + ((-3*I*a^2 + 2*I*b^2)*cos(d*x + c)^2 + 3*I*a^2 -
2*I*b^2)*sqrt(1/2*I*e)*sin(d*x + c)*weierstrassZeta(4, 0, weierstrassPInve
rse(4, 0, cos(d*x + c) - I*sin(d*x + c))) + ((3*a^2 - 2*b^2)*cos(d*x + c)^
3 - 2*a*b - (4*a^2 - b^2)*cos(d*x + c))*sqrt(e*sin(d*x + c)))/((d*e^4*cos(
d*x + c)^2 - d*e^4)*sin(d*x + c))
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**2/(e*sin(d*x+c))**(7/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{7/2}}\,dx = \int \frac{(b\cos(dx+c)+a)^2}{(e\sin(dx+c))^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^2/(e*sin(d*x + c))^(7/2), x)
```

## Giac [F]

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{7/2}}\,dx = \int \frac{(b\cos(dx+c)+a)^2}{(e\sin(dx+c))^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^2/(e*sin(d*x + c))^(7/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{7/2}}\,dx = \int \frac{(a+b\,\cos(c+d\,x))^2}{(e\,\sin(c+d\,x))^{7/2}}\,dx$$

input
```
int((a + b*cos(c + d*x))^2/(e*sin(c + d*x))^(7/2),x)
```

output
```
int((a + b*cos(c + d*x))^2/(e*sin(c + d*x))^(7/2), x)
```

## Reduce [F]

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{7/2}}\,dx = \frac{\sqrt{e}\left(-4\sqrt{\sin(dx+c)}\,ab+5\left(\int\frac{\sqrt{\sin(dx+c)}}{\sin(dx+c)^4}dx\right)\sin(dx+c)^3a^2d+5\left(\int\frac{\sqrt{\sin(dx-}}{\sin}\right.\right.}{5\sin(dx+c)^3\,d\,e^4}$$

input
```
int((a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(7/2),x)
```

output
```
(sqrt(e)*( - 4*sqrt(sin(c + d*x))*a*b + 5*int(sqrt(sin(c + d*x))/sin(c + d
*x)**4,x)*sin(c + d*x)**3*a**2*d + 5*int((sqrt(sin(c + d*x))*cos(c + d*x)*
*2)/sin(c + d*x)**4,x)*sin(c + d*x)**3*b**2*d))/(5*sin(c + d*x)**3*d*e**4)
```

# 3.49 $\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 242

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}\,dx = \frac{10a(11a^2+6b^2)\,e^4\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{231d\sqrt{e\sin(c+dx)}} - \frac{10a(11a^2+6b^2)\,e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{231d} - \frac{2a(11a^2+6b^2)\,e\cos(c+dx)(e\sin(c+dx))^{5/2}}{77d} + \frac{2b(177a^2+44b^2)\,(e\sin(c+dx))^{9/2}}{1287de} + \frac{34ab(a+b\cos(c+dx))(e\sin(c+dx))^{9/2}}{143de} + \frac{2b(a+b\cos(c+dx))^2(e\sin(c+dx))^{9/2}}{13de}$$

output
```
10/231*a*(11*a^2+6*b^2)*e^4*InverseJacobiAM(1/2*c-1/4*Pi+1/2*d*x,2^(1/2))*
sin(d*x+c)^(1/2)/d/(e*sin(d*x+c))^(1/2)-10/231*a*(11*a^2+6*b^2)*e^3*cos(d*
x+c)*(e*sin(d*x+c))^(1/2)/d-2/77*a*(11*a^2+6*b^2)*e*cos(d*x+c)*(e*sin(d*x+
c))^(5/2)/d+2/1287*b*(177*a^2+44*b^2)*(e*sin(d*x+c))^(9/2)/d/e+34/143*a*b*
(a+b*cos(d*x+c))*(e*sin(d*x+c))^(9/2)/d/e+2/13*b*(a+b*cos(d*x+c))^2*(e*sin
(d*x+c))^(9/2)/d/e
```

## Mathematica [A] (verified)

Time = 4.03 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.85

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}\,dx = \frac{\left(154b(78a^2+11b^2)\csc^3(c+dx)+\frac{1}{3}(-156a(506a^2+213b^2)\cos(c+dx)-77b(624a^2+73b\right.}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^3*(e*Sin[c + d*x])^(7/2),x]
```

output
```
((154*b*(78*a^2 + 11*b^2)*Csc[c + d*x]^3 + ((-156*a*(506*a^2 + 213*b^2)*Co
s[c + d*x] - 77*b*(624*a^2 + 73*b^2)*Cos[2*(c + d*x)] + 234*a*(44*a^2 - 39
*b^2)*Cos[3*(c + d*x)] - 154*b*(-78*a^2 + b^2)*Cos[4*(c + d*x)] + 4914*a*b
^2*Cos[5*(c + d*x)] + 693*b^3*Cos[6*(c + d*x)])*Csc[c + d*x]^3)/3 - (2080*
a*(11*a^2 + 6*b^2)*EllipticF[(-2*c + Pi - 2*d*x)/4, 2])/Sin[c + d*x]^(7/2)
)*(e*Sin[c + d*x])^(7/2))/(48048*d)
```

## Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.98, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3042, 3171, 27, 3042, 3341, 27, 3042, 3148, 3042, 3115, 3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e\sin(c+dx))^{7/2}(a+b\cos(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{7/2}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{3171}$$

$$\frac{2}{13}\int \frac{1}{2}(a+b\cos(c+dx))\left(13a^2+17b\cos(c+dx)a+4b^2\right)(e\sin(c+dx))^{7/2}dx+\frac{2b(e\sin(c+dx))^{9/2}(a+b\cos(c+dx))^2}{13de}$$

$\Big\downarrow$ 27

$$\frac{1}{13}\int (a+b\cos(c+dx))\left(13a^2+17b\cos(c+dx)a+4b^2\right)(e\sin(c+dx))^{7/2}dx+\frac{2b(e\sin(c+dx))^{9/2}(a+b\cos(c+dx))^2}{13de}$$

$\Big\downarrow$ 3042

$$\frac{1}{13}\int \left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(13a^2+17b\sin\left(c+dx+\frac{\pi}{2}\right)a+4b^2\right)dx+\frac{2b(e\sin(c+dx))^{9/2}(a+b\cos(c+dx))^2}{13de}$$

$\Big\downarrow$ 3341

$$\frac{1}{13}\left(\frac{2}{11}\int \frac{1}{2}\left(13a\left(11a^2+6b^2\right)+b\left(177a^2+44b^2\right)\cos(c+dx)\right)(e\sin(c+dx))^{7/2}dx+\frac{34ab(e\sin(c+dx))^{9/2}(a+}{11de}\right.$$
$$\frac{2b(e\sin(c+dx))^{9/2}(a+b\cos(c+dx))^2}{13de}$$

$\Big\downarrow$ 27

$$\frac{1}{13}\left(\frac{1}{11}\int \left(13a\left(11a^2+6b^2\right)+b\left(177a^2+44b^2\right)\cos(c+dx)\right)(e\sin(c+dx))^{7/2}dx+\frac{34ab(e\sin(c+dx))^{9/2}(a+b}{11de}\right.$$
$$\frac{2b(e\sin(c+dx))^{9/2}(a+b\cos(c+dx))^2}{13de}$$

$\Big\downarrow$ 3042

$$\frac{1}{13}\left(\frac{1}{11}\int \left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{7/2}\left(13a\left(11a^2+6b^2\right)-b\left(177a^2+44b^2\right)\sin\left(c+dx-\frac{\pi}{2}\right)\right)dx+\frac{34ab(e\sin(c+}{}\right.$$
$$\frac{2b(e\sin(c+dx))^{9/2}(a+b\cos(c+dx))^2}{13de}$$

$\Big\downarrow$ 3148

$$\frac{1}{13}\left(\frac{1}{11}\left(13a\left(11a^2+6b^2\right)\int (e\sin(c+dx))^{7/2}dx+\frac{2b\left(177a^2+44b^2\right)(e\sin(c+dx))^{9/2}}{9de}\right)+\frac{34ab(e\sin(c+dx))^{9}}{11}\right.$$
$$\frac{2b(e\sin(c+dx))^{9/2}(a+b\cos(c+dx))^2}{13de}$$

$\downarrow$ 3042

$$\frac{1}{13}\left(\frac{1}{11}\left(13a\left(11a^2+6b^2\right)\int (e\sin(c+dx))^{7/2}dx+\frac{2b\left(177a^2+44b^2\right)(e\sin(c+dx))^{9/2}}{9de}\right)+\frac{34ab(e\sin(c+dx))^{9}}{11}\right.$$
$$\frac{2b(e\sin(c+dx))^{9/2}(a+b\cos(c+dx))^2}{13de}$$

$\downarrow$ 3115

$$\frac{1}{13}\left(\frac{1}{11}\left(13a\left(11a^2+6b^2\right)\left(\frac{5}{7}e^2\int (e\sin(c+dx))^{3/2}dx-\frac{2e\cos(c+dx)(e\sin(c+dx))^{5/2}}{7d}\right)+\frac{2b\left(177a^2+44b^2\right)}{9d}\right.\right.$$
$$\frac{2b(e\sin(c+dx))^{9/2}(a+b\cos(c+dx))^2}{13de}$$

$\downarrow$ 3042

$$\frac{1}{13}\left(\frac{1}{11}\left(13a\left(11a^2+6b^2\right)\left(\frac{5}{7}e^2\int (e\sin(c+dx))^{3/2}dx-\frac{2e\cos(c+dx)(e\sin(c+dx))^{5/2}}{7d}\right)+\frac{2b\left(177a^2+44b^2\right)}{9d}\right.\right.$$
$$\frac{2b(e\sin(c+dx))^{9/2}(a+b\cos(c+dx))^2}{13de}$$

$\downarrow$ 3115

$$\frac{1}{13}\left(\frac{1}{11}\left(13a\left(11a^2+6b^2\right)\left(\frac{5}{7}e^2\left(\frac{1}{3}e^2\int \frac{1}{\sqrt{e\sin(c+dx)}}dx-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)-\frac{2e\cos(c+dx)(e}{7}\right.\right.\right.$$
$$\frac{2b(e\sin(c+dx))^{9/2}(a+b\cos(c+dx))^2}{13de}$$

$\downarrow$ 3042

$$\frac{1}{13}\left(\frac{1}{11}\left(13a\left(11a^2+6b^2\right)\left(\frac{5}{7}e^2\left(\frac{1}{3}e^2\int \frac{1}{\sqrt{e\sin(c+dx)}}dx-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)-\frac{2e\cos(c+dx)(e}{7}\right.\right.\right.$$
$$\frac{2b(e\sin(c+dx))^{9/2}(a+b\cos(c+dx))^2}{13de}$$

$\downarrow$ 3121

$$\frac{1}{13}\left(\frac{1}{11}\left(13a(11a^2+6b^2)\left(\frac{5}{7}e^2\left(\frac{e^2\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{3\sqrt{e\sin(c+dx)}}-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)-\frac{2e\cos(c\right.\right.\right.$$
$$\frac{2b(e\sin(c+dx))^{9/2}(a+b\cos(c+dx))^2}{13de}$$

$\downarrow$ 3042

$$\frac{1}{13}\left(\frac{1}{11}\left(13a(11a^2+6b^2)\left(\frac{5}{7}e^2\left(\frac{e^2\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{3\sqrt{e\sin(c+dx)}}-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)-\frac{2e\cos(c\right.\right.\right.$$
$$\frac{2b(e\sin(c+dx))^{9/2}(a+b\cos(c+dx))^2}{13de}$$

$\downarrow$ 3120

$$\frac{1}{13}\left(\frac{1}{11}\left(13a(11a^2+6b^2)\left(\frac{5}{7}e^2\left(\frac{2e^2\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{3d\sqrt{e\sin(c+dx)}}-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx}}{3d}\right.\right.\right.\right.$$
$$\frac{2b(e\sin(c+dx))^{9/2}(a+b\cos(c+dx))^2}{13de}$$

input
```
Int[(a + b*Cos[c + d*x])^3*(e*Sin[c + d*x])^(7/2),x]
```

output
```
(2*b*(a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(9/2))/(13*d*e) + ((34*a*b*(a
 + b*Cos[c + d*x])*(e*Sin[c + d*x])^(9/2))/(11*d*e) + ((2*b*(177*a^2 + 44*
b^2)*(e*Sin[c + d*x])^(9/2))/(9*d*e) + 13*a*(11*a^2 + 6*b^2)*((-2*e*Cos[c
+ d*x]*(e*Sin[c + d*x])^(5/2))/(7*d) + (5*e^2*((2*e^2*EllipticF[(c - Pi/2
+ d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*d*Sqrt[e*Sin[c + d*x]]) - (2*e*Cos[c +
 d*x]*Sqrt[e*Sin[c + d*x]])/(3*d)))/7))/11)/13
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3171
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[1/(m + p)   Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
 && GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

rule 3341
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[1/(m + p + 1)   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Sim
p[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 &&  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && S
implerQ[c + d*x, a + b*x])
```

## Maple [A] (verified)

Time = 43.71 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.14

| method | result |
|---|---|
| default | $\frac{\frac{2b(e\sin(dx+c))^{\frac{9}{2}}\left(9\cos(dx+c)^2b^2+39a^2+4b^2\right)}{117e}-\frac{e^4a\left(-126b^2\cos(dx+c)^6\sin(dx+c)-66a^2\cos(dx+c)^4\sin(dx+c)+216b^2\cos(dx+c)^4\sin(dx+c)+5\right.}{}}{}$ |
| parts | $-\frac{a^3e^4\left(-6\sin(dx+c)^5+5\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sqrt{\sin(dx+c)}\ \mathrm{EllipticF}\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-4\sin(dx+c)^3+10\sin(dx\right.}{21\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$ |

input
```
int((a+cos(d*x+c)*b)^3*(e*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
(2/117/e*b*(e*sin(d*x+c))^(9/2)*(9*cos(d*x+c)^2*b^2+39*a^2+4*b^2)-1/231*e^
4*a*(-126*b^2*cos(d*x+c)^6*sin(d*x+c)-66*a^2*cos(d*x+c)^4*sin(d*x+c)+216*b
^2*cos(d*x+c)^4*sin(d*x+c)+55*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*
sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2+30*(1-sin
(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*
x+c))^(1/2),1/2*2^(1/2))*b^2+176*a^2*cos(d*x+c)^2*sin(d*x+c)-30*b^2*cos(d*
x+c)^2*sin(d*x+c))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.02

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}\,dx = \frac{2\left(195\left(11\,a^3+6\,ab^2\right)\sqrt{-\frac{1}{2}i\,e}e^3\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)+i\,\sin(dx+c))+195\right.}{}$$

input
```
integrate((a+b*cos(d*x+c))^3*(e*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
2/9009*(195*(11*a^3 + 6*a*b^2)*sqrt(-1/2*I*e)*e^3*weierstrassPInverse(4, 0
, cos(d*x + c) + I*sin(d*x + c)) + 195*(11*a^3 + 6*a*b^2)*sqrt(1/2*I*e)*e^
3*weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)) + (693*b^3*e^3*
cos(d*x + c)^6 + 2457*a*b^2*e^3*cos(d*x + c)^5 + 77*(39*a^2*b - 14*b^3)*e^
3*cos(d*x + c)^4 + 117*(11*a^3 - 36*a*b^2)*e^3*cos(d*x + c)^3 - 77*(78*a^2
*b - b^3)*e^3*cos(d*x + c)^2 - 39*(88*a^3 - 15*a*b^2)*e^3*cos(d*x + c) + 7
7*(39*a^2*b + 4*b^3)*e^3)*sqrt(e*sin(d*x + c)))/d
```

## Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**3*(e*sin(d*x+c))**(7/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}\,dx = \int (b\cos(dx+c)+a)^3(e\sin(dx+c))^{\frac{7}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3*(e*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^3*(e*sin(d*x + c))^(7/2), x)
```

## Giac [F]

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}\,dx=\int (b\cos(dx+c)+a)^3(e\sin(dx+c))^{\frac{7}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3*(e*sin(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^3*(e*sin(d*x + c))^(7/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}\,dx=\int (e\,\sin(c+d\,x))^{7/2}\,(a+b\,\cos(c+d\,x))^3\,dx$$

input
```
int((e*sin(c + d*x))^(7/2)*(a + b*cos(c + d*x))^3,x)
```

output
```
int((e*sin(c + d*x))^(7/2)*(a + b*cos(c + d*x))^3, x)
```

## Reduce [F]

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}\,dx=\frac{\sqrt{e}\,e^3\left(2\sqrt{\sin(dx+c)}\,\sin(dx+c)^4a^2b+3\left(\int\sqrt{\sin(dx+c)}\,\cos(dx+c)^3\sin(dx+c)^3dx\right)\right.}{}$$

input
```
int((a+b*cos(d*x+c))^3*(e*sin(d*x+c))^(7/2),x)
```

output
```
(sqrt(e)*e**3*(2*sqrt(sin(c + d*x))*sin(c + d*x)**4*a**2*b + 3*int(sqrt(si
n(c + d*x))*cos(c + d*x)**3*sin(c + d*x)**3,x)*b**3*d + 9*int(sqrt(sin(c +
 d*x))*cos(c + d*x)**2*sin(c + d*x)**3,x)*a*b**2*d + 3*int(sqrt(sin(c + d*
x))*sin(c + d*x)**3,x)*a**3*d))/(3*d)
```

# 3.50 $\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 202

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}\,dx = \frac{2a(3a^2+2b^2)\,e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}} - \frac{2a(3a^2+2b^2)\,e\cos(c+dx)(e\sin(c+dx))^{3/2}}{15d} + \frac{2b(43a^2+12b^2)\,(e\sin(c+dx))^{7/2}}{231de} + \frac{10ab(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}{33de} + \frac{2b(a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}}{11de}$$

output
```
-2/5*a*(3*a^2+2*b^2)*e^2*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*s
in(d*x+c))^(1/2)/d/sin(d*x+c)^(1/2)-2/15*a*(3*a^2+2*b^2)*e*cos(d*x+c)*(e*s
in(d*x+c))^(3/2)/d+2/231*b*(43*a^2+12*b^2)*(e*sin(d*x+c))^(7/2)/d/e+10/33*
a*b*(a+b*cos(d*x+c))*(e*sin(d*x+c))^(7/2)/d/e+2/11*b*(a+b*cos(d*x+c))^2*(e
*sin(d*x+c))^(7/2)/d/e
```

## Mathematica [A] (verified)

Time = 2.49 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.74

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}\,dx =$$
$$-\frac{(e\sin(c+dx))^{5/2}\left(1848(3a^3+2ab^2)\,E\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\,2\right)+(462a(4a^2+b^2)\cos(c+dx)+5b(-396\right.}{4620d\sin}$$

input
```
Integrate[(a + b*Cos[c + d*x])^3*(e*Sin[c + d*x])^(5/2),x]
```

output
```
-1/4620*((e*Sin[c + d*x])^(5/2)*(1848*(3*a^3 + 2*a*b^2)*EllipticE[(-2*c +
Pi - 2*d*x)/4, 2] + (462*a*(4*a^2 + b^2)*Cos[c + d*x] + 5*b*(-396*a^2 - 69
*b^2 + 12*(33*a^2 + 4*b^2)*Cos[2*(c + d*x)] + 154*a*b*Cos[3*(c + d*x)] + 2
1*b^2*Cos[4*(c + d*x)]))*Sin[c + d*x]^(3/2)))/(d*Sin[c + d*x]^(5/2))
```

## Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3042, 3171, 27, 3042, 3341, 27, 3042, 3148, 3042, 3115, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^3\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \ 3171$$

$$\frac{2}{11}\int \frac{1}{2}(a+b\cos(c+dx))\left(11a^2+15b\cos(c+dx)a+4b^2\right)(e\sin(c+dx))^{5/2}dx+\frac{2b(e\sin(c+dx))^{7/2}(a+b\cos(c+dx))^2}{11de}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{11}\int (a+b\cos(c+dx))\left(11a^2+15b\cos(c+dx)a+4b^2\right)(e\sin(c+dx))^{5/2}dx+\frac{2b(e\sin(c+dx))^{7/2}(a+b\cos(c+dx))^2}{11de}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{11}\int \left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(11a^2+15b\sin\left(c+dx+\frac{\pi}{2}\right)a+4b^2\right)dx+\frac{2b(e\sin(c+dx))^{7/2}(a+b\cos(c+dx))^2}{11de}$$

$$\Big\downarrow \; 3341$$

$$\frac{1}{11}\left(\frac{2}{9}\int \frac{3}{2}\left(11a\left(3a^2+2b^2\right)+b\left(43a^2+12b^2\right)\cos(c+dx)\right)(e\sin(c+dx))^{5/2}dx+\frac{10ab(e\sin(c+dx))^{7/2}(a+b\,\mathrm{co}}{3de}\right.$$
$$\frac{2b(e\sin(c+dx))^{7/2}(a+b\cos(c+dx))^2}{11de}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{11}\left(\frac{1}{3}\int \left(11a\left(3a^2+2b^2\right)+b\left(43a^2+12b^2\right)\cos(c+dx)\right)(e\sin(c+dx))^{5/2}dx+\frac{10ab(e\sin(c+dx))^{7/2}(a+b\cos}{3de}\right.$$
$$\frac{2b(e\sin(c+dx))^{7/2}(a+b\cos(c+dx))^2}{11de}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{11}\left(\frac{1}{3}\int \left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2}\left(11a\left(3a^2+2b^2\right)-b\left(43a^2+12b^2\right)\sin\left(c+dx-\frac{\pi}{2}\right)\right)dx+\frac{10ab(e\sin(c+dx}{}\right.$$
$$\frac{2b(e\sin(c+dx))^{7/2}(a+b\cos(c+dx))^2}{11de}$$

$$\Big\downarrow \; 3148$$

$$\frac{1}{11}\left(\frac{1}{3}\left(11a\left(3a^2+2b^2\right)\int (e\sin(c+dx))^{5/2}dx+\frac{2b\left(43a^2+12b^2\right)(e\sin(c+dx))^{7/2}}{7de}\right)+\frac{10ab(e\sin(c+dx))^{7/2}(a}{3de}\right.$$
$$\frac{2b(e\sin(c+dx))^{7/2}(a+b\cos(c+dx))^2}{11de}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{11}\left(\frac{1}{3}\left(11a\left(3a^2+2b^2\right)\int(e\sin(c+dx))^{5/2}dx+\frac{2b\left(43a^2+12b^2\right)(e\sin(c+dx))^{7/2}}{7de}\right)+\frac{10ab(e\sin(c+dx))^{7/2}(c}{3de}\right.$$

$$\frac{2b(e\sin(c+dx))^{7/2}(a+b\cos(c+dx))^2}{11de}$$

$\big\downarrow$ 3115

$$\frac{1}{11}\left(\frac{1}{3}\left(11a\left(3a^2+2b^2\right)\left(\frac{3}{5}e^2\int\sqrt{e\sin(c+dx)}dx-\frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d}\right)+\frac{2b\left(43a^2+12b^2\right)(e\sin}{7de}\right.\right.$$

$$\frac{2b(e\sin(c+dx))^{7/2}(a+b\cos(c+dx))^2}{11de}$$

$\big\downarrow$ 3042

$$\frac{1}{11}\left(\frac{1}{3}\left(11a\left(3a^2+2b^2\right)\left(\frac{3}{5}e^2\int\sqrt{e\sin(c+dx)}dx-\frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d}\right)+\frac{2b\left(43a^2+12b^2\right)(e\sin}{7de}\right.\right.$$

$$\frac{2b(e\sin(c+dx))^{7/2}(a+b\cos(c+dx))^2}{11de}$$

$\big\downarrow$ 3121

$$\frac{1}{11}\left(\frac{1}{3}\left(11a\left(3a^2+2b^2\right)\left(\frac{3e^2\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{5\sqrt{\sin(c+dx)}}-\frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d}\right)+\frac{2b\left(43a^2\right.}{}\right.\right.$$

$$\frac{2b(e\sin(c+dx))^{7/2}(a+b\cos(c+dx))^2}{11de}$$

$\big\downarrow$ 3042

$$\frac{1}{11}\left(\frac{1}{3}\left(11a\left(3a^2+2b^2\right)\left(\frac{3e^2\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{5\sqrt{\sin(c+dx)}}-\frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d}\right)+\frac{2b\left(43a^2\right.}{}\right.\right.$$

$$\frac{2b(e\sin(c+dx))^{7/2}(a+b\cos(c+dx))^2}{11de}$$

$\big\downarrow$ 3119

$$\frac{1}{11}\left(\frac{1}{3}\left(11a\left(3a^2+2b^2\right)\left(\frac{6e^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}}-\frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d}\right)+\frac{2b(43a}{}\right.\right.$$

$$\frac{2b(e\sin(c+dx))^{7/2}(a+b\cos(c+dx))^2}{11de}$$

input
```
Int[(a + b*Cos[c + d*x])^3*(e*Sin[c + d*x])^(5/2),x]
```

output
```
(2*b*(a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(7/2))/(11*d*e) + ((10*a*b*(a
 + b*Cos[c + d*x])*(e*Sin[c + d*x])^(7/2))/(3*d*e) + ((2*b*(43*a^2 + 12*b^
2)*(e*Sin[c + d*x])^(7/2))/(7*d*e) + 11*a*(3*a^2 + 2*b^2)*((6*e^2*Elliptic
E[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(5*d*Sqrt[Sin[c + d*x]]) -
(2*e*Cos[c + d*x]*(e*Sin[c + d*x])^(3/2))/(5*d)))/3)/11
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3171
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[1/(m + p)   Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
 && GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

rule 3341
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[1/(m + p + 1)   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Sim
p[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && !LtQ[p, -1] && IntegerQ[2*m] && !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && S
implerQ[c + d*x, a + b*x])
```

## Maple [A] (verified)

Time = 43.23 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.76

| method | result |
|---|---|
| default | $\frac{\frac{2b(e\sin(dx+c))^{\frac{7}{2}}\left(7\cos(dx+c)^2b^2+33a^2+4b^2\right)}{77e}-\frac{e^3a\left(10\sin(dx+c)^6b^2+18\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sqrt{\sin(dx+c)}\ \text{EllipticE}\left(\sqrt{1-\sin(dx+c)}\right.\right.}{}}{}$ |
| parts | $-\frac{a^3e^3\left(6\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sqrt{\sin(dx+c)}\ \text{EllipticE}\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-3\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sqrt{\sin(\right.}{5\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$ |

input
```
int((a+cos(d*x+c)*b)^3*(e*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(2/77/e*b*(e*sin(d*x+c))^(7/2)*(7*cos(d*x+c)^2*b^2+33*a^2+4*b^2)-1/15*e^3*
a*(10*sin(d*x+c)^6*b^2+18*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(
d*x+c)^(1/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2+12*(1-sin(d*x
+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticE((1-sin(d*x+c)
)^(1/2),1/2*2^(1/2))*b^2-9*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin
(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2-6*(1-sin(d*x
+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c)
)^(1/2),1/2*2^(1/2))*b^2-6*a^2*sin(d*x+c)^4-14*sin(d*x+c)^4*b^2+6*a^2*sin(
d*x+c)^2+4*b^2*sin(d*x+c)^2)/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.02

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}\,dx =$$

$$-\frac{2\left(-231i\left(3a^3+2ab^2\right)\sqrt{-\frac{1}{2}i\,e}e^2\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx\right.}{}$$

input
```
integrate((a+b*cos(d*x+c))^3*(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-2/1155*(-231*I*(3*a^3 + 2*a*b^2)*sqrt(-1/2*I*e)*e^2*weierstrassZeta(4, 0,
 weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) + 231*I*(3*a^3
+ 2*a*b^2)*sqrt(1/2*I*e)*e^2*weierstrassZeta(4, 0, weierstrassPInverse(4,
0, cos(d*x + c) - I*sin(d*x + c))) + (105*b^3*e^2*cos(d*x + c)^4 + 385*a*b
^2*e^2*cos(d*x + c)^3 + 45*(11*a^2*b - b^3)*e^2*cos(d*x + c)^2 + 231*(a^3
- a*b^2)*e^2*cos(d*x + c) - 15*(33*a^2*b + 4*b^3)*e^2)*sqrt(e*sin(d*x + c)
)*sin(d*x + c))/d
```

## Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**3*(e*sin(d*x+c))**(5/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}\,dx = \int (b\cos(dx+c)+a)^3(e\sin(dx+c))^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3*(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^3*(e*sin(d*x + c))^(5/2), x)
```

## Giac [F]

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}\,dx = \int (b\cos(dx+c)+a)^3(e\sin(dx+c))^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3*(e*sin(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^3*(e*sin(d*x + c))^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}\,dx=\int (e\,\sin\left(c+d\,x\right))^{5/2}\,\left(a+b\,\cos\left(c+d\,x\right)\right)^3\,dx$$

input
```
int((e*sin(c + d*x))^(5/2)*(a + b*cos(c + d*x))^3,x)
```

output
```
int((e*sin(c + d*x))^(5/2)*(a + b*cos(c + d*x))^3, x)
```

## Reduce [F]

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}\,dx=\frac{\sqrt{e}\,e^2\left(6\sqrt{\sin\left(dx+c\right)}\,\sin\left(dx+c\right)^3a^2b+7\left(\int\sqrt{\sin\left(dx+c\right)}\,\cos\left(dx+c\right)^3\sin\left(dx+c\right)^2dx\right)\right.}{}$$

input
```
int((a+b*cos(d*x+c))^3*(e*sin(d*x+c))^(5/2),x)
```

output
```
(sqrt(e)*e**2*(6*sqrt(sin(c + d*x))*sin(c + d*x)**3*a**2*b + 7*int(sqrt(si
n(c + d*x))*cos(c + d*x)**3*sin(c + d*x)**2,x)*b**3*d + 21*int(sqrt(sin(c
+ d*x))*cos(c + d*x)**2*sin(c + d*x)**2,x)*a*b**2*d + 7*int(sqrt(sin(c + d
*x))*sin(c + d*x)**2,x)*a**3*d))/(7*d)
```

# 3.51 $\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 202

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}\,dx = \frac{2a(7a^2+6b^2)\,e^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{21d\sqrt{e\sin(c+dx)}} - \frac{2a(7a^2+6b^2)\,e\cos(c+dx)\sqrt{e\sin(c+dx)}}{21d} + \frac{2b(89a^2+28b^2)\,(e\sin(c+dx))^{5/2}}{315de} + \frac{26ab(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}{63de} + \frac{2b(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}}{9de}$$

output
```
2/21*a*(7*a^2+6*b^2)*e^2*InverseJacobiAM(1/2*c-1/4*Pi+1/2*d*x,2^(1/2))*sin
(d*x+c)^(1/2)/d/(e*sin(d*x+c))^(1/2)-2/21*a*(7*a^2+6*b^2)*e*cos(d*x+c)*(e*
sin(d*x+c))^(1/2)/d+2/315*b*(89*a^2+28*b^2)*(e*sin(d*x+c))^(5/2)/d/e+26/63
*a*b*(a+b*cos(d*x+c))*(e*sin(d*x+c))^(5/2)/d/e+2/9*b*(a+b*cos(d*x+c))^2*(e
*sin(d*x+c))^(5/2)/d/e
```

## Mathematica [A] (verified)

Time = 2.17 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.73

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}\,dx = \frac{\left(-20a(28a^2+15b^2)\cot(c+dx)-\frac{2}{3}b(-756a^2-147b^2+28(27a^2+4b^2)\cos(2(c+dx))+2\right.}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^3*(e*Sin[c + d*x])^(3/2),x]
```

output
```
((-20*a*(28*a^2 + 15*b^2)*Cot[c + d*x] - (2*b*(-756*a^2 - 147*b^2 + 28*(27
*a^2 + 4*b^2)*Cos[2*(c + d*x)] + 270*a*b*Cos[3*(c + d*x)] + 35*b^2*Cos[4*(
c + d*x)])*Csc[c + d*x])/3 - (80*a*(7*a^2 + 6*b^2)*EllipticF[(-2*c + Pi -
2*d*x)/4, 2])/Sin[c + d*x]^(3/2))*(e*Sin[c + d*x])^(3/2))/(840*d)
```

## Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3042, 3171, 27, 3042, 3341, 27, 3042, 3148, 3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^3\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \ 3171$$

$$\frac{2}{9}\int \frac{1}{2}(a+b\cos(c+dx))\left(9a^2+13b\cos(c+dx)a+4b^2\right)(e\sin(c+dx))^{3/2}dx+\frac{2b(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}{9de}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9}\int (a+b\cos(c+dx))\left(9a^2+13b\cos(c+dx)a+4b^2\right)(e\sin(c+dx))^{3/2}dx+\frac{2b(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}{9de}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\int \left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(9a^2+13b\sin\left(c+dx+\frac{\pi}{2}\right)a+4b^2\right)dx+\frac{2b(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}{9de}$$

$$\Big\downarrow \text{3341}$$

$$\frac{1}{9}\left(\frac{2}{7}\int \frac{1}{2}\left(9a\left(7a^2+6b^2\right)+b\left(89a^2+28b^2\right)\cos(c+dx)\right)(e\sin(c+dx))^{3/2}dx+\frac{26ab(e\sin(c+dx))^{5/2}(a+b\cos(}{7de}\right.$$
$$\frac{2b(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}{9de}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9}\left(\frac{1}{7}\int \left(9a\left(7a^2+6b^2\right)+b\left(89a^2+28b^2\right)\cos(c+dx)\right)(e\sin(c+dx))^{3/2}dx+\frac{26ab(e\sin(c+dx))^{5/2}(a+b\cos(c}{7de}\right.$$
$$\frac{2b(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}{9de}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\left(\frac{1}{7}\int \left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}\left(9a\left(7a^2+6b^2\right)-b\left(89a^2+28b^2\right)\sin\left(c+dx-\frac{\pi}{2}\right)\right)dx+\frac{26ab(e\sin(c+dx))^{5}}{7}\right.$$
$$\frac{2b(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}{9de}$$

$$\Big\downarrow \text{3148}$$

$$\frac{1}{9}\left(\frac{1}{7}\left(9a\left(7a^2+6b^2\right)\int (e\sin(c+dx))^{3/2}dx+\frac{2b\left(89a^2+28b^2\right)(e\sin(c+dx))^{5/2}}{5de}\right)+\frac{26ab(e\sin(c+dx))^{5/2}(a+}{7de}\right.$$
$$\frac{2b(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}{9de}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\left(\frac{1}{7}\left(9a\left(7a^2+6b^2\right)\int(e\sin(c+dx))^{3/2}dx+\frac{2b\left(89a^2+28b^2\right)(e\sin(c+dx))^{5/2}}{5de}\right)+\frac{26ab(e\sin(c+dx))^{5/2}(a+}{7de}\right.$$
$$\frac{2b(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}{9de}$$

$\Big\downarrow$ 3115

$$\frac{1}{9}\left(\frac{1}{7}\left(9a\left(7a^2+6b^2\right)\left(\frac{1}{3}e^2\int\frac{1}{\sqrt{e\sin(c+dx)}}dx-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)+\frac{2b\left(89a^2+28b^2\right)(e\sin(c+}{5de}\right.\right.$$
$$\frac{2b(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}{9de}$$

$\Big\downarrow$ 3042

$$\frac{1}{9}\left(\frac{1}{7}\left(9a\left(7a^2+6b^2\right)\left(\frac{1}{3}e^2\int\frac{1}{\sqrt{e\sin(c+dx)}}dx-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)+\frac{2b\left(89a^2+28b^2\right)(e\sin(c+}{5de}\right.\right.$$
$$\frac{2b(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}{9de}$$

$\Big\downarrow$ 3121

$$\frac{1}{9}\left(\frac{1}{7}\left(9a\left(7a^2+6b^2\right)\left(\frac{e^2\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{3\sqrt{e\sin(c+dx)}}-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)+\frac{2b\left(89a^2+28b^2\right)(e}{5de}\right.\right.$$
$$\frac{2b(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}{9de}$$

$\Big\downarrow$ 3042

$$\frac{1}{9}\left(\frac{1}{7}\left(9a\left(7a^2+6b^2\right)\left(\frac{e^2\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{3\sqrt{e\sin(c+dx)}}-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)+\frac{2b\left(89a^2+28b^2\right)(e}{5de}\right.\right.$$
$$\frac{2b(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}{9de}$$

$\Big\downarrow$ 3120

$$\frac{1}{9}\left(\frac{1}{7}\left(9a\left(7a^2+6b^2\right)\left(\frac{2e^2\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{3d\sqrt{e\sin(c+dx)}}-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)+\frac{2b(8}{}\right.\right.$$
$$\frac{2b(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}{9de}$$

input
```
Int[(a + b*Cos[c + d*x])^3*(e*Sin[c + d*x])^(3/2),x]
```

output
```
(2*b*(a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(5/2))/(9*d*e) + ((26*a*b*(a
+ b*Cos[c + d*x])*(e*Sin[c + d*x])^(5/2))/(7*d*e) + ((2*b*(89*a^2 + 28*b^2
)*(e*Sin[c + d*x])^(5/2))/(5*d*e) + 9*a*(7*a^2 + 6*b^2)*((2*e^2*EllipticF[
(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*d*Sqrt[e*Sin[c + d*x]]) - (2
*e*Cos[c + d*x]*Sqrt[e*Sin[c + d*x]])/(3*d)))/7)/9
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3171
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[1/(m + p)   Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
 && GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

rule 3341
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[1/(m + p + 1)   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Sim
p[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 &&  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && S
implerQ[c + d*x, a + b*x])
```

## Maple [A] (verified)

Time = 3.83 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.35

| method | result |
|---|---|
| parts | $-\frac{a^{3} e^{2} \left(\sqrt{1-\sin \left(d x+c\right)}\, \sqrt{2+2 \sin \left(d x+c\right)}\, \sqrt{\sin \left(d x+c\right)}\, \operatorname{EllipticF}\left(\sqrt{1-\sin \left(d x+c\right)}, \frac{\sqrt{2}}{2}\right)-2 \sin \left(d x+c\right)^{3}+2 \sin \left(d x+c\right)\right)}{3 \cos \left(d x+c\right) \sqrt{e \sin \left(d x+c\right)}\, d}-\frac{2 b^{3} \left(\frac{(e \sin}{}\right.}{}$ |
| default | $-\frac{e^{2} \left(70 b^{3} \cos \left(d x+c\right)^{5} \sin \left(d x+c\right)+270 a \,b^{2} \cos \left(d x+c\right)^{4} \sin \left(d x+c\right)+105 \sqrt{1-\sin \left(d x+c\right)}\, \sqrt{2+2 \sin \left(d x+c\right)}\, \sqrt{\sin \left(d x+c\right)}\, \operatorname{EllipticF}\left(\sqrt{}\right.\right.}{}$ |

input
```
int((a+cos(d*x+c)*b)^3*(e*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*a^3*e^2*((1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)
*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-2*sin(d*x+c)^3+2*sin(d*x+c))/
cos(d*x+c)/(e*sin(d*x+c))^(1/2)/d-2*b^3/d/e^3*(1/9*(e*sin(d*x+c))^(9/2)-1/
5*e^2*(e*sin(d*x+c))^(5/2))+6/5*a^2*b*(e*sin(d*x+c))^(5/2)/e/d-2/7*b^2*a*e
^2*(3*sin(d*x+c)^5+(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^
(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-5*sin(d*x+c)^3+2*sin(d*x
+c))/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/d
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.91

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}\,dx = \frac{2\left(15\left(7a^3+6ab^2\right)\sqrt{-\frac{1}{2}i\,e}e\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\,\sin(dx+c))+15\left(7a\right.\right.}{}$$

input
```
integrate((a+b*cos(d*x+c))^3*(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
2/315*(15*(7*a^3 + 6*a*b^2)*sqrt(-1/2*I*e)*e*weierstrassPInverse(4, 0, cos
(d*x + c) + I*sin(d*x + c)) + 15*(7*a^3 + 6*a*b^2)*sqrt(1/2*I*e)*e*weierst
rassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)) - (35*b^3*e*cos(d*x + c)
^4 + 135*a*b^2*e*cos(d*x + c)^3 + 7*(27*a^2*b - b^3)*e*cos(d*x + c)^2 + 15
*(7*a^3 - 3*a*b^2)*e*cos(d*x + c) - 7*(27*a^2*b + 4*b^3)*e)*sqrt(e*sin(d*x
 + c)))/d
```

## Sympy [F]

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}\,dx = \int (e\sin(c+dx))^{\frac{3}{2}}\,(a+b\cos(c+dx))^3\,dx$$

input
```
integrate((a+b*cos(d*x+c))**3*(e*sin(d*x+c))**(3/2),x)
```

output
```
Integral((e*sin(c + d*x))**(3/2)*(a + b*cos(c + d*x))**3, x)
```

## Maxima [F]

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}\,dx = \int (b\cos(dx+c)+a)^3(e\sin(dx+c))^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3*(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^3*(e*sin(d*x + c))^(3/2), x)
```

## Giac [F]

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}\,dx = \int (b\cos(dx+c)+a)^3(e\sin(dx+c))^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3*(e*sin(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^3*(e*sin(d*x + c))^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}\,dx = \int (e\,\sin(c+d\,x))^{3/2}\,(a+b\,\cos(c+d\,x))^3\,dx$$

input
```
int((e*sin(c + d*x))^(3/2)*(a + b*cos(c + d*x))^3,x)
```

output
```
int((e*sin(c + d*x))^(3/2)*(a + b*cos(c + d*x))^3, x)
```

## Reduce [F]

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}\,dx = \frac{\sqrt{e}\,e\left(6\sqrt{\sin(dx+c)}\,\sin(dx+c)^2 a^2b+5\left(\int\sqrt{\sin(dx+c)}\,\cos(dx+c)^3\sin(dx+c)\,dx\right)b^3\right.}{}$$

input

```
int((a+b*cos(d*x+c))^3*(e*sin(d*x+c))^(3/2),x)
```

output

```
(sqrt(e)*e*(6*sqrt(sin(c + d*x))*sin(c + d*x)**2*a**2*b + 5*int(sqrt(sin(c
 + d*x))*cos(c + d*x)**3*sin(c + d*x),x)*b**3*d + 15*int(sqrt(sin(c + d*x)
)*cos(c + d*x)**2*sin(c + d*x),x)*a*b**2*d + 5*int(sqrt(sin(c + d*x))*sin(
c + d*x),x)*a**3*d))/(5*d)
```

# 3.52 $\int (a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 161

$$\begin{aligned}\int (a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}\,dx &= \frac{2a(5a^2+6b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}} \\ &\quad+\frac{2b(57a^2+20b^2)\,(e\sin(c+dx))^{3/2}}{105de}+\frac{22ab(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}{35de} \\ &\quad+\frac{2b(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}}{7de}\end{aligned}$$

output
```
-2/5*a*(5*a^2+6*b^2)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d
*x+c))^(1/2)/d/sin(d*x+c)^(1/2)+2/105*b*(57*a^2+20*b^2)*(e*sin(d*x+c))^(3/
2)/d/e+22/35*a*b*(a+b*cos(d*x+c))*(e*sin(d*x+c))^(3/2)/d/e+2/7*b*(a+b*cos(
d*x+c))^2*(e*sin(d*x+c))^(3/2)/d/e
```

## Mathematica [A] (verified)

Time = 1.45 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.65

$$\int (a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}\,dx$$
$$=\frac{\sqrt{e\sin(c+dx)}\left(-42(5a^3+6ab^2)\,E\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\,2\right)+b(210a^2+55b^2+126ab\cos(c+dx)+15b\right.}{105d\sqrt{\sin(c+dx)}}$$

input
```
Integrate[(a + b*Cos[c + d*x])^3*Sqrt[e*Sin[c + d*x]],x]
```

output
```
(Sqrt[e*Sin[c + d*x]]*(-42*(5*a^3 + 6*a*b^2)*EllipticE[(-2*c + Pi - 2*d*x)
/4, 2] + b*(210*a^2 + 55*b^2 + 126*a*b*Cos[c + d*x] + 15*b^2*Cos[2*(c + d*
x)])*Sin[c + d*x]^(3/2)))/(105*d*Sqrt[Sin[c + d*x]])
```

## Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 3171, 27, 3042, 3341, 27, 3042, 3148, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{3171}$$

$$\frac{2}{7}\int \frac{1}{2}(a+b\cos(c+dx))\left(7a^2+11b\cos(c+dx)a+4b^2\right)\sqrt{e\sin(c+dx)}dx+\frac{2b(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}{7de}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\int (a+b\cos(c+dx))\left(7a^2+11b\cos(c+dx)a+4b^2\right)\sqrt{e\sin(c+dx)}dx+\frac{2b(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}{7de}$$

$\big\downarrow$ 3042

$$\frac{1}{7}\int \sqrt{-e\cos\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(7a^2+11b\sin\left(c+dx+\frac{\pi}{2}\right)a+4b^2\right)dx+\frac{2b(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}{7de}$$

$\big\downarrow$ 3341

$$\frac{1}{7}\left(\frac{2}{5}\int \frac{1}{2}\left(7a\left(5a^2+6b^2\right)+b\left(57a^2+20b^2\right)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}dx+\frac{22ab(e\sin(c+dx))^{3/2}(a+b\cos(c-\right.$$
$$\frac{2b(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}{7de}$$

$\big\downarrow$ 27

$$\frac{1}{7}\left(\frac{1}{5}\int \left(7a\left(5a^2+6b^2\right)+b\left(57a^2+20b^2\right)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}dx+\frac{22ab(e\sin(c+dx))^{3/2}(a+b\cos(c+\right.$$
$$\frac{2b(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}{7de}$$

$\big\downarrow$ 3042

$$\frac{1}{7}\left(\frac{1}{5}\int \sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(7a\left(5a^2+6b^2\right)-b\left(57a^2+20b^2\right)\sin\left(c+dx-\frac{\pi}{2}\right)\right)dx+\frac{22ab(e\sin(c+dx))^{3/2}(\right.$$
$$\frac{2b(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}{7de}$$

$\big\downarrow$ 3148

$$\frac{1}{7}\left(\frac{1}{5}\left(7a\left(5a^2+6b^2\right)\int \sqrt{e\sin(c+dx)}dx+\frac{2b\left(57a^2+20b^2\right)(e\sin(c+dx))^{3/2}}{3de}\right)+\frac{22ab(e\sin(c+dx))^{3/2}(a+b\right.$$
$$\frac{2b(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}{7de}$$

$\big\downarrow$ 3042

$$\frac{1}{7}\left(\frac{1}{5}\left(7a\left(5a^2+6b^2\right)\int\sqrt{e\sin(c+dx)}dx+\frac{2b\left(57a^2+20b^2\right)(e\sin(c+dx))^{3/2}}{3de}\right)+\frac{22ab(e\sin(c+dx))^{3/2}(a+b}{5de}\right.$$
$$\frac{2b(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}{7de}$$

$$\Big\downarrow \text{ 3121}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{7a\left(5a^2+6b^2\right)\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}+\frac{2b\left(57a^2+20b^2\right)(e\sin(c+dx))^{3/2}}{3de}\right)+\frac{22ab(e\sin(c+}{}\right.$$
$$\frac{2b(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}{7de}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{7a\left(5a^2+6b^2\right)\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}+\frac{2b\left(57a^2+20b^2\right)(e\sin(c+dx))^{3/2}}{3de}\right)+\frac{22ab(e\sin(c+}{}\right.$$
$$\frac{2b(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}{7de}$$

$$\Big\downarrow \text{ 3119}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{2b\left(57a^2+20b^2\right)(e\sin(c+dx))^{3/2}}{3de}+\frac{14a\left(5a^2+6b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}\right)+\frac{22ab(e\sin}{}\right.$$
$$\frac{2b(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}{7de}$$

input
```
Int[(a + b*Cos[c + d*x])^3*Sqrt[e*Sin[c + d*x]],x]
```

output
```
(2*b*(a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(3/2))/(7*d*e) + ((22*a*b*(a
+ b*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2))/(5*d*e) + ((14*a*(5*a^2 + 6*b^2)
*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(d*Sqrt[Sin[c + d*
x]]) + (2*b*(57*a^2 + 20*b^2)*(e*Sin[c + d*x])^(3/2))/(3*d*e))/5)/7
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3171
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[1/(m + p)   Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
 && GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

rule 3341
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[1/(m + p + 1)   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Sim
p[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 &&  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && S
implerQ[c + d*x, a + b*x])
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(143) = 286$.

Time = 4.27 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.95

| method | result |
|---|---|
| parts | $-\frac{a^{3} e \sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sqrt{\sin(dx+c)}\left(2\,\mathrm{EllipticE}\left(\sqrt{1-\sin(dx+c)}, \frac{\sqrt{2}}{2}\right)-\mathrm{EllipticF}\left(\sqrt{1-\sin(dx+c)}, \frac{\sqrt{2}}{2}\right)\right)}{\cos(dx+c)\sqrt{e\sin(dx+c)}\,d} - \frac{2b^{3}}{}$ |
| default | $\frac{\frac{2b(e\sin(dx+c))^{\frac{3}{2}}\left(3\cos(dx+c)^{2}b^{2}+21a^{2}+4b^{2}\right)}{21e} - \frac{ae\left(10\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sqrt{\sin(dx+c)}\,\mathrm{EllipticE}\left(\sqrt{1-\sin(dx+c)}, \frac{\sqrt{2}}{2}\right)a^{2}+12\sqrt{1-\mathrm{si}}\right.}{}}{}$ |

input
```
int((a+cos(d*x+c)*b)^3*(e*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-a^3*e*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*(2*Ell
ipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-EllipticF((1-sin(d*x+c))^(1/2),1/
2*2^(1/2)))/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/d-2*b^3/d/e^3*(1/7*(e*sin(d*x+
c))^(7/2)-1/3*e^2*(e*sin(d*x+c))^(3/2))-6/5*b^2*a*e*(2*(1-sin(d*x+c))^(1/2
)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticE((1-sin(d*x+c))^(1/2),1
/2*2^(1/2))-(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*E
llipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))+cos(d*x+c)^4-cos(d*x+c)^2)/cos(
d*x+c)/(e*sin(d*x+c))^(1/2)/d+2*a^2*b*(e*sin(d*x+c))^(3/2)/e/d
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.87

$$\int (a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}\,dx$$
$$= \frac{2\left(\left(15\,b^3\cos\left(dx+c\right)^2+63\,ab^2\cos\left(dx+c\right)+105\,a^2b+20\,b^3\right)\sqrt{e\sin\left(dx+c\right)}\sin\left(dx+c\right)-21\left(-5i\,a^3\right.\right.}{}$$

input
```
integrate((a+b*cos(d*x+c))^3*(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
2/105*((15*b^3*cos(d*x + c)^2 + 63*a*b^2*cos(d*x + c) + 105*a^2*b + 20*b^3
)*sqrt(e*sin(d*x + c))*sin(d*x + c) - 21*(-5*I*a^3 - 6*I*a*b^2)*sqrt(-1/2*
I*e)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(
d*x + c))) - 21*(5*I*a^3 + 6*I*a*b^2)*sqrt(1/2*I*e)*weierstrassZeta(4, 0,
weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

## Sympy [F]

$$\int (a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}\,dx = \int \sqrt{e\sin\left(c+dx\right)}\left(a+b\cos\left(c+dx\right)\right)^3\,dx$$

input
```
integrate((a+b*cos(d*x+c))**3*(e*sin(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(e*sin(c + d*x))*(a + b*cos(c + d*x))**3, x)
```

## Maxima [F]

$$\int (a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}\,dx=\int (b\cos{(dx+c)}+a)^3\sqrt{e\sin{(dx+c)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3*(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^3*sqrt(e*sin(d*x + c)), x)
```

## Giac [F]

$$\int (a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}\,dx=\int (b\cos{(dx+c)}+a)^3\sqrt{e\sin{(dx+c)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3*(e*sin(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^3*sqrt(e*sin(d*x + c)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}\,dx=\int \sqrt{e\,\sin{(c+d\,x)}}\,(a+b\,\cos{(c+d\,x)})^3\,dx$$

input
```
int((e*sin(c + d*x))^(1/2)*(a + b*cos(c + d*x))^3,x)
```

output
```
int((e*sin(c + d*x))^(1/2)*(a + b*cos(c + d*x))^3, x)
```

## Reduce [F]

$$\int (a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}\,dx$$
$$= \frac{\sqrt{e}\left(2\sqrt{\sin(dx+c)}\,\sin(dx+c)\,a^2b+\left(\int\sqrt{\sin(dx+c)}dx\right)a^3d+\left(\int\sqrt{\sin(dx+c)}\,\cos(dx+c)^3\,dx\right)\right.}{d}$$

input
```
int((a+b*cos(d*x+c))^3*(e*sin(d*x+c))^(1/2),x)
```

output
```
(sqrt(e)*(2*sqrt(sin(c + d*x))*sin(c + d*x)*a**2*b + int(sqrt(sin(c + d*x)
),x)*a**3*d + int(sqrt(sin(c + d*x))*cos(c + d*x)**3,x)*b**3*d + 3*int(sqr
t(sin(c + d*x))*cos(c + d*x)**2,x)*a*b**2*d))/d
```

# 3.53 $\int \frac{(a+b\cos(c+dx))^3}{\sqrt{e\sin(c+dx)}}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 157

$$\int \frac{(a+b\cos(c+dx))^3}{\sqrt{e\sin(c+dx)}}\,dx = \frac{2a(a^2+2b^2)\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{d\sqrt{e\sin(c+dx)}} + \frac{2b(11a^2+4b^2)\sqrt{e\sin(c+dx)}}{5de} + \frac{6ab(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}{5de} + \frac{2b(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}}{5de}$$

output

```
2*a*(a^2+2*b^2)*InverseJacobiAM(1/2*c-1/4*Pi+1/2*d*x,2^(1/2))*sin(d*x+c)^(
1/2)/d/(e*sin(d*x+c))^(1/2)+2/5*b*(11*a^2+4*b^2)*(e*sin(d*x+c))^(1/2)/d/e+
6/5*a*b*(a+b*cos(d*x+c))*(e*sin(d*x+c))^(1/2)/d/e+2/5*b*(a+b*cos(d*x+c))^2
*(e*sin(d*x+c))^(1/2)/d/e
```

## Mathematica [A] (verified)

Time = 1.63 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.62

$$\int \frac{(a+b\cos(c+dx))^3}{\sqrt{e\sin(c+dx)}}\,dx$$
$$= \frac{-10a(a^2+2b^2)\,\text{EllipticF}\left(\frac{1}{4}(-2c+\pi-2dx),2\right)\sqrt{\sin(c+dx)}+b(30a^2+9b^2+10ab\cos(c+dx)+b^2\,\text{c}}{5d\sqrt{e\sin(c+dx)}}$$

input
```
Integrate[(a + b*Cos[c + d*x])^3/Sqrt[e*Sin[c + d*x]],x]
```

output
```
(-10*a*(a^2 + 2*b^2)*EllipticF[(-2*c + Pi - 2*d*x)/4, 2]*Sqrt[Sin[c + d*x]
] + b*(30*a^2 + 9*b^2 + 10*a*b*Cos[c + d*x] + b^2*Cos[2*(c + d*x)])*Sin[c
+ d*x])/(5*d*Sqrt[e*Sin[c + d*x]])
```

## Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 3171, 27, 3042, 3341, 27, 3042, 3148, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^3}{\sqrt{e\sin(c+dx)}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}dx$$
$$\Big\downarrow \text{3171}$$
$$\frac{2}{5}\int \frac{(a+b\cos(c+dx))\left(5a^2+9b\cos(c+dx)a+4b^2\right)}{2\sqrt{e\sin(c+dx)}}dx+\frac{2b\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}{5de}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{5}\int \frac{(a+b\cos(c+dx))\left(5a^2+9b\cos(c+dx)a+4b^2\right)}{\sqrt{e\sin(c+dx)}}dx+\frac{2b\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}{5de}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{5}\int \frac{\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)\left(5a^2-9b\sin\left(c+dx-\frac{\pi}{2}\right)a+4b^2\right)}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}dx+\frac{2b\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}{5de}$$

$$\Big\downarrow \text{ 3341}$$

$$\frac{1}{5}\left(\frac{2}{3}\int \frac{3\left(5a\left(a^2+2b^2\right)+b\left(11a^2+4b^2\right)\cos(c+dx)\right)}{2\sqrt{e\sin(c+dx)}}dx+\frac{6ab\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}{de}\right)+\frac{2b\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}{5de}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{5}\left(\int \frac{5a\left(a^2+2b^2\right)+b\left(11a^2+4b^2\right)\cos(c+dx)}{\sqrt{e\sin(c+dx)}}dx+\frac{6ab\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}{de}\right)+\frac{2b\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}{5de}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{5}\left(\int \frac{5a\left(a^2+2b^2\right)-b\left(11a^2+4b^2\right)\sin\left(c+dx-\frac{\pi}{2}\right)}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}dx+\frac{6ab\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}{de}\right)+\frac{2b\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}{5de}$$

$$\Big\downarrow \text{ 3148}$$

$$\frac{1}{5}\left(5a\left(a^2+2b^2\right)\int \frac{1}{\sqrt{e\sin(c+dx)}}dx+\frac{2b\left(11a^2+4b^2\right)\sqrt{e\sin(c+dx)}}{de}+\frac{6ab\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}{de}\right)+\frac{2b\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}{5de}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{5}\left(5a\left(a^2+2b^2\right)\int\frac{1}{\sqrt{e\sin(c+dx)}}dx+\frac{2b(11a^2+4b^2)\sqrt{e\sin(c+dx)}}{de}+\frac{6ab\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}{de}\right.$$
$$\frac{2b\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}{5de}$$

$\big\downarrow$ 3121

$$\frac{1}{5}\left(\frac{5a\left(a^2+2b^2\right)\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{\sqrt{e\sin(c+dx)}}+\frac{2b(11a^2+4b^2)\sqrt{e\sin(c+dx)}}{de}+\frac{6ab\sqrt{e\sin(c+dx)}(a+b\cos(}{de}\right.$$
$$\frac{2b\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}{5de}$$

$\big\downarrow$ 3042

$$\frac{1}{5}\left(\frac{5a\left(a^2+2b^2\right)\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{\sqrt{e\sin(c+dx)}}+\frac{2b(11a^2+4b^2)\sqrt{e\sin(c+dx)}}{de}+\frac{6ab\sqrt{e\sin(c+dx)}(a+b\cos(}{de}\right.$$
$$\frac{2b\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}{5de}$$

$\big\downarrow$ 3120

$$\frac{1}{5}\left(\frac{2b(11a^2+4b^2)\sqrt{e\sin(c+dx)}}{de}+\frac{10a\left(a^2+2b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}+6ab\sqrt{e\sin(c+\right.$$
$$\frac{2b\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}{5de}$$

input
```
Int[(a + b*Cos[c + d*x])^3/Sqrt[e*Sin[c + d*x]],x]
```

output
```
(2*b*(a + b*Cos[c + d*x])^2*Sqrt[e*Sin[c + d*x]])/(5*d*e) + ((10*a*(a^2 +
2*b^2)*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(d*Sqrt[e*Sin[
c + d*x]]) + (2*b*(11*a^2 + 4*b^2)*Sqrt[e*Sin[c + d*x]])/(d*e) + (6*a*b*(a
 + b*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]])/(d*e))/5
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3171
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[1/(m + p)   Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
 && GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

rule 3341
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[1/(m + p + 1)   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Sim
p[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 &&  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && S
implerQ[c + d*x, a + b*x])
```

## Maple [A] (verified)

Time = 4.01 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.34

| method | result |
|---|---|
| default | $-\frac{5\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sqrt{\sin(dx+c)}\,\operatorname{EllipticF}\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)a^{3}+10\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sqrt{\sin(dx}}{5}$ |
| parts | $-\frac{a^{3}\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sqrt{\sin(dx+c)}\,\operatorname{EllipticF}\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)}{\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}-\frac{2b^{3}\left(\frac{(e\sin(dx+c))^{\frac{5}{2}}}{5}-e^{2}\sqrt{e\sin(dx+c)}\right)}{d\,e^{3}}+$ |

input
```
int((a+cos(d*x+c)*b)^3/(e*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*(5*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+
c))^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^3
+10*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticF
((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a*b^2-2*b^3*cos(d*x+c)^3*sin(d*x+c)-10*
a*b^2*cos(d*x+c)^2*sin(d*x+c)-30*a^2*b*cos(d*x+c)*sin(d*x+c)-8*b^3*cos(d*x
+c)*sin(d*x+c))/d
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.80

$$\int \frac{(a+b\cos(c+dx))^3}{\sqrt{e\sin(c+dx)}}\,dx$$
$$= \frac{2\left(5\left(a^3+2\,ab^2\right)\sqrt{-\frac{1}{2}i\,e}\text{weierstrassPInverse}(4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+5\left(a^3+2\,ab^2\right)\sqrt{\frac{1}{2}i\,e}\text{we}}{}$$

input
```
integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
2/5*(5*(a^3 + 2*a*b^2)*sqrt(-1/2*I*e)*weierstrassPInverse(4, 0, cos(d*x +
c) + I*sin(d*x + c)) + 5*(a^3 + 2*a*b^2)*sqrt(1/2*I*e)*weierstrassPInverse
(4, 0, cos(d*x + c) - I*sin(d*x + c)) + (b^3*cos(d*x + c)^2 + 5*a*b^2*cos(
d*x + c) + 15*a^2*b + 4*b^3)*sqrt(e*sin(d*x + c)))/(d*e)
```

## Sympy [F]

$$\int \frac{(a+b\cos(c+dx))^3}{\sqrt{e\sin(c+dx)}}\,dx = \int \frac{\left(a+b\cos\left(c+dx\right)\right)^3}{\sqrt{e\sin\left(c+dx\right)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**3/(e*sin(d*x+c))**(1/2),x)
```

output
```
Integral((a + b*cos(c + d*x))**3/sqrt(e*sin(c + d*x)), x)
```

## Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^3}{\sqrt{e\sin(c+dx)}}\,dx = \int \frac{(b\cos(dx+c)+a)^3}{\sqrt{e\sin(dx+c)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^3/sqrt(e*sin(d*x + c)), x)
```

## Giac [F]

$$\int \frac{(a+b\cos(c+dx))^3}{\sqrt{e\sin(c+dx)}}\,dx = \int \frac{(b\cos(dx+c)+a)^3}{\sqrt{e\sin(dx+c)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^3/sqrt(e*sin(d*x + c)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^3}{\sqrt{e\sin(c+dx)}}\,dx = \int \frac{(a+b\cos(c+d\,x))^3}{\sqrt{e\sin(c+d\,x)}}\,dx$$

input
```
int((a + b*cos(c + d*x))^3/(e*sin(c + d*x))^(1/2),x)
```

output
```
int((a + b*cos(c + d*x))^3/(e*sin(c + d*x))^(1/2), x)
```

## Reduce [F]

$$\int \frac{(a+b\cos(c+dx))^3}{\sqrt{e\sin(c+dx)}}\,dx$$
$$=\frac{\sqrt{e}\left(6\sqrt{\sin(dx+c)}\,a^2b+\left(\int\frac{\sqrt{\sin(dx+c)}}{\sin(dx+c)}dx\right)a^3d+\left(\int\frac{\sqrt{\sin(dx+c)}\cos(dx+c)^3}{\sin(dx+c)}dx\right)b^3d+3\left(\int\frac{\sqrt{\sin(dx+c)}\cos(dx+c)^2}{\sin(dx+c)}dx\right)ab^2d\right)}{de}$$

input
```
int((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(1/2),x)
```

output
```
(sqrt(e)*(6*sqrt(sin(c + d*x))*a**2*b + int(sqrt(sin(c + d*x))/sin(c + d*x
),x)*a**3*d + int((sqrt(sin(c + d*x))*cos(c + d*x)**3)/sin(c + d*x),x)*b**
3*d + 3*int((sqrt(sin(c + d*x))*cos(c + d*x)**2)/sin(c + d*x),x)*a*b**2*d)
)/(d*e)
```

# 3.54 $\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{3/2}}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 165

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{3/2}}\,dx = -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))^2}{de\sqrt{e\sin(c+dx)}} - \frac{2a(a^2+6b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{de^2\sqrt{\sin(c+dx)}} - \frac{2b(3a^2+4b^2)\,(e\sin(c+dx))^{3/2}}{3de^3} - \frac{2ab(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}{de^3}$$

output
```
-2*(b+a*cos(d*x+c))*(a+b*cos(d*x+c))^2/d/e/(e*sin(d*x+c))^(1/2)+2*a*(a^2+6
*b^2)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/d/
e^2/sin(d*x+c)^(1/2)-2/3*b*(3*a^2+4*b^2)*(e*sin(d*x+c))^(3/2)/d/e^3-2*a*b*
(a+b*cos(d*x+c))*(e*sin(d*x+c))^(3/2)/d/e^3
```

## Mathematica [A] (verified)

Time = 1.23 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.61

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{3/2}}\,dx =$$

$$-\frac{2\left(9a^2b+3b^3+3a(a^2+3b^2)\cos(c+dx)-3a(a^2+6b^2)\,E\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\,2\right)\sqrt{\sin(c+dx)}+b^3\sin^2\right.}{3de\sqrt{e\sin(c+dx)}}$$

input
```
Integrate[(a + b*Cos[c + d*x])^3/(e*Sin[c + d*x])^(3/2),x]
```

output
```
(-2*(9*a^2*b + 3*b^3 + 3*a*(a^2 + 3*b^2)*Cos[c + d*x] - 3*a*(a^2 + 6*b^2)*
EllipticE[(-2*c + Pi - 2*d*x)/4, 2]*Sqrt[Sin[c + d*x]] + b^3*Sin[c + d*x]^
2))/(3*d*e*Sqrt[e*Sin[c + d*x]])
```

## Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {3042, 3170, 27, 3042, 3341, 27, 3042, 3148, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{3/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \; 3170$$

$$-\frac{2\int \frac{1}{2}(a+b\cos(c+dx))\left(a^2+5b\cos(c+dx)a+4b^2\right)\sqrt{e\sin(c+dx)}dx}{e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{de\sqrt{e\sin(c+dx)}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int (a+b\cos(c+dx))\left(a^2+5b\cos(c+dx)a+4b^2\right)\sqrt{e\sin(c+dx)}dx}{e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{de\sqrt{e\sin(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\int \sqrt{-e\cos\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(a^2+5b\sin\left(c+dx+\frac{\pi}{2}\right)a+4b^2\right)dx}{e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{de\sqrt{e\sin(c+dx)}}$$

$$\Big\downarrow \; 3341$$

$$-\frac{\frac{2}{5}\int \frac{5}{2}\left(a\left(a^2+6b^2\right)+b\left(3a^2+4b^2\right)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}dx+\frac{2ab(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}{de}}{e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{de\sqrt{e\sin(c+dx)}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \left(a\left(a^2+6b^2\right)+b\left(3a^2+4b^2\right)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}dx+\frac{2ab(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}{de}}{e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{de\sqrt{e\sin(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\int \sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a\left(a^2+6b^2\right)-b\left(3a^2+4b^2\right)\sin\left(c+dx-\frac{\pi}{2}\right)\right)dx+\frac{2ab(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}{de}}{e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{de\sqrt{e\sin(c+dx)}}$$

$$\Big\downarrow \; 3148$$

$$-\frac{a\left(a^2+6b^2\right)\int \sqrt{e\sin(c+dx)}dx+\frac{2b\left(3a^2+4b^2\right)(e\sin(c+dx))^{3/2}}{3de}+\frac{2ab(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}{de}}{e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{de\sqrt{e\sin(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{a(a^2+6b^2)\int\sqrt{e\sin(c+dx)}dx+\frac{2b(3a^2+4b^2)(e\sin(c+dx))^{3/2}}{3de}+\frac{2ab(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}{de}}{e^2}}{}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{de\sqrt{e\sin(c+dx)}}$$

$\downarrow$ 3121

$$-\frac{\frac{a(a^2+6b^2)\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}+\frac{2b(3a^2+4b^2)(e\sin(c+dx))^{3/2}}{3de}+\frac{2ab(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}{de}}{e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{de\sqrt{e\sin(c+dx)}}$$

$\downarrow$ 3042

$$-\frac{\frac{a(a^2+6b^2)\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}+\frac{2b(3a^2+4b^2)(e\sin(c+dx))^{3/2}}{3de}+\frac{2ab(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}{de}}{e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{de\sqrt{e\sin(c+dx)}}$$

$\downarrow$ 3119

$$-\frac{\frac{2b(3a^2+4b^2)(e\sin(c+dx))^{3/2}}{3de}+\frac{2a(a^2+6b^2)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}+\frac{2ab(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}{de}}{e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{de\sqrt{e\sin(c+dx)}}$$

input
```
Int[(a + b*Cos[c + d*x])^3/(e*Sin[c + d*x])^(3/2),x]
```

output
```
(-2*(b + a*Cos[c + d*x])*(a + b*Cos[c + d*x])^2)/(d*e*Sqrt[e*Sin[c + d*x]]
) - ((2*a*(a^2 + 6*b^2)*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*
x]])/(d*Sqrt[Sin[c + d*x]]) + (2*b*(3*a^2 + 4*b^2)*(e*Sin[c + d*x])^(3/2))
/(3*d*e) + (2*a*b*(a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2))/(d*e))/e^2
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3170
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Simp[1/(g^2*(p + 1))
  Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) +
a^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g
}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*
p] || IntegerQ[m])
```

rule 3341
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[1/(m + p + 1)   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Sim
p[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 &&  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && S
implerQ[c + d*x, a + b*x])
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 312 vs. $2(153) = 306$.

Time = 4.81 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.90

| method | result |
|---|---|
| default | $\frac{6\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sqrt{\sin(dx+c)}\,\mathrm{EllipticE}\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)a^3+36\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sqrt{\sin(dx+c)}}{}$ |
| parts | $\frac{a^3\left(2\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sqrt{\sin(dx+c)}\,\mathrm{EllipticE}\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sqrt{\sin(dx+c)}\right.}{e\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$ |

input
```
int((a+cos(d*x+c)*b)^3/(e*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/e/(e*sin(d*x+c))^(1/2)/cos(d*x+c)*(6*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x
+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^
3+36*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*Elliptic
E((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a*b^2-3*(1-sin(d*x+c))^(1/2)*(2+2*sin(
d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))
*a^3-18*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*Ellip
ticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a*b^2+2*cos(d*x+c)^3*b^3-6*a^3*cos(
d*x+c)^2-18*a*b^2*cos(d*x+c)^2-18*a^2*b*cos(d*x+c)-8*b^3*cos(d*x+c))/d
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.99

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{3/2}}\,dx =$$
$$-\frac{2\left(3\left(i\,a^3+6i\,ab^2\right)\sqrt{-\frac{1}{2}i\,e}\sin\left(dx+c\right)\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos\left(dx+c\right)+i\,\sin\right.}{}$$

input
```
integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(3*(I*a^3 + 6*I*a*b^2)*sqrt(-1/2*I*e)*sin(d*x + c)*weierstrassZeta(4,
 0, weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*(-I*a^3
- 6*I*a*b^2)*sqrt(1/2*I*e)*sin(d*x + c)*weierstrassZeta(4, 0, weierstrassP
Inverse(4, 0, cos(d*x + c) - I*sin(d*x + c))) - (b^3*cos(d*x + c)^2 - 9*a^
2*b - 4*b^3 - 3*(a^3 + 3*a*b^2)*cos(d*x + c))*sqrt(e*sin(d*x + c)))/(d*e^2
*sin(d*x + c))
```

## Sympy [F]

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{3/2}}\,dx = \int \frac{(a+b\cos\left(c+dx\right))^3}{(e\sin\left(c+dx\right))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**3/(e*sin(d*x+c))**(3/2),x)
```

output
```
Integral((a + b*cos(c + d*x))**3/(e*sin(c + d*x))**(3/2), x)
```

## Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{3/2}}\,dx = \int \frac{(b\cos(dx+c)+a)^3}{(e\sin(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^3/(e*sin(d*x + c))^(3/2), x)
```

## Giac [F]

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{3/2}}\,dx = \int \frac{(b\cos(dx+c)+a)^3}{(e\sin(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^3/(e*sin(d*x + c))^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{3/2}}\,dx = \int \frac{(a+b\cos(c+d\,x))^3}{(e\sin(c+d\,x))^{3/2}}\,dx$$

input
```
int((a + b*cos(c + d*x))^3/(e*sin(c + d*x))^(3/2),x)
```

output
```
int((a + b*cos(c + d*x))^3/(e*sin(c + d*x))^(3/2), x)
```

## Reduce [F]

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{3/2}}\,dx = \frac{\sqrt{e}\left(-6\sqrt{\sin\left(dx+c\right)}\,\cos\left(dx+c\right)^2 b^3 - 8\sqrt{\sin\left(dx+c\right)}\,\sin\left(dx+c\right)^2 b^3 - 18\right.}{}$$

input
```
int((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(3/2),x)
```

output
```
(sqrt(e)*( - 6*sqrt(sin(c + d*x))*cos(c + d*x)**2*b**3 - 8*sqrt(sin(c + d*
x))*sin(c + d*x)**2*b**3 - 18*sqrt(sin(c + d*x))*a**2*b + 3*int(sqrt(sin(c
 + d*x))/sin(c + d*x)**2,x)*sin(c + d*x)*a**3*d + 9*int((sqrt(sin(c + d*x)
)*cos(c + d*x)**2)/sin(c + d*x)**2,x)*sin(c + d*x)*a*b**2*d))/(3*sin(c + d
*x)*d*e**2)
```

# 3.55 $\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{5/2}}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 169

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{5/2}}\,dx = -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))^2}{3de(e\sin(c+dx))^{3/2}} + \frac{2a(a^2-6b^2)\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3de^2\sqrt{e\sin(c+dx)}} - \frac{2b(a^2+4b^2)\sqrt{e\sin(c+dx)}}{3de^3} - \frac{2ab(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}{3de^3}$$

output
```
-2/3*(b+a*cos(d*x+c))*(a+b*cos(d*x+c))^2/d/e/(e*sin(d*x+c))^(3/2)+2/3*a*(a
^2-6*b^2)*InverseJacobiAM(1/2*c-1/4*Pi+1/2*d*x,2^(1/2))*sin(d*x+c)^(1/2)/d
/e^2/(e*sin(d*x+c))^(1/2)-2/3*b*(a^2+4*b^2)*(e*sin(d*x+c))^(1/2)/d/e^3-2/3
*a*b*(a+b*cos(d*x+c))*(e*sin(d*x+c))^(1/2)/d/e^3
```

## Mathematica [A] (verified)

Time = 2.11 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.60

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{5/2}}\,dx = -\frac{6a^2b+5b^3+2a(a^2+3b^2)\cos(c+dx)-3b^3\cos(2(c+dx))+2a(a^2-6b^2)\,\text{EllipticF}\left(\frac{1}{4}(-2c+\pi-2dx),\right.}{3de(e\sin(c+dx))^{3/2}}$$

input
```
Integrate[(a + b*Cos[c + d*x])^3/(e*Sin[c + d*x])^(5/2),x]
```

output
```
-1/3*(6*a^2*b + 5*b^3 + 2*a*(a^2 + 3*b^2)*Cos[c + d*x] - 3*b^3*Cos[2*(c +
d*x)] + 2*a*(a^2 - 6*b^2)*EllipticF[(-2*c + Pi - 2*d*x)/4, 2]*Sin[c + d*x]
^(3/2))/(d*e*(e*Sin[c + d*x])^(3/2))
```

## Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 3170, 27, 3042, 3341, 27, 3042, 3148, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 3170}$$

$$-\frac{2\int -\frac{(a+b\cos(c+dx))\left(a^2-3b\cos(c+dx)a-4b^2\right)}{2\sqrt{e\sin(c+dx)}}dx}{3e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{3de(e\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{(a+b\cos(c+dx))(a^2-3b\cos(c+dx)a-4b^2)}{\sqrt{e\sin(c+dx)}}dx}{3e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{3de(e\sin(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\int \frac{(a-b\sin(c+dx-\frac{\pi}{2}))(a^2+3b\sin(c+dx-\frac{\pi}{2})a-4b^2)}{\sqrt{e\cos(c+dx-\frac{\pi}{2})}}dx}{3e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{3de(e\sin(c+dx))^{3/2}}$$

$\downarrow$ 3341

$$\frac{\frac{2}{3}\int \frac{3(a(a^2-6b^2)-b(a^2+4b^2)\cos(c+dx))}{2\sqrt{e\sin(c+dx)}}dx-\frac{2ab\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}{de}}{3e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{3de(e\sin(c+dx))^{3/2}}$$

$\downarrow$ 27

$$\frac{\int \frac{a(a^2-6b^2)-b(a^2+4b^2)\cos(c+dx)}{\sqrt{e\sin(c+dx)}}dx-\frac{2ab\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}{de}}{3e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{3de(e\sin(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\int \frac{a(a^2-6b^2)+b(a^2+4b^2)\sin(c+dx-\frac{\pi}{2})}{\sqrt{e\cos(c+dx-\frac{\pi}{2})}}dx-\frac{2ab\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}{de}}{3e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{3de(e\sin(c+dx))^{3/2}}$$

$\downarrow$ 3148

$$\frac{a(a^2-6b^2)\int \frac{1}{\sqrt{e\sin(c+dx)}}dx-\frac{2b(a^2+4b^2)\sqrt{e\sin(c+dx)}}{de}-\frac{2ab\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}{de}}{3e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{3de(e\sin(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{a(a^2-6b^2)\int \frac{1}{\sqrt{e\sin(c+dx)}}dx-\frac{2b(a^2+4b^2)\sqrt{e\sin(c+dx)}}{de}-\frac{2ab\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}{de}}{3e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{3de(e\sin(c+dx))^{3/2}}$$

$\downarrow$ 3121

$$\frac{\frac{a(a^2-6b^2)\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}}dx}{\sqrt{e\sin(c+dx)}}-\frac{2b(a^2+4b^2)\sqrt{e\sin(c+dx)}}{de}-\frac{2ab\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}{de}}{3e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{3de(e\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{a(a^2-6b^2)\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}}dx}{\sqrt{e\sin(c+dx)}}-\frac{2b(a^2+4b^2)\sqrt{e\sin(c+dx)}}{de}-\frac{2ab\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}{de}}{3e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{3de(e\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{-\frac{2b(a^2+4b^2)\sqrt{e\sin(c+dx)}}{de}+\frac{2a(a^2-6b^2)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-\frac{2ab\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}{de}}{3e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{3de(e\sin(c+dx))^{3/2}}$$

input
```
Int[(a + b*Cos[c + d*x])^3/(e*Sin[c + d*x])^(5/2),x]
```

output
```
(-2*(b + a*Cos[c + d*x])*(a + b*Cos[c + d*x])^2)/(3*d*e*(e*Sin[c + d*x])^(
3/2)) + ((2*a*(a^2 - 6*b^2)*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c +
d*x]])/(d*Sqrt[e*Sin[c + d*x]]) - (2*b*(a^2 + 4*b^2)*Sqrt[e*Sin[c + d*x]])
/(d*e) - (2*a*b*(a + b*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]])/(d*e))/(3*e^2)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3170
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Simp[1/(g^2*(p + 1))
  Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) +
a^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g
}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*
p] || IntegerQ[m])
```

rule 3341
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[1/(m + p + 1)   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Sim
p[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 &&  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && S
implerQ[c + d*x, a + b*x])
```

## Maple [A] (verified)

Time = 4.48 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.34

| method | result |
|---|---|
| default | $\frac{-\frac{2b\left(-3\cos(dx+c)^2b^2+3a^2+4b^2\right)}{3e(e\sin(dx+c))^{\frac{3}{2}}}-\frac{a\left(\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sin(dx+c)^{\frac{5}{2}}\,\text{EllipticF}\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)a^2-6b^2\sqrt{1-\sin(dx+c)}\,\sqrt{2+2}\right.}{3e^2\sin(dx+c)^2\cos}}{d}$ |
| parts | $-\frac{a^3\left(\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sin(dx+c)^{\frac{5}{2}}\,\text{EllipticF}\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-2\sin(dx+c)^3+2\sin(dx+c)\right)}{3e^2\sin(dx+c)^2\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}-\frac{2b^3\left(\sqrt{e\sin}\right.}{}$ |

input
```
int((a+cos(d*x+c)*b)^3/(e*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(-2/3*b/e/(e*sin(d*x+c))^(3/2)*(-3*cos(d*x+c)^2*b^2+3*a^2+4*b^2)-1/3*a/e^2
*((1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(5/2)*EllipticF((
1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2-6*b^2*(1-sin(d*x+c))^(1/2)*(2+2*sin(d
*x+c))^(1/2)*sin(d*x+c)^(5/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))+
2*a^2*cos(d*x+c)^2*sin(d*x+c)+6*b^2*cos(d*x+c)^2*sin(d*x+c))/sin(d*x+c)^2/
cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.11

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{5/2}}\,dx =$$
$$-\frac{2\left(\left(a^3-6\,ab^2-(a^3-6\,ab^2)\cos\left(dx+c\right)^2\right)\sqrt{-\frac{1}{2}i\,e}\,\text{weierstrassPInverse}(4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right.}{}$$

input
```
integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-2/3*((a^3 - 6*a*b^2 - (a^3 - 6*a*b^2)*cos(d*x + c)^2)*sqrt(-1/2*I*e)*weie
rstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c)) + (a^3 - 6*a*b^2 - (a
^3 - 6*a*b^2)*cos(d*x + c)^2)*sqrt(1/2*I*e)*weierstrassPInverse(4, 0, cos(
d*x + c) - I*sin(d*x + c)) + (3*b^3*cos(d*x + c)^2 - 3*a^2*b - 4*b^3 - (a^
3 + 3*a*b^2)*cos(d*x + c))*sqrt(e*sin(d*x + c)))/(d*e^3*cos(d*x + c)^2 - d
*e^3)
```

## Sympy [F]

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{5/2}}\,dx = \int \frac{(a+b\cos{(c+dx)})^3}{(e\sin{(c+dx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**3/(e*sin(d*x+c))**(5/2),x)
```

output
```
Integral((a + b*cos(c + d*x))**3/(e*sin(c + d*x))**(5/2), x)
```

## Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{5/2}}\,dx = \int \frac{(b\cos{(dx+c)}+a)^3}{(e\sin{(dx+c)})^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^3/(e*sin(d*x + c))^(5/2), x)
```

## Giac [F]

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{5/2}}\,dx = \int \frac{(b\cos(dx+c)+a)^3}{(e\sin(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^3/(e*sin(d*x + c))^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{5/2}}\,dx = \int \frac{(a+b\cos(c+d\,x))^3}{(e\,\sin(c+d\,x))^{5/2}}\,dx$$

input
```
int((a + b*cos(c + d*x))^3/(e*sin(c + d*x))^(5/2),x)
```

output
```
int((a + b*cos(c + d*x))^3/(e*sin(c + d*x))^(5/2), x)
```

## Reduce [F]

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{5/2}}\,dx = \frac{\sqrt{e}\left(-2\sqrt{\sin(dx+c)}\cos(dx+c)^2 b^3 - 8\sqrt{\sin(dx+c)}\sin(dx+c)^2 b^3 - 6\sqrt{}\right.}{}$$

input
```
int((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(5/2),x)
```

output
```
(sqrt(e)*( - 2*sqrt(sin(c + d*x))*cos(c + d*x)**2*b**3 - 8*sqrt(sin(c + d*
x))*sin(c + d*x)**2*b**3 - 6*sqrt(sin(c + d*x))*a**2*b + 3*int(sqrt(sin(c
+ d*x))/sin(c + d*x)**3,x)*sin(c + d*x)**2*a**3*d + 9*int((sqrt(sin(c + d*
x))*cos(c + d*x)**2)/sin(c + d*x)**3,x)*sin(c + d*x)**2*a*b**2*d))/(3*sin(
c + d*x)**2*d*e**3)
```

# 3.56 $\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{7/2}}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 192

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{7/2}}\,dx = -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))^2}{5de(e\sin(c+dx))^{5/2}} + \frac{2(a+b\cos(c+dx))\left(ab-(3a^2-4b^2)\cos(c+dx)\right)}{5de^3\sqrt{e\sin(c+dx)}} - \frac{6a(a^2-2b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5de^4\sqrt{\sin(c+dx)}} - \frac{2b(3a^2-4b^2)\,(e\sin(c+dx))^{3/2}}{5de^5}$$

output
```
-2/5*(b+a*cos(d*x+c))*(a+b*cos(d*x+c))^2/d/e/(e*sin(d*x+c))^(5/2)+2/5*(a+b
*cos(d*x+c))*(a*b-(3*a^2-4*b^2)*cos(d*x+c))/d/e^3/(e*sin(d*x+c))^(1/2)+6/5
*a*(a^2-2*b^2)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))
^(1/2)/d/e^4/sin(d*x+c)^(1/2)-2/5*b*(3*a^2-4*b^2)*(e*sin(d*x+c))^(3/2)/d/e
^5
```

## Mathematica [A] (verified)

Time = 1.95 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.68

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{7/2}}\,dx = -\frac{12a^2b-6b^3+a(7a^2+6b^2)\cos(c+dx)+10b^3\cos(2(c+dx))-3a^3\cos(3(c+dx))+6ab^2\cos(3(c+dx)}{10de(e\sin(c+dx))^{5/2}}$$

input
```
Integrate[(a + b*Cos[c + d*x])^3/(e*Sin[c + d*x])^(7/2),x]
```

output
```
-1/10*(12*a^2*b - 6*b^3 + a*(7*a^2 + 6*b^2)*Cos[c + d*x] + 10*b^3*Cos[2*(c
 + d*x)] - 3*a^3*Cos[3*(c + d*x)] + 6*a*b^2*Cos[3*(c + d*x)] - 12*a*(a^2 -
 2*b^2)*EllipticE[(-2*c + Pi - 2*d*x)/4, 2]*Sin[c + d*x]^(5/2))/(d*e*(e*Si
n[c + d*x])^(5/2))
```

## Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 3170, 27, 3042, 3340, 27, 3042, 3148, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{7/2}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{7/2}}dx$$

$$\Big\downarrow \ 3170$$

$$-\frac{2\int -\frac{(a+b\cos(c+dx))(3a^2-b\cos(c+dx)a-4b^2)}{2(e\sin(c+dx))^{3/2}}dx}{5e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{5de(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{(a+b\cos(c+dx))(3a^2-b\cos(c+dx)a-4b^2)}{(e\sin(c+dx))^{3/2}}dx}{5e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{5de(e\sin(c+dx))^{5/2}}$$

$\big\downarrow$ 3042

$$\frac{\int \frac{\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)\left(3a^2+b\sin\left(c+dx-\frac{\pi}{2}\right)a-4b^2\right)}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}}dx}{5e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{5de(e\sin(c+dx))^{5/2}}$$

$\big\downarrow$ 3340

$$\frac{\frac{2(a+b\cos(c+dx))(ab-(3a^2-4b^2)\cos(c+dx))}{de\sqrt{e\sin(c+dx)}}-\frac{2\int \frac{3}{2}\left(a(a^2-2b^2)+b(3a^2-4b^2)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}dx}{e^2}}{5e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{5de(e\sin(c+dx))^{5/2}}$$

$\big\downarrow$ 27

$$\frac{\frac{2(a+b\cos(c+dx))(ab-(3a^2-4b^2)\cos(c+dx))}{de\sqrt{e\sin(c+dx)}}-\frac{3\int \left(a(a^2-2b^2)+b(3a^2-4b^2)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}dx}{e^2}}{5e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{5de(e\sin(c+dx))^{5/2}}$$

$\big\downarrow$ 3042

$$\frac{\frac{2(a+b\cos(c+dx))(ab-(3a^2-4b^2)\cos(c+dx))}{de\sqrt{e\sin(c+dx)}}-\frac{3\int \sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a(a^2-2b^2)-b(3a^2-4b^2)\sin\left(c+dx-\frac{\pi}{2}\right)\right)dx}{e^2}}{5e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{5de(e\sin(c+dx))^{5/2}}$$

$\big\downarrow$ 3148

$$\frac{\frac{2(a+b\cos(c+dx))(ab-(3a^2-4b^2)\cos(c+dx))}{de\sqrt{e\sin(c+dx)}}-\frac{3\left(a(a^2-2b^2)\int \sqrt{e\sin(c+dx)}dx+\frac{2b(3a^2-4b^2)(e\sin(c+dx))^{3/2}}{3de}\right)}{e^2}}{5e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{5de(e\sin(c+dx))^{5/2}}$$

$\big\downarrow$ 3042

$$\frac{\frac{2(a+b\cos(c+dx))(ab-(3a^2-4b^2)\cos(c+dx))}{de\sqrt{e\sin(c+dx)}}-\frac{3\left(a(a^2-2b^2)\int \sqrt{e\sin(c+dx)}dx+\frac{2b(3a^2-4b^2)(e\sin(c+dx))^{3/2}}{3de}\right)}{e^2}}{5e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{5de(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3121}$$

$$\frac{\frac{2(a+b\cos(c+dx))\left(ab-\left(3a^2-4b^2\right)\cos(c+dx)\right)}{de\sqrt{e\sin(c+dx)}}-\frac{3\left(\frac{a\left(a^2-2b^2\right)\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}+\frac{2b\left(3a^2-4b^2\right)(e\sin(c+dx))^{3/2}}{3de}\right)}{e^2}}{5e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{5de(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2(a+b\cos(c+dx))\left(ab-\left(3a^2-4b^2\right)\cos(c+dx)\right)}{de\sqrt{e\sin(c+dx)}}-\frac{3\left(\frac{a\left(a^2-2b^2\right)\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}+\frac{2b\left(3a^2-4b^2\right)(e\sin(c+dx))^{3/2}}{3de}\right)}{e^2}}{5e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{5de(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{2(a+b\cos(c+dx))\left(ab-\left(3a^2-4b^2\right)\cos(c+dx)\right)}{de\sqrt{e\sin(c+dx)}}-\frac{3\left(\frac{2b\left(3a^2-4b^2\right)(e\sin(c+dx))^{3/2}}{3de}+\frac{2a\left(a^2-2b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}\right)}{e^2}}{5e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{5de(e\sin(c+dx))^{5/2}}$$

input

```
Int[(a + b*Cos[c + d*x])^3/(e*Sin[c + d*x])^(7/2),x]
```

output

```
(-2*(b + a*Cos[c + d*x])*(a + b*Cos[c + d*x])^2)/(5*d*e*(e*Sin[c + d*x])^(
5/2)) + ((2*(a + b*Cos[c + d*x])*(a*b - (3*a^2 - 4*b^2)*Cos[c + d*x]))/(d*
e*Sqrt[e*Sin[c + d*x]]) - (3*((2*a*(a^2 - 2*b^2)*EllipticE[(c - Pi/2 + d*x
)/2, 2]*Sqrt[e*Sin[c + d*x]])/(d*Sqrt[Sin[c + d*x]]) + (2*b*(3*a^2 - 4*b^2
)*(e*Sin[c + d*x])^(3/2))/(3*d*e)))/e^2)/(5*e^2)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3170
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Simp[1/(g^2*(p + 1))
  Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) +
a^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g
}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*
p] || IntegerQ[m])
```

rule 3340
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(g*
Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x])^m*((d + c*Sin[e + f*x])/(f*g*(p
 + 1))), x] + Simp[1/(g^2*(p + 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Si
n[e + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ
[m, 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0]
&& SimplerQ[c + d*x, a + b*x])
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 374 vs. $2(174) = 348$.

Time = 4.68 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.95

| method | result |
|---|---|
| default | $\frac{-\frac{2b\left(5\cos(dx+c)^2b^2+3a^2-4b^2\right)}{5e(e\sin(dx+c))^{\frac{5}{2}}}+\frac{a\left(6\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sin(dx+c)^{\frac{7}{2}}\,\text{EllipticE}\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)a^2-12\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\text{s}}}{}}{}$ |
| parts | $\frac{a^3\left(6\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sin(dx+c)^{\frac{7}{2}}\,\text{EllipticE}\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-3\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sin(dx+c)\right.}{5e^3\sin(dx+c)^3\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$ |

input
```
int((a+cos(d*x+c)*b)^3/(e*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
(-2/5*b/e/(e*sin(d*x+c))^(5/2)*(5*cos(d*x+c)^2*b^2+3*a^2-4*b^2)+1/5*a/e^3*
(6*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(7/2)*EllipticE(
(1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2-12*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x
+c))^(1/2)*sin(d*x+c)^(7/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^
2-3*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(7/2)*EllipticF
((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2+6*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x
+c))^(1/2)*sin(d*x+c)^(7/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^
2+6*a^2*cos(d*x+c)^4*sin(d*x+c)-12*b^2*cos(d*x+c)^4*sin(d*x+c)-8*a^2*cos(d
*x+c)^2*sin(d*x+c)+6*b^2*cos(d*x+c)^2*sin(d*x+c))/sin(d*x+c)^3/cos(d*x+c)/
(e*sin(d*x+c))^(1/2))/d
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.27

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{7/2}}\,dx =$$

$$-\frac{2\left(3\left(-i\,a^3+2i\,ab^2+\left(i\,a^3-2i\,ab^2\right)\cos\left(dx+c\right)^2\right)\sqrt{-\frac{1}{2}i\,e}\sin\left(dx+c\right)\text{weierstrassZeta}(4,0,\text{weierstrassP}}{}$$

input
```
integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
-2/5*(3*(-I*a^3 + 2*I*a*b^2 + (I*a^3 - 2*I*a*b^2)*cos(d*x + c)^2)*sqrt(-1/
2*I*e)*sin(d*x + c)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*
x + c) + I*sin(d*x + c))) + 3*(I*a^3 - 2*I*a*b^2 + (-I*a^3 + 2*I*a*b^2)*co
s(d*x + c)^2)*sqrt(1/2*I*e)*sin(d*x + c)*weierstrassZeta(4, 0, weierstrass
PInverse(4, 0, cos(d*x + c) - I*sin(d*x + c))) - (5*b^3*cos(d*x + c)^2 - 3
*(a^3 - 2*a*b^2)*cos(d*x + c)^3 + 3*a^2*b - 4*b^3 + (4*a^3 - 3*a*b^2)*cos(
d*x + c))*sqrt(e*sin(d*x + c)))/((d*e^4*cos(d*x + c)^2 - d*e^4)*sin(d*x +
c))
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**3/(e*sin(d*x+c))**(7/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{7/2}}\,dx = \int \frac{(b\cos(dx+c)+a)^3}{(e\sin(dx+c))^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^3/(e*sin(d*x + c))^(7/2), x)
```

## Giac [F]

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{7/2}}\,dx = \int \frac{(b\cos(dx+c)+a)^3}{(e\sin(dx+c))^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^3/(e*sin(d*x + c))^(7/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{7/2}}\,dx = \int \frac{(a+b\,\cos(c+d\,x))^3}{(e\,\sin(c+d\,x))^{7/2}}\,dx$$

input
```
int((a + b*cos(c + d*x))^3/(e*sin(c + d*x))^(7/2),x)
```

output
```
int((a + b*cos(c + d*x))^3/(e*sin(c + d*x))^(7/2), x)
```

## Reduce [F]

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{7/2}}\,dx = \frac{\sqrt{e}\left(-2\sqrt{\sin\left(dx+c\right)}\,\cos\left(dx+c\right)^2 b^3+8\sqrt{\sin\left(dx+c\right)}\,\sin\left(dx+c\right)^2 b^3-6\sqrt{\right.}}{}$$

input
```
int((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(7/2),x)
```

output
```
(sqrt(e)*( - 2*sqrt(sin(c + d*x))*cos(c + d*x)**2*b**3 + 8*sqrt(sin(c + d*
x))*sin(c + d*x)**2*b**3 - 6*sqrt(sin(c + d*x))*a**2*b + 5*int(sqrt(sin(c
+ d*x))/sin(c + d*x)**4,x)*sin(c + d*x)**3*a**3*d + 15*int((sqrt(sin(c + d
*x))*cos(c + d*x)**2)/sin(c + d*x)**4,x)*sin(c + d*x)**3*a*b**2*d))/(5*sin
(c + d*x)**3*d*e**4)
```

# 3.57 $\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{9/2}}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 193

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{9/2}}\,dx = -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))^2}{7de(e\sin(c+dx))^{7/2}} - \frac{2(a+b\cos(c+dx))\left(ab+(5a^2-4b^2)\cos(c+dx)\right)}{21de^3(e\sin(c+dx))^{3/2}} + \frac{2a(5a^2-6b^2)\operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{21de^4\sqrt{e\sin(c+dx)}} - \frac{2b(5a^2-4b^2)\sqrt{e\sin(c+dx)}}{21de^5}$$

output
```
-2/7*(b+a*cos(d*x+c))*(a+b*cos(d*x+c))^2/d/e/(e*sin(d*x+c))^(7/2)-2/21*(a+
b*cos(d*x+c))*(a*b+(5*a^2-4*b^2)*cos(d*x+c))/d/e^3/(e*sin(d*x+c))^(3/2)+2/
21*a*(5*a^2-6*b^2)*InverseJacobiAM(1/2*c-1/4*Pi+1/2*d*x,2^(1/2))*sin(d*x+c
)^(1/2)/d/e^4/(e*sin(d*x+c))^(1/2)-2/21*b*(5*a^2-4*b^2)*(e*sin(d*x+c))^(1/
2)/d/e^5
```

## Mathematica [A] (verified)

Time = 2.26 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.75

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{9/2}}\,dx =$$
$$-\frac{2\csc^4(c+dx)\sqrt{e\sin(c+dx)}\left(\frac{1}{4}(36a^2b-2b^3+a(17a^2+30b^2)\cos(c+dx)+14b^3\cos(2(c+dx))-5a^3\text{c}\right.}{21de^5}$$

input
```
Integrate[(a + b*Cos[c + d*x])^3/(e*Sin[c + d*x])^(9/2),x]
```

output
```
(-2*Csc[c + d*x]^4*Sqrt[e*Sin[c + d*x]]*((36*a^2*b - 2*b^3 + a*(17*a^2 + 3
0*b^2)*Cos[c + d*x] + 14*b^3*Cos[2*(c + d*x)] - 5*a^3*Cos[3*(c + d*x)] + 6
*a*b^2*Cos[3*(c + d*x)])/4 + a*(5*a^2 - 6*b^2)*EllipticF[(-2*c + Pi - 2*d*
x)/4, 2]*Sin[c + d*x]^(7/2)))/(21*d*e^5)
```

## Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 3170, 27, 3042, 3340, 27, 3042, 3148, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{9/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{9/2}}dx$$

$$\Big\downarrow \text{3170}$$

$$-\frac{2\int -\frac{(a+b\cos(c+dx))(5a^2+b\cos(c+dx)a-4b^2)}{2(e\sin(c+dx))^{5/2}}dx}{7e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{7de(e\sin(c+dx))^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(a+b\cos(c+dx))(5a^2+b\cos(c+dx)a-4b^2)}{(e\sin(c+dx))^{5/2}}dx}{7e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{7de(e\sin(c+dx))^{7/2}}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{(a-b\sin(c+dx-\frac{\pi}{2}))(5a^2-b\sin(c+dx-\frac{\pi}{2})a-4b^2)}{(e\cos(c+dx-\frac{\pi}{2}))^{5/2}}dx}{7e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{7de(e\sin(c+dx))^{7/2}}$$

$\Big\downarrow$ 3340

$$\frac{-\frac{2\int -\frac{a(5a^2-6b^2)-b(5a^2-4b^2)\cos(c+dx)}{2\sqrt{e\sin(c+dx)}}dx}{3e^2}-\frac{2(a+b\cos(c+dx))((5a^2-4b^2)\cos(c+dx)+ab)}{3de(e\sin(c+dx))^{3/2}}}{7e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{7de(e\sin(c+dx))^{7/2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{a(5a^2-6b^2)-b(5a^2-4b^2)\cos(c+dx)}{\sqrt{e\sin(c+dx)}}dx}{3e^2}-\frac{2(a+b\cos(c+dx))((5a^2-4b^2)\cos(c+dx)+ab)}{3de(e\sin(c+dx))^{3/2}}}{7e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{7de(e\sin(c+dx))^{7/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{a(5a^2-6b^2)+b(5a^2-4b^2)\sin(c+dx-\frac{\pi}{2})}{\sqrt{e\cos(c+dx-\frac{\pi}{2})}}dx}{3e^2}-\frac{2(a+b\cos(c+dx))((5a^2-4b^2)\cos(c+dx)+ab)}{3de(e\sin(c+dx))^{3/2}}}{7e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{7de(e\sin(c+dx))^{7/2}}$$

$\Big\downarrow$ 3148

$$\frac{\frac{a(5a^2-6b^2)\int \frac{1}{\sqrt{e\sin(c+dx)}}dx-\frac{2b(5a^2-4b^2)\sqrt{e\sin(c+dx)}}{de}}{3e^2}-\frac{2(a+b\cos(c+dx))((5a^2-4b^2)\cos(c+dx)+ab)}{3de(e\sin(c+dx))^{3/2}}}{7e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{7de(e\sin(c+dx))^{7/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{a(5a^2-6b^2)\int \frac{1}{\sqrt{e\sin(c+dx)}}dx-\frac{2b(5a^2-4b^2)\sqrt{e\sin(c+dx)}}{de}}{3e^2}-\frac{2(a+b\cos(c+dx))((5a^2-4b^2)\cos(c+dx)+ab)}{3de(e\sin(c+dx))^{3/2}}}{7e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{7de(e\sin(c+dx))^{7/2}}$$

$$\Big\downarrow \textbf{3121}$$

$$\frac{\frac{\frac{a(5a^2-6b^2)\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}}dx}{\sqrt{e\sin(c+dx)}}-\frac{2b(5a^2-4b^2)\sqrt{e\sin(c+dx)}}{de}}{3e^2}-\frac{2(a+b\cos(c+dx))((5a^2-4b^2)\cos(c+dx)+ab)}{3de(e\sin(c+dx))^{3/2}}}{7e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{7de(e\sin(c+dx))^{7/2}}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\frac{\frac{a(5a^2-6b^2)\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}}dx}{\sqrt{e\sin(c+dx)}}-\frac{2b(5a^2-4b^2)\sqrt{e\sin(c+dx)}}{de}}{3e^2}-\frac{2(a+b\cos(c+dx))((5a^2-4b^2)\cos(c+dx)+ab)}{3de(e\sin(c+dx))^{3/2}}}{7e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{7de(e\sin(c+dx))^{7/2}}$$

$$\Big\downarrow \textbf{3120}$$

$$\frac{\frac{\frac{2a(5a^2-6b^2)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-\frac{2b(5a^2-4b^2)\sqrt{e\sin(c+dx)}}{de}}{3e^2}-\frac{2(a+b\cos(c+dx))((5a^2-4b^2)\cos(c+dx)+ab)}{3de(e\sin(c+dx))^{3/2}}}{7e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{7de(e\sin(c+dx))^{7/2}}$$

input
```
Int[(a + b*Cos[c + d*x])^3/(e*Sin[c + d*x])^(9/2),x]
```

output
```
(-2*(b + a*Cos[c + d*x])*(a + b*Cos[c + d*x])^2)/(7*d*e*(e*Sin[c + d*x])^(
7/2)) + ((-2*(a + b*Cos[c + d*x])*(a*b + (5*a^2 - 4*b^2)*Cos[c + d*x]))/(3
*d*e*(e*Sin[c + d*x])^(3/2)) + ((2*a*(5*a^2 - 6*b^2)*EllipticF[(c - Pi/2 +
 d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(d*Sqrt[e*Sin[c + d*x]]) - (2*b*(5*a^2 - 4
*b^2)*Sqrt[e*Sin[c + d*x]])/(d*e))/(3*e^2))/(7*e^2)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3170
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Simp[1/(g^2*(p + 1))
  Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) +
a^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g
}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*
p] || IntegerQ[m])
```

rule 3340
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(g*
Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x])^m*((d + c*Sin[e + f*x])/(f*g*(p
 + 1))), x] + Simp[1/(g^2*(p + 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Si
n[e + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ
[m, 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0]
&& SimplerQ[c + d*x, a + b*x])
```

## Maple [A] (verified)

Time = 4.95 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.37

| method | result |
|---|---|
| default | $\frac{-\frac{2b\left(7\cos(dx+c)^2b^2+9a^2-4b^2\right)}{21e(e\sin(dx+c))^{\frac{7}{2}}}-\frac{a\left(5\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sin(dx+c)^{\frac{9}{2}}\operatorname{EllipticF}\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)a^2-6\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin}\right.}{}}{}$ |
| parts | $-\frac{a^3\left(5\sqrt{1-\sin(dx+c)}\,\sqrt{2+2\sin(dx+c)}\,\sin(dx+c)^{\frac{9}{2}}\operatorname{EllipticF}\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-10\sin(dx+c)^5+4\sin(dx+c)^3+6\sin(dx+c)\right)}{21e^4\sin(dx+c)^4\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$ |

input
```
int((a+cos(d*x+c)*b)^3/(e*sin(d*x+c))^(9/2),x,method=_RETURNVERBOSE)
```

output
```
(-2/21*b/e/(e*sin(d*x+c))^(7/2)*(7*cos(d*x+c)^2*b^2+9*a^2-4*b^2)-1/21*a/e^
4*(5*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(9/2)*Elliptic
F((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2-6*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*
x+c))^(1/2)*sin(d*x+c)^(9/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b
^2-10*a^2*cos(d*x+c)^4*sin(d*x+c)+12*b^2*cos(d*x+c)^4*sin(d*x+c)+16*a^2*co
s(d*x+c)^2*sin(d*x+c)+6*b^2*cos(d*x+c)^2*sin(d*x+c))/sin(d*x+c)^4/cos(d*x+
c)/(e*sin(d*x+c))^(1/2))/d
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.42

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{9/2}}\,dx = \frac{2\left(\left((5\,a^3-6\,ab^2)\cos{(dx+c)}^4+5\,a^3-6\,ab^2-2\,(5\,a^3-6\,ab^2)\cos{(dx+c)}^2\right)\right.}{}$$

input
```
integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(9/2),x, algorithm="fricas")
```

output
```
2/21*(((5*a^3 - 6*a*b^2)*cos(d*x + c)^4 + 5*a^3 - 6*a*b^2 - 2*(5*a^3 - 6*a
*b^2)*cos(d*x + c)^2)*sqrt(-1/2*I*e)*weierstrassPInverse(4, 0, cos(d*x + c
) + I*sin(d*x + c)) + ((5*a^3 - 6*a*b^2)*cos(d*x + c)^4 + 5*a^3 - 6*a*b^2
- 2*(5*a^3 - 6*a*b^2)*cos(d*x + c)^2)*sqrt(1/2*I*e)*weierstrassPInverse(4,
 0, cos(d*x + c) - I*sin(d*x + c)) - (7*b^3*cos(d*x + c)^2 - (5*a^3 - 6*a*
b^2)*cos(d*x + c)^3 + 9*a^2*b - 4*b^3 + (8*a^3 + 3*a*b^2)*cos(d*x + c))*sq
rt(e*sin(d*x + c)))/(d*e^5*cos(d*x + c)^4 - 2*d*e^5*cos(d*x + c)^2 + d*e^5
)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**3/(e*sin(d*x+c))**(9/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{9/2}}\,dx = \int \frac{(b\cos(dx+c)+a)^3}{(e\sin(dx+c))^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(9/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^3/(e*sin(d*x + c))^(9/2), x)
```

## Giac [F]

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{9/2}}\,dx = \int \frac{(b\cos(dx+c)+a)^3}{(e\sin(dx+c))^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(9/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^3/(e*sin(d*x + c))^(9/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{9/2}}\,dx = \int \frac{(a+b\cos(c+d\,x))^3}{(e\sin(c+d\,x))^{9/2}}\,dx$$

input
```
int((a + b*cos(c + d*x))^3/(e*sin(c + d*x))^(9/2),x)
```

output
```
int((a + b*cos(c + d*x))^3/(e*sin(c + d*x))^(9/2), x)
```

## Reduce [F]

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{9/2}}\,dx = \frac{\sqrt{e}\left(-6\sqrt{\sin(dx+c)}\,\cos(dx+c)^2 b^3+8\sqrt{\sin(dx+c)}\,\sin(dx+c)^2 b^3-18\right.}{}$$

input
```
int((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(9/2),x)
```

output
```
(sqrt(e)*( - 6*sqrt(sin(c + d*x))*cos(c + d*x)**2*b**3 + 8*sqrt(sin(c + d*
x))*sin(c + d*x)**2*b**3 - 18*sqrt(sin(c + d*x))*a**2*b + 21*int(sqrt(sin(
c + d*x))/sin(c + d*x)**5,x)*sin(c + d*x)**4*a**3*d + 63*int((sqrt(sin(c +
 d*x))*cos(c + d*x)**2)/sin(c + d*x)**5,x)*sin(c + d*x)**4*a*b**2*d))/(21*
sin(c + d*x)**4*d*e**5)
```

# 3.58 $\int \frac{(e\sin(c+dx))^{11/2}}{a+b\cos(c+dx)}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 544

$$\begin{aligned}\int \frac{(e\sin(c+dx))^{11/2}}{a+b\cos(c+dx)}\,dx &= \frac{(-a^2+b^2)^{9/4}\,e^{11/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{11/2}d} \\ &+ \frac{(-a^2+b^2)^{9/4}\,e^{11/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{11/2}d} \\ &+ \frac{2a(21a^4-49a^2b^2+33b^4)\,e^6\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{21b^6d\sqrt{e\sin(c+dx)}} \\ &- \frac{a(a^2-b^2)^3\,e^6\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b^6\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}} \\ &- \frac{a(a^2-b^2)^3\,e^6\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b^6\left(a^2-b\left(b+\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}} \\ &- \frac{2e^5\left(21(a^2-b^2)^2-ab(7a^2-12b^2)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{21b^5d} \\ &+ \frac{2e^3(7(a^2-b^2)-5ab\cos(c+dx))\,(e\sin(c+dx))^{5/2}}{35b^3d} - \frac{2e(e\sin(c+dx))^{9/2}}{9bd}\end{aligned}$$

output
```
(-a^2+b^2)^(9/4)*e^(11/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(
1/4)/e^(1/2))/b^(11/2)/d+(-a^2+b^2)^(9/4)*e^(11/2)*arctanh(b^(1/2)*(e*sin(
d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(11/2)/d+2/21*a*(21*a^4-49*a^2*b
^2+33*b^4)*e^6*InverseJacobiAM(1/2*c-1/4*Pi+1/2*d*x,2^(1/2))*sin(d*x+c)^(1
/2)/b^6/d/(e*sin(d*x+c))^(1/2)+a*(a^2-b^2)^3*e^6*EllipticPi(cos(1/2*c+1/4*
Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^6/(a^2-b*
(b-(-a^2+b^2)^(1/2)))/d/(e*sin(d*x+c))^(1/2)+a*(a^2-b^2)^3*e^6*EllipticPi(
cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/
2)/b^6/(a^2-b*(b+(-a^2+b^2)^(1/2)))/d/(e*sin(d*x+c))^(1/2)-2/21*e^5*(21*(a
^2-b^2)^2-a*b*(7*a^2-12*b^2)*cos(d*x+c))*(e*sin(d*x+c))^(1/2)/b^5/d+2/35*e
^3*(7*a^2-7*b^2-5*a*b*cos(d*x+c))*(e*sin(d*x+c))^(5/2)/b^3/d-2/9*e*(e*sin(
d*x+c))^(9/2)/b/d
```

## Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 19.00 (sec) , antiderivative size = 2035, normalized size of antiderivative = 3.74

$$\int \frac{(e\sin(c+dx))^{11/2}}{a+b\cos(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[(e*Sin[c + d*x])^(11/2)/(a + b*Cos[c + d*x]),x]
```

output
```
(((a*(28*a^2 - 51*b^2)*Cos[c + d*x])/(42*b^4) + ((-9*a^2 + 14*b^2)*Cos[2*(
c + d*x)])/(45*b^3) + (a*Cos[3*(c + d*x)])/(14*b^2) - Cos[4*(c + d*x)]/(36
*b))*Csc[c + d*x]^5*(e*Sin[c + d*x])^(11/2))/d - ((e*Sin[c + d*x])^(11/2)*
((2*(392*a^3*b - 722*a*b^3)*Cos[c + d*x]^2*(a + b*Sqrt[1 - Sin[c + d*x]^2]
)*((a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4
)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)]
- Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x
]] + b*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(
1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)
^(3/4)) + (5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^
2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]^2
])/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c
 + d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Sin[c + d*
x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2,
1, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)])*Sin[c + d*x]^2
)*(a^2 + b^2*(-1 + Sin[c + d*x]^2)))))/((a + b*Cos[c + d*x])*(1 - Sin[c +
d*x]^2)) + (2*(-280*a^4 + 636*a^2*b^2 - 721*b^4)*Cos[c + d*x]*(a + b*Sqrt[
1 - Sin[c + d*x]^2])*(((-1/8 + I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]
*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*S
qrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)...
```

## Rubi [A] (warning: unable to verify)

Time = 2.93 (sec) , antiderivative size = 555, normalized size of antiderivative = 1.02, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.960$, Rules used = {3042, 3174, 25, 3042, 3344, 27, 3042, 3344, 27, 3042, 3346, 3042, 3121, 3042, 3120, 3181, 266, 756, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sin(c+dx))^{11/2}}{a+b\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{11/2}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3174}$$

$$-\frac{e^2\int -\frac{(b+a\cos(c+dx))(e\sin(c+dx))^{7/2}}{a+b\cos(c+dx)}dx}{b}-\frac{2e(e\sin(c+dx))^{9/2}}{9bd}$$

$\Big\downarrow$ 25

$$\frac{e^2\int \frac{(b+a\cos(c+dx))(e\sin(c+dx))^{7/2}}{a+b\cos(c+dx)}dx}{b}-\frac{2e(e\sin(c+dx))^{9/2}}{9bd}$$

$\Big\downarrow$ 3042

$$\frac{e^2\int \frac{\left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{2e(e\sin(c+dx))^{9/2}}{9bd}$$

$\Big\downarrow$ 3344

$$\frac{e^2\left(\frac{2e^2\int -\frac{\left(b\left(2a^2-7b^2\right)+a\left(7a^2-12b^2\right)\cos(c+dx)\right)(e\sin(c+dx))^{3/2}}{2(a+b\cos(c+dx))}dx}{7b^2}+\frac{2e(e\sin(c+dx))^{5/2}\left(7\left(a^2-b^2\right)-5ab\cos(c+dx)\right)}{35b^2d}\right)}{b}-\frac{2e(e\sin(c+dx))^{9/2}}{9bd}$$

$\Big\downarrow$ 27

$$\frac{e^2\left(\frac{2e(e\sin(c+dx))^{5/2}\left(7\left(a^2-b^2\right)-5ab\cos(c+dx)\right)}{35b^2d}-\frac{e^2\int \frac{\left(b\left(2a^2-7b^2\right)+a\left(7a^2-12b^2\right)\cos(c+dx)\right)(e\sin(c+dx))^{3/2}}{a+b\cos(c+dx)}dx}{7b^2}\right)}{b}-\frac{2e(e\sin(c+dx))^{9/2}}{9bd}$$

$\Big\downarrow$ 3042

$$\frac{e^2\left(\frac{2e(e\sin(c+dx))^{5/2}\left(7\left(a^2-b^2\right)-5ab\cos(c+dx)\right)}{35b^2d}-\frac{e^2\int \frac{\left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(b\left(2a^2-7b^2\right)+a\left(7a^2-12b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{7b^2}\right)}{b}-\frac{2e(e\sin(c+dx))^{9/2}}{9bd}$$

$\Big\downarrow$ 3344

$$e^2\left(\frac{2e(e\sin(c+dx))^{5/2}\left(7\left(a^2-b^2\right)-5ab\cos(c+dx)\right)}{35b^2d}-\frac{e^2\left(\frac{2e^2\int-\frac{b\left(14a^4-30b^2a^2+21b^4\right)+a\left(21a^4-49b^2a^2+33b^4\right)\cos(c+dx)}{2(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{3b^2}+\frac{2e\sqrt{e\sin(c+dx)}\left(21\right.}{}\right.}{7b^2}\right.}{b}$$

$$\frac{2e(e\sin(c+dx))^{9/2}}{9bd}$$

$\downarrow$ 27

$$e^2\left(\frac{2e(e\sin(c+dx))^{5/2}\left(7\left(a^2-b^2\right)-5ab\cos(c+dx)\right)}{35b^2d}-\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(21\left(a^2-b^2\right)^2-ab\left(7a^2-12b^2\right)\cos(c+dx)\right)}{3b^2d}-\frac{e^2\int\frac{b\left(14a^4-30b^2a^2+21b^4\right)+}{(a+b\cos(c}}{}\right.}{7b^2}\right.}{b}$$

$$\frac{2e(e\sin(c+dx))^{9/2}}{9bd}$$

$\downarrow$ 3042

$$e^2\left(\frac{2e(e\sin(c+dx))^{5/2}\left(7\left(a^2-b^2\right)-5ab\cos(c+dx)\right)}{35b^2d}-\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(21\left(a^2-b^2\right)^2-ab\left(7a^2-12b^2\right)\cos(c+dx)\right)}{3b^2d}-\frac{e^2\int\frac{b\left(14a^4-30b^2a^2+21b^4\right)-}{\sqrt{e\cos\left(c+dx\right.}}}{}\right.}{7b^2}\right.}{b}$$

$$\frac{2e(e\sin(c+dx))^{9/2}}{9bd}$$

$\downarrow$ 3346

$$\frac{e^2\left(\frac{2e(e\sin(c+dx))^{5/2}\left(7\left(a^2-b^2\right)-5ab\cos(c+dx)\right)}{35b^2d}-\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(21\left(a^2-b^2\right)^2-ab\left(7a^2-12b^2\right)\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{a\left(21a^4-49a^2b^2+33b^4\right)\int}{b}\right.}{}\right.}{7b^2}\right.}{b}$$

$$\frac{2e(e\sin(c+dx))^{9/2}}{9bd}$$

$\downarrow$ 3042

$$\frac{e^2\left(\frac{2e(e\sin(c+dx))^{5/2}\left(7\left(a^2-b^2\right)-5ab\cos(c+dx)\right)}{35b^2d}-\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(21\left(a^2-b^2\right)^2-ab\left(7a^2-12b^2\right)\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{a\left(21a^4-49a^2b^2+33b^4\right)\int}{b}\right.}{}\right.}{7b^2}\right.}{b}$$

$$\frac{2e(e\sin(c+dx))^{9/2}}{9bd}$$

$\downarrow$ 3121

$$\frac{e^2\left(\frac{2e(e\sin(c+dx))^{5/2}(7(a^2-b^2)-5ab\cos(c+dx))}{35b^2d}-\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(21\left(a^2-b^2\right)^2-ab\left(7a^2-12b^2\right)\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{a\left(21a^4-49a^2b^2+33b^4\right)\sqrt{\cdots}}{b\sqrt{e\sin\cdots}}\cdots\right)}{\cdots}\right)}{7b^2\cdots}\right)}{b\cdots}$$

$$\frac{2e(e\sin(c+dx))^{9/2}}{9bd}$$

$\downarrow$ 3042

$$\frac{e^2\left(\frac{2e(e\sin(c+dx))^{5/2}(7(a^2-b^2)-5ab\cos(c+dx))}{35b^2d}-\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(21\left(a^2-b^2\right)^2-ab\left(7a^2-12b^2\right)\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{a\left(21a^4-49a^2b^2+33b^4\right)\sqrt{\cdots}}{b\sqrt{e\sin\cdots}}\cdots\right)}{\cdots}\right)}{7b^2\cdots}\right)}{b\cdots}$$

$$\frac{2e(e\sin(c+dx))^{9/2}}{9bd}$$

$\downarrow$ 3120

$$e^2\left(\frac{2e(e\sin(c+dx))^{5/2}\left(7\left(a^2-b^2\right)-5ab\cos(c+dx)\right)}{35b^2d}-\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(21\left(a^2-b^2\right)^2-ab\left(7a^2-12b^2\right)\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{2a\left(21a^4-49a^2b^2+33b^4\right)\sqrt{}}{ba}\right.}{}\right.}{}\right.$$

$$b$$

$$\frac{2e(e\sin(c+dx))^{9/2}}{9bd}$$

$\downarrow$ 3181

$$e^2\left(\frac{2e(e\sin(c+dx))^{5/2}\left(7\left(a^2-b^2\right)-5ab\cos(c+dx)\right)}{35b^2d}-\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(21\left(a^2-b^2\right)^2-ab\left(7a^2-12b^2\right)\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{2a\left(21a^4-49a^2b^2+33b^4\right)\sqrt{}}{ba}\right.}{}\right.}{}\right.$$

$$\frac{2e(e\sin(c+dx))^{9/2}}{9bd}$$

$\downarrow$ 266

$$e^2 \frac{2a\left(21a^4-49a^2b^2+33b^4\right)\sqrt{}}{b}$$

$$e^2 \left( \frac{2e\sqrt{e\sin(c+dx)}\left(21\left(a^2-b^2\right)^2-ab\left(7a^2-12b^2\right)\cos(c+dx)\right)}{3b^2d} - \right.$$

$$e^2 \left( \frac{2e(e\sin(c+dx))^{5/2}\left(7\left(a^2-b^2\right)-5ab\cos(c+dx)\right)}{35b^2d} - \right.$$

$$\frac{2e(e\sin(c+dx))^{9/2}}{9bd}$$

$\downarrow$ 756

$$e^2\left(\frac{2e(e\sin(c+dx))^{5/2}(7(a^2-b^2)-5ab\cos(c+dx))}{35b^2d}-\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(21\left(a^2-b^2\right)^2-ab\left(7a^2-12b^2\right)\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{2a\left(21a^4-49a^2b^2+33b^4\right)\sqrt{\cdots}}{bd\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

$$\frac{2e(e\sin(c+dx))^{9/2}}{9bd}$$

$\downarrow$ 218

$$e^2 \left( \frac{2a\left(21a^4-49a^2b^2+33b^4\right)\sqrt{}}{bd} \right.$$

$$e^2 \left( \frac{2e\sqrt{e\sin(c+dx)}\left(21\left(a^2-b^2\right)^2-ab\left(7a^2-12b^2\right)\cos(c+dx)\right)}{3b^2d} - \right.$$

$$e^2 \left( \frac{2e(e\sin(c+dx))^{5/2}\left(7\left(a^2-b^2\right)-5ab\cos(c+dx)\right)}{35b^2d} - \right.$$

$$\Big\downarrow \; 221$$

$$e^2 \left( \frac{2e(e\sin(c+dx))^{5/2}\left(7\left(a^2-b^2\right)-5ab\cos(c+dx)\right)}{35b^2d} - \frac{e^2 \left( \frac{2e\sqrt{e\sin(c+dx)}\left(21\left(a^2-b^2\right)^2-ab\left(7a^2-12b^2\right)\cos(c+dx)\right)}{3b^2d} - \frac{e^2 \left( \frac{2a\left(21a^4-49a^2b^2+33b^4\right)\sqrt{\cdots}}{bd\cdots} \cdots \right.}{\cdots} \right.}{\cdots} \right.$$

↓ 3042

$$e^2\left(\frac{2a\left(21a^4-49a^2b^2+33b^4\right)\sqrt{\ldots}}{b\ldots}\right.$$

$$e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(21\left(a^2-b^2\right)^2-ab\left(7a^2-12b^2\right)\cos(c+dx)\right)}{3b^2d}-\frac{\ldots}{\ldots}\right.$$

$$e^2\left(\frac{2e(e\sin(c+dx))^{5/2}\left(7\left(a^2-b^2\right)-5ab\cos(c+dx)\right)}{35b^2d}-\frac{\ldots}{\ldots}\right.$$

↓ 3286

$$e^2\left(\frac{2a\left(21a^4-49a^2b^2+33b^4\right)\sqrt{}}{bd}\right.$$

$$e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(21\left(a^2-b^2\right)^2-ab\left(7a^2-12b^2\right)\cos(c+dx)\right)}{3b^2d}-\frac{}{}\right.$$

$$e^2\left(\frac{2e(e\sin(c+dx))^{5/2}(7(a^2-b^2)-5ab\cos(c+dx))}{35b^2d}-\frac{}{}\right.$$

↓ 3042

$$e^2\left(\frac{2a\left(21a^4-49a^2b^2+33b^4\right)\sqrt{\phantom{x}}}{ba}\right.$$

$$e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(21\left(a^2-b^2\right)^2-ab\left(7a^2-12b^2\right)\cos(c+dx)\right)}{3b^2d}-\right.$$

$$e^2\left(\frac{2e(e\sin(c+dx))^{5/2}\left(7\left(a^2-b^2\right)-5ab\cos(c+dx)\right)}{35b^2d}-\right.$$

↓ 3284

$$e^2 \left( \frac{2a\left(21a^4-49a^2b^2+33b^4\right)\sqrt{\cdots}}{bd\cdots} \right.$$

$$e^2 \left( \frac{2e\sqrt{e\sin(c+dx)}\left(21\left(a^2-b^2\right)^2-ab\left(7a^2-12b^2\right)\cos(c+dx)\right)}{3b^2d} - \frac{\cdots}{\cdots} \right.$$

$$e^2 \left( \frac{2e(e\sin(c+dx))^{5/2}\left(7\left(a^2-b^2\right)-5ab\cos(c+dx)\right)}{35b^2d} - \frac{\cdots}{\cdots} \right.$$

input
```
Int[(e*Sin[c + d*x])^(11/2)/(a + b*Cos[c + d*x]),x]
```

output
```
(-2*e*(e*Sin[c + d*x])^(9/2))/(9*b*d) + (e^2*((2*e*(7*(a^2 - b^2) - 5*a*b*
Cos[c + d*x])*(e*Sin[c + d*x])^(5/2))/(35*b^2*d) - (e^2*((2*e*(21*(a^2 - b
^2)^2 - a*b*(7*a^2 - 12*b^2)*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]])/(3*b^2*d)
 - (e^2*((2*a*(21*a^4 - 49*a^2*b^2 + 33*b^4)*EllipticF[(c - Pi/2 + d*x)/2,
 2]*Sqrt[Sin[c + d*x]])/(b*d*Sqrt[e*Sin[c + d*x]]) - (21*(a^2 - b^2)^3*((-
2*b*e*(-1/2*ArcTan[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/(Sqr
t[b]*(-a^2 + b^2)^(3/4)*e^(3/2)) - ArcTanh[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/
(-a^2 + b^2)^(1/4)]/(2*Sqrt[b]*(-a^2 + b^2)^(3/4)*e^(3/2))))/d + (a*Ellipt
icPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x
]])/(Sqrt[-a^2 + b^2]*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]]) - (a*
EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c
 + d*x]])/(Sqrt[-a^2 + b^2]*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]])
))/b))/(3*b^2)))/(7*b^2)))/b
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3174
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Simp[g^2*((p - 1)/(b*(m + p)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; F
reeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
 0] && IntegersQ[2*m, 2*p]
```

rule 3181
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[-a/(2*q)   Int[1/(
Sqrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Simp[b*(g/f)   Subst[
Int[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - S
imp[a/(2*q)   Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3344
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Simp[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Si
n[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^
2*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1
, 0] && IntegerQ[2*m]
```

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

## Maple [A] (warning: unable to verify)

Time = 3.99 (sec) , antiderivative size = 931, normalized size of antiderivative = 1.71

| method | result | size |
|---|---|---|
| default | Expression too large to display | 931 |

input
```
int((e*sin(d*x+c))^(11/2)/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)
```

output
```
(-2*e*b*(1/45/b^6*(e*sin(d*x+c))^(1/2)*e^4*(5*b^4*cos(d*x+c)^4+9*cos(d*x+c
)^2*a^2*b^2-19*b^4*cos(d*x+c)^2+45*a^4-99*a^2*b^2+59*b^4)-1/8*e^6*(a^6-3*a
^4*b^2+3*a^2*b^4-b^6)/b^6*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1
/2)*(ln((e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/
2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*s
in(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arctan(2^(1/2)/(e^2
*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2)/(e^2*(a^2-b
^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)))+(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)
*e^6*a*(-1/21/b^6/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*(-6*b^4*cos(d*x+c)^4*s
in(d*x+c)+21*a^4*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1
/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-49*a^2*b^2*(1-sin(d*x+c))^
(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/
2),1/2*2^(1/2))+33*b^4*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x
+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-14*a^2*b^2*cos(d*x+c
)^2*sin(d*x+c)+30*b^4*cos(d*x+c)^2*sin(d*x+c))-(a^6-3*a^4*b^2+3*a^2*b^4-b^
6)/b^6*(-1/2/b/(-a^2+b^2)^(1/2)*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2
)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b
)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/
2/b/(-a^2+b^2)^(1/2)*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c
)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*Ellipt...
```

## Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{11/2}}{a+b\cos(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))^(11/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{11/2}}{a+b\cos(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))**(11/2)/(a+b*cos(d*x+c)),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(e\sin(c+dx))^{11/2}}{a+b\cos(c+dx)}\,dx = \int \frac{(e\sin(dx+c))^{\frac{11}{2}}}{b\cos(dx+c)+a}\,dx$$

input
```
integrate((e*sin(d*x+c))^(11/2)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((e*sin(d*x + c))^(11/2)/(b*cos(d*x + c) + a), x)
```

## Giac [F]

$$\int \frac{(e\sin(c+dx))^{11/2}}{a+b\cos(c+dx)}\,dx = \int \frac{(e\sin(dx+c))^{\frac{11}{2}}}{b\cos(dx+c)+a}\,dx$$

input
```
integrate((e*sin(d*x+c))^(11/2)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*sin(d*x + c))^(11/2)/(b*cos(d*x + c) + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{11/2}}{a+b\cos(c+dx)}\,dx = \int \frac{(e\ \sin(c+d\,x))^{11/2}}{a+b\ \cos(c+d\,x)}\,dx$$

input
```
int((e*sin(c + d*x))^(11/2)/(a + b*cos(c + d*x)),x)
```

output
```
int((e*sin(c + d*x))^(11/2)/(a + b*cos(c + d*x)), x)
```

## Reduce [F]

$$\int \frac{(e\sin(c+dx))^{11/2}}{a+b\cos(c+dx)}\,dx = \sqrt{e}\left(\int \frac{\sqrt{\sin(dx+c)}\,\sin(dx+c)^5}{\cos(dx+c)\,b+a}dx\right)e^5$$

input
```
int((e*sin(d*x+c))^(11/2)/(a+b*cos(d*x+c)),x)
```

output
```
sqrt(e)*int((sqrt(sin(c + d*x))*sin(c + d*x)**5)/(cos(c + d*x)*b + a),x)*e
**5
```

# 3.59 $\int \frac{(e\sin(c+dx))^{9/2}}{a+b\cos(c+dx)}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 461

$$\begin{aligned}\int \frac{(e\sin(c+dx))^{9/2}}{a+b\cos(c+dx)}\,dx &= -\frac{(-a^2+b^2)^{7/4}\,e^{9/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{9/2}d} \\ &+ \frac{(-a^2+b^2)^{7/4}\,e^{9/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{9/2}d} \\ &+ \frac{a(a^2-b^2)^2\,e^5\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b^5\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} \\ &+ \frac{a(a^2-b^2)^2\,e^5\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b^5\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} \\ &- \frac{2a(5a^2-8b^2)\,e^4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{e\sin(c+dx)}}{5b^4d\sqrt{\sin(c+dx)}} \\ &+ \frac{2e^3(5(a^2-b^2)-3ab\cos(c+dx))\,(e\sin(c+dx))^{3/2}}{15b^3d} - \frac{2e(e\sin(c+dx))^{7/2}}{7bd}\end{aligned}$$

output
```
-(-a^2+b^2)^(7/4)*e^(9/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(
1/4)/e^(1/2))/b^(9/2)/d+(-a^2+b^2)^(7/4)*e^(9/2)*arctanh(b^(1/2)*(e*sin(d*
x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(9/2)/d-a*(a^2-b^2)^2*e^5*Elliptic
Pi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^
(1/2)/b^5/(b-(-a^2+b^2)^(1/2))/d/(e*sin(d*x+c))^(1/2)-a*(a^2-b^2)^2*e^5*El
lipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d
*x+c)^(1/2)/b^5/(b+(-a^2+b^2)^(1/2))/d/(e*sin(d*x+c))^(1/2)+2/5*a*(5*a^2-8
*b^2)*e^4*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2
)/b^4/d/sin(d*x+c)^(1/2)+2/15*e^3*(5*a^2-5*b^2-3*a*b*cos(d*x+c))*(e*sin(d*
x+c))^(3/2)/b^3/d-2/7*e*(e*sin(d*x+c))^(7/2)/b/d
```

## Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 17.37 (sec) , antiderivative size = 834, normalized size of antiderivative = 1.81

$$\int \frac{(e\sin(c+dx))^{9/2}}{a+b\cos(c+dx)}\,dx = -\frac{(e\sin(c+dx))^{9/2}\left(\frac{(5a^3-8ab^2)\cos^2(c+dx)\left(3\sqrt{2}a(a^2-b^2)^{3/4}\left(2\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-2\arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-\log\left(\cdots\right.\right.\right.}{\cdots}\right.}{\cdots} + \frac{\csc^4(c+dx)(e\sin(c+dx))^{9/2}\left(-\frac{(-28a^2+37b^2)\sin(c+dx)}{42b^3}-\frac{a\sin(2(c+dx))}{5b^2}+\frac{\sin(3(c+dx))}{14b}\right)}{d}$$

input
```
Integrate[(e*Sin[c + d*x])^(9/2)/(a + b*Cos[c + d*x]),x]
```

output
```
-1/5*((e*Sin[c + d*x])^(9/2)*(((5*a^3 - 8*a*b^2)*Cos[c + d*x]^2*(3*Sqrt[2]
*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a
^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2
- b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sq
rt[Sin[c + d*x]] + b*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]
*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]]) + 8*b^(5/2)*Appel
lF1[3/4, -1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*
Sin[c + d*x]^(3/2))*(a + b*Sqrt[1 - Sin[c + d*x]^2]))/(12*b^(3/2)*(-a^2 +
b^2)*(a + b*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) + (2*(2*a^2*b - 5*b^3)*Cos
[c + d*x]*(((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])
/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(
-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(
1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 +
I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]]))/(Sq
rt[b]*(-a^2 + b^2)^(1/4)) + (a*AppellF1[3/4, 1/2, 1, 7/4, Sin[c + d*x]^2,
(b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2))/(3*(a^2 - b^2)))*(a
 + b*Sqrt[1 - Sin[c + d*x]^2]))/((a + b*Cos[c + d*x])*Sqrt[1 - Sin[c + d*x
]^2])))/(b^3*d*Sin[c + d*x]^(9/2)) + (Csc[c + d*x]^4*(e*Sin[c + d*x])^(9/2
)*(-1/42*((-28*a^2 + 37*b^2)*Sin[c + d*x])/b^3 - (a*Sin[2*(c + d*x)])/(5*b
^2) + Sin[3*(c + d*x)]/(14*b)))/d
```

## Rubi [A] (warning: unable to verify)

Time = 2.16 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.840$, Rules used = {3042, 3174, 25, 3042, 3344, 27, 3042, 3346, 3042, 3121, 3042, 3119, 3180, 266, 827, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sin(c+dx))^{9/2}}{a+b\cos(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{9/2}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 3174}$$

$$-\frac{e^2\int -\frac{(b+a\cos(c+dx))(e\sin(c+dx))^{5/2}}{a+b\cos(c+dx)}dx}{b}-\frac{2e(e\sin(c+dx))^{7/2}}{7bd}$$

$$\Big\downarrow \ 25$$

$$\frac{e^2\int \frac{(b+a\cos(c+dx))(e\sin(c+dx))^{5/2}}{a+b\cos(c+dx)}dx}{b}-\frac{2e(e\sin(c+dx))^{7/2}}{7bd}$$

$$\Big\downarrow \ 3042$$

$$\frac{e^2\int \frac{\left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{2e(e\sin(c+dx))^{7/2}}{7bd}$$

$$\Big\downarrow \ 3344$$

$$\frac{e^2\left(\frac{2e^2\int -\frac{\left(b\left(2a^2-5b^2\right)+a\left(5a^2-8b^2\right)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{2(a+b\cos(c+dx))}dx}{5b^2}+\frac{2e(e\sin(c+dx))^{3/2}\left(5\left(a^2-b^2\right)-3ab\cos(c+dx)\right)}{15b^2d}\right)}{b}-\frac{2e(e\sin(c+dx))^{7/2}}{7bd}$$

$$\Big\downarrow \ 27$$

$$\frac{e^2\left(\frac{2e(e\sin(c+dx))^{3/2}\left(5\left(a^2-b^2\right)-3ab\cos(c+dx)\right)}{15b^2d}-\frac{e^2\int \frac{\left(b\left(2a^2-5b^2\right)+a\left(5a^2-8b^2\right)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx}{5b^2}\right)}{b}-\frac{2e(e\sin(c+dx))^{7/2}}{7bd}$$

$$\Big\downarrow \ 3042$$

$$\frac{e^2\left(\frac{2e(e\sin(c+dx))^{3/2}\left(5\left(a^2-b^2\right)-3ab\cos(c+dx)\right)}{15b^2d}-\frac{e^2\int \frac{\sqrt{-e\cos\left(c+dx+\frac{\pi}{2}\right)}\left(b\left(2a^2-5b^2\right)+a\left(5a^2-8b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5b^2}\right)}{b}-\frac{2e(e\sin(c+dx))^{7/2}}{7bd}$$

$$\Big\downarrow \ 3346$$

$$\frac{e^2\left(\frac{2e(e\sin(c+dx))^{3/2}(5(a^2-b^2)-3ab\cos(c+dx))}{15b^2d}-\frac{e^2\left(\frac{a\left(5a^2-8b^2\right)\int\sqrt{e\sin(c+dx)}dx}{b}-\frac{5\left(a^2-b^2\right)^2\int\frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx}{b}\right)}{5b^2}\right)}{b}-\frac{2e(e\sin(c+dx))^{7/2}}{7bd}$$

$\Big\downarrow$ 3042

$$\frac{e^2\left(\frac{2e(e\sin(c+dx))^{3/2}(5(a^2-b^2)-3ab\cos(c+dx))}{15b^2d}-\frac{e^2\left(\frac{a\left(5a^2-8b^2\right)\int\sqrt{e\sin(c+dx)}dx}{b}-\frac{5\left(a^2-b^2\right)^2\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{b}\right)}{5b^2}\right)}{b}-\frac{2e(e\sin(c+dx))^{7/2}}{7bd}$$

$\Big\downarrow$ 3121

$$\frac{e^2\left(\frac{2e(e\sin(c+dx))^{3/2}(5(a^2-b^2)-3ab\cos(c+dx))}{15b^2d}-\frac{e^2\left(\frac{a\left(5a^2-8b^2\right)\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{b\sqrt{\sin(c+dx)}}-\frac{5\left(a^2-b^2\right)^2\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{b}\right)}{5b^2}\right)}{b}-\frac{2e(e\sin(c+dx))^{7/2}}{7bd}$$

$\Big\downarrow$ 3042

$$\frac{e^2\left(\frac{2e(e\sin(c+dx))^{3/2}\left(5\left(a^2-b^2\right)-3ab\cos(c+dx)\right)}{15b^2d}-\frac{e^2\left(\frac{a\left(5a^2-8b^2\right)\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{b\sqrt{\sin(c+dx)}}-\frac{5\left(a^2-b^2\right)^2\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{b}\right)}{5b^2}\right)}{b}-\frac{2e(e\sin(c+dx))^{7/2}}{7bd}$$

$\Big\downarrow$ 3119

$$\frac{e^2\left(\frac{2e(e\sin(c+dx))^{3/2}\left(5\left(a^2-b^2\right)-3ab\cos(c+dx)\right)}{15b^2d}-\frac{e^2\left(\frac{2a\left(5a^2-8b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{5\left(a^2-b^2\right)^2\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{b}\right)}{5b^2}\right)}{b}-\frac{2e(e\sin(c+dx))^{7/2}}{7bd}$$

$\Big\downarrow$ 3180

$$\frac{e^2\left(\frac{2e(e\sin(c+dx))^{3/2}\left(5\left(a^2-b^2\right)-3ab\cos(c+dx)\right)}{15b^2d}-\frac{e^2\left(\frac{2a\left(5a^2-8b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{5\left(a^2-b^2\right)^2\left(-\frac{be\int\frac{\sqrt{e\sin(c+dx)}}{b^2\sin^2(c+dx)e^2+\left(a^2 \cdots\right.}}{d}\cdots\right)}{\cdots}\right)}{\cdots}\right)}{b}-\frac{2e(e\sin(c+dx))^{7/2}}{7bd}$$

$\Big\downarrow$ 266

$$e^2\left(\frac{2e(e\sin(c+dx))^{3/2}(5(a^2-b^2)-3ab\cos(c+dx))}{15b^2d}-\frac{e^2\left(\frac{2a\left(5a^2-8b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{5\left(a^2-b^2\right)^2\left(-\frac{2be\int\frac{e^2\sin^2(c+dx}{b^2e^4\sin^4(c+dx)+\left(a\right.}}{d}\right.}{}\right.}{b}\right.$$

$$\frac{2e(e\sin(c+dx))^{7/2}}{7bd}$$

$\downarrow$ 827

$$e^2\left(\frac{2e(e\sin(c+dx))^{3/2}(5(a^2-b^2)-3ab\cos(c+dx))}{15b^2d}-\frac{e^2\left(\frac{2a\left(5a^2-8b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{5\left(a^2-b^2\right)^2\left(-\frac{2be\left(\int\frac{1}{be^2\sin^2(c+dx)+\sqrt{}}\right.}{}\right.}{}\right.}{}\right.$$

$$\frac{2e(e\sin(c+dx))^{7/2}}{7bd}$$

$\downarrow$ 218

$$e^2\left(\frac{2e(e\sin(c+dx))^{3/2}\left(5\left(a^2-b^2\right)-3ab\cos(c+dx)\right)}{15b^2d}-\frac{e^2\left(\frac{2a\left(5a^2-8b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{5\left(a^2-b^2\right)^2\left(-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+}{\sqrt[4]{b^2-}}\right.}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-}}\right.}{}\right.}{}\right)}{}\right)}{b}$$

$$\frac{2e(e\sin(c+dx))^{7/2}}{7bd}$$

$\downarrow$ 221

$$e^2\left(\frac{2e(e\sin(c+dx))^{3/2}(5(a^2-b^2)-3ab\cos(c+dx))}{15b^2d}-\frac{e^2\left(\frac{2a(5a^2-8b^2)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{5(a^2-b^2)^2\left(-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}\ldots\right.}}{2b}\ldots\right.}{\ldots}\right)}{\ldots}\right)}{b}$$

$$\frac{2e(e\sin(c+dx))^{7/2}}{7bd}$$

$\downarrow$ 3042

$$e^2 \left( \frac{2e(e\sin(c+dx))^{3/2}\left(5\left(a^2-b^2\right)-3ab\cos(c+dx)\right)}{15b^2d} - \frac{e^2 \left( \frac{2a\left(5a^2-8b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}} - \frac{5\left(a^2-b^2\right)^2 \left( -\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-\cdots}\right.}}{2b} \cdots \right.}{\cdots} \right.}{\cdots} \right.$$

$$\cdots \qquad b$$

$$\frac{2e(e\sin(c+dx))^{7/2}}{7bd}$$

$$\Big\downarrow \ 3286$$

$$e^2\left(\frac{2e(e\sin(c+dx))^{3/2}\left(5\left(a^2-b^2\right)-3ab\cos(c+dx)\right)}{15b^2d}-\frac{e^2\left(\frac{2a\left(5a^2-8b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{5\left(a^2-b^2\right)^2\left(-\frac{ae\sqrt{\sin(c+dx)}\int\frac{}{\sqrt{\sin(c+d}}}{2b\sqrt{e\,s}}\right.}{}\right.}{}\right.$$

$$\frac{2e(e\sin(c+dx))^{7/2}}{7bd}$$

$\downarrow$ 3042

$$e^2\left(\frac{2e(e\sin(c+dx))^{3/2}\left(5\left(a^2-b^2\right)-3ab\cos(c+dx)\right)}{15b^2d}-\frac{e^2\left(\frac{2a\left(5a^2-8b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{5\left(a^2-b^2\right)^2\left(-\frac{ae\sqrt{\sin(c+dx)}\int\frac{}{\sqrt{\sin(c+d}}}{2b\sqrt{e\,s}}\right.}{}\right.}{}\right.$$

$$\frac{2e(e\sin(c+dx))^{7/2}}{7bd}$$

$\big\downarrow$ 3284

$$e^2\left(\frac{2e(e\sin(c+dx))^{3/2}(5(a^2-b^2)-3ab\cos(c+dx))}{15b^2d}-\frac{e^2\left(\frac{2a(5a^2-8b^2)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{5(a^2-b^2)^2\left(-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+}{\sqrt[4]{b^2-}}\right.}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-}}\right.}{}\right.}{}\right.}{}\right.}{}$$

$$\frac{2e(e\sin(c+dx))^{7/2}}{7bd}$$

input
```
Int[(e*Sin[c + d*x])^(9/2)/(a + b*Cos[c + d*x]),x]
```

output
```
(-2*e*(e*Sin[c + d*x])^(7/2))/(7*b*d) + (e^2*((2*e*(5*(a^2 - b^2) - 3*a*b*
Cos[c + d*x])*(e*Sin[c + d*x])^(3/2))/(15*b^2*d) - (e^2*((2*a*(5*a^2 - 8*b
^2)*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(b*d*Sqrt[Sin[c
 + d*x]]) - (5*(a^2 - b^2)^2*((-2*b*e*(ArcTan[(Sqrt[b]*Sqrt[e]*Sin[c + d*x
])/(-a^2 + b^2)^(1/4)]/(2*b^(3/2)*(-a^2 + b^2)^(1/4)*Sqrt[e]) - ArcTanh[(S
qrt[b]*Sqrt[e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/(2*b^(3/2)*(-a^2 + b^2)^(
1/4)*Sqrt[e])))/d + (a*e*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/
2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(b*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[
c + d*x]]) + (a*e*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x
)/2, 2]*Sqrt[Sin[c + d*x]])/(b*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x
]])))/b))/(5*b^2)))/b
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3174
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Simp[g^2*((p - 1)/(b*(m + p)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; F
reeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
 0] && IntegersQ[2*m, 2*p]
```

rule 3180
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[a*(g/(2*b))   Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Simp[a*(g/(2*b))   In
t[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Simp[b*(g/f)   Su
bst[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3344
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Simp[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Si
n[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^
2*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1
, 0] && IntegerQ[2*m]
```

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 850 vs. $2(404) = 808$.

Time = 3.64 (sec) , antiderivative size = 851, normalized size of antiderivative = 1.85

| method | result |
|---|---|
| default | $$-2eb\left(-\frac{(e\sin(dx+c))^{\frac{3}{2}}e^2\left(3\cos(dx+c)^2b^2+7a^2-10b^2\right)}{21b^4}+\frac{e^4\left(a^4-2a^2b^2+b^4\right)\sqrt{2}\left(\ln\left(\frac{e\sin(dx+c)-\left(\frac{e^2\left(a^2-b^2\right)}{b^2}\right)^{\frac{1}{4}}\sqrt{e\sin(dx+c)}\sqrt{2}+\sqrt{\frac{e^2(}{}}}{e\sin(dx+c)+\left(\frac{e^2\left(a^2-b^2\right)}{b^2}\right)^{\frac{1}{4}}\sqrt{e\sin(dx+c)}\sqrt{2}+\sqrt{\frac{e^2(}{}}}\right.\right.}{8b^6\left(\frac{e^2\left(a^2\right.}{b^2}\right.}\right.$$ |

input
```
int((e*sin(d*x+c))^(9/2)/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)
```

output
```
(-2*e*b*(-1/21/b^4*(e*sin(d*x+c))^(3/2)*e^2*(3*cos(d*x+c)^2*b^2+7*a^2-10*b
^2)+1/8*e^4*(a^4-2*a^2*b^2+b^4)/b^6/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*(ln(
(e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*
(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c
))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arctan(2^(1/2)/(e^2*(a^2-b^
2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)
^(1/4)*(e*sin(d*x+c))^(1/2)-1)))+(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*e^5*a*(
1/5/b^4/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*(10*(1-sin(d*x+c))^(1/2)*(2+2*si
n(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2
))*a^2-16*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*Ell
ipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^2-5*(1-sin(d*x+c))^(1/2)*(2+2*s
in(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/
2))*a^2+8*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*Ell
ipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^2+2*b^2*cos(d*x+c)^4-2*cos(d*x+
c)^2*b^2)+(a^4-2*a^2*b^2+b^4)/b^4*(-1/2/b^2*(1-sin(d*x+c))^(1/2)*(2+2*sin(
d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+
b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2
*2^(1/2))-1/2/b^2*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(
1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((
1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))))/cos(d*x+c)/...
```

## Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{9/2}}{a+b\cos(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))^(9/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{9/2}}{a+b\cos(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))**(9/2)/(a+b*cos(d*x+c)),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(e\sin(c+dx))^{9/2}}{a+b\cos(c+dx)}\,dx = \int \frac{(e\sin(dx+c))^{\frac{9}{2}}}{b\cos(dx+c)+a}\,dx$$

input
```
integrate((e*sin(d*x+c))^(9/2)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((e*sin(d*x + c))^(9/2)/(b*cos(d*x + c) + a), x)
```

## Giac [F]

$$\int \frac{(e\sin(c+dx))^{9/2}}{a+b\cos(c+dx)}\,dx=\int \frac{(e\sin(dx+c))^{\frac{9}{2}}}{b\cos(dx+c)+a}\,dx$$

input
```
integrate((e*sin(d*x+c))^(9/2)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*sin(d*x + c))^(9/2)/(b*cos(d*x + c) + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{9/2}}{a+b\cos(c+dx)}\,dx=\int \frac{(e\,\sin(c+d\,x))^{9/2}}{a+b\,\cos(c+d\,x)}\,dx$$

input
```
int((e*sin(c + d*x))^(9/2)/(a + b*cos(c + d*x)),x)
```

output
```
int((e*sin(c + d*x))^(9/2)/(a + b*cos(c + d*x)), x)
```

## Reduce [F]

$$\int \frac{(e\sin(c+dx))^{9/2}}{a+b\cos(c+dx)}\,dx=\sqrt{e}\left(\int \frac{\sqrt{\sin(dx+c)}\,\sin(dx+c)^4}{\cos(dx+c)\,b+a}dx\right)e^4$$

input
```
int((e*sin(d*x+c))^(9/2)/(a+b*cos(d*x+c)),x)
```

output
```
sqrt(e)*int((sqrt(sin(c + d*x))*sin(c + d*x)**4)/(cos(c + d*x)*b + a),x)*e
**4
```

# 3.60 $\int \frac{(e\sin(c+dx))^{7/2}}{a+b\cos(c+dx)}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 474

$$
\begin{aligned}
\int \frac{(e\sin(c+dx))^{7/2}}{a+b\cos(c+dx)}\,dx &= \frac{(-a^2+b^2)^{5/4}\,e^{7/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{7/2}d} \\
&+ \frac{(-a^2+b^2)^{5/4}\,e^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{7/2}d} \\
&- \frac{2a(3a^2-4b^2)\,e^4\,\operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3b^4d\sqrt{e\sin(c+dx)}} \\
&+ \frac{a(a^2-b^2)^2\,e^4\,\operatorname{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b^4\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}} \\
&+ \frac{a(a^2-b^2)^2\,e^4\,\operatorname{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b^4\left(a^2-b\left(b+\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}} \\
&+ \frac{2e^3(3(a^2-b^2)-ab\cos(c+dx))\sqrt{e\sin(c+dx)}}{3b^3d} - \frac{2e(e\sin(c+dx))^{5/2}}{5bd}
\end{aligned}
$$

output
```
(-a^2+b^2)^(5/4)*e^(7/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1
/4)/e^(1/2))/b^(7/2)/d+(-a^2+b^2)^(5/4)*e^(7/2)*arctanh(b^(1/2)*(e*sin(d*x
+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(7/2)/d-2/3*a*(3*a^2-4*b^2)*e^4*Inv
erseJacobiAM(1/2*c-1/4*Pi+1/2*d*x,2^(1/2))*sin(d*x+c)^(1/2)/b^4/d/(e*sin(d
*x+c))^(1/2)-a*(a^2-b^2)^2*e^4*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b
-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^4/(a^2-b*(b-(-a^2+b^2)^(1/2
)))/d/(e*sin(d*x+c))^(1/2)-a*(a^2-b^2)^2*e^4*EllipticPi(cos(1/2*c+1/4*Pi+1
/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^4/(a^2-b*(b+(
-a^2+b^2)^(1/2)))/d/(e*sin(d*x+c))^(1/2)+2/3*e^3*(3*a^2-3*b^2-a*b*cos(d*x+
c))*(e*sin(d*x+c))^(1/2)/b^3/d-2/5*e*(e*sin(d*x+c))^(5/2)/b/d
```

## Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 17.63 (sec) , antiderivative size = 1955, normalized size of antiderivative = 4.12

$$\int \frac{(e \sin(c+dx))^{7/2}}{a+b\cos(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[(e*Sin[c + d*x])^(7/2)/(a + b*Cos[c + d*x]),x]
```

output
```
(((-2*a*Cos[c + d*x])/(3*b^2) + Cos[2*(c + d*x)]/(5*b))*Csc[c + d*x]^3*(e*
Sin[c + d*x])^(7/2))/d + ((e*Sin[c + d*x])^(7/2)*((28*a*b*Cos[c + d*x]^2*(
a + b*Sqrt[1 - Sin[c + d*x]^2])*((a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[S
in[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[
c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2
 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] +
 Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]]))/
(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)) + (5*b*(a^2 - b^2)*AppellF1[1/4, -1/
2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Sin[c +
 d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5
/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1
[5/4, -1/2, 2, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] + (
a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)
/(-a^2 + b^2)])*Sin[c + d*x]^2)*(a^2 + b^2*(-1 + Sin[c + d*x]^2)))))/((a +
 b*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) + (2*(-10*a^2 + 27*b^2)*Cos[c + d*x
]*(a + b*Sqrt[1 - Sin[c + d*x]^2])*(((-1/8 + I/8)*Sqrt[b]*(2*ArcTan[1 - ((
1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1
+ I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2
] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*
x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Si...
```

## Rubi [A] (warning: unable to verify)

Time = 2.21 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.01, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.840$, Rules used = {3042, 3174, 25, 3042, 3344, 27, 3042, 3346, 3042, 3121, 3042, 3120, 3181, 266, 756, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sin(c+dx))^{7/2}}{a+b\cos(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{7/2}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 3174}$$

$$-\frac{e^2\int -\frac{(b+a\cos(c+dx))(e\sin(c+dx))^{3/2}}{a+b\cos(c+dx)}dx}{b}-\frac{2e(e\sin(c+dx))^{5/2}}{5bd}$$

$$\Big\downarrow \text{25}$$

$$\frac{e^2\int \frac{(b+a\cos(c+dx))(e\sin(c+dx))^{3/2}}{a+b\cos(c+dx)}dx}{b}-\frac{2e(e\sin(c+dx))^{5/2}}{5bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e^2\int \frac{\left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{2e(e\sin(c+dx))^{5/2}}{5bd}$$

$$\Big\downarrow \text{3344}$$

$$\frac{e^2\left(\frac{2e^2\int -\frac{b\left(2a^2-3b^2\right)+a\left(3a^2-4b^2\right)\cos(c+dx)}{2(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{3b^2}+\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(a^2-b^2\right)-ab\cos(c+dx)\right)}{3b^2d}\right)}{b}-\frac{2e(e\sin(c+dx))^{5/2}}{5bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(a^2-b^2\right)-ab\cos(c+dx)\right)}{3b^2d}-\frac{e^2\int \frac{b\left(2a^2-3b^2\right)+a\left(3a^2-4b^2\right)\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{3b^2}\right)}{b}-\frac{2e(e\sin(c+dx))^{5/2}}{5bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(a^2-b^2\right)-ab\cos(c+dx)\right)}{3b^2d}-\frac{e^2\int \frac{b\left(2a^2-3b^2\right)-a\left(3a^2-4b^2\right)\sin\left(c+dx-\frac{\pi}{2}\right)}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{3b^2}\right)}{b}-\frac{2e(e\sin(c+dx))^{5/2}}{5bd}$$

$$\Big\downarrow \text{3346}$$

$$\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(a^2-b^2\right)-ab\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{a\left(3a^2-4b^2\right)\int\frac{1}{\sqrt{e\sin(c+dx)}}dx}{b}-\frac{3\left(a^2-b^2\right)^2\int\frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{b}\right)}{3b^2}\right)}{b}-\frac{2e(e\sin(c+dx))^{5/2}}{5bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(a^2-b^2\right)-ab\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{a\left(3a^2-4b^2\right)\int\frac{1}{\sqrt{e\sin(c+dx)}}dx}{b}-\frac{3\left(a^2-b^2\right)^2\int\frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{b}\right)}{3b^2}\right)}{b}-\frac{2e(e\sin(c+dx))^{5/2}}{5bd}$$

$$\Big\downarrow \; 3121$$

$$\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(a^2-b^2\right)-ab\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{a\left(3a^2-4b^2\right)\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{b\sqrt{e\sin(c+dx)}}-\frac{3\left(a^2-b^2\right)^2\int\frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{b}\right)}{3b^2}\right)}{b}-\frac{2e(e\sin(c+dx))^{5/2}}{5bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(a^2-b^2\right)-ab\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{a\left(3a^2-4b^2\right)\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{b\sqrt{e\sin(c+dx)}}-\frac{3\left(a^2-b^2\right)^2\int\frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{b}\right)}{3b^2}\right)}{b}-\frac{2e(e\sin(c+dx))^{5/2}}{5bd}$$

$\downarrow$ 3120

$$e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(a^2-b^2\right)-ab\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{2a\left(3a^2-4b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{3\left(a^2-b^2\right)^2\int\frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\right.}}{b}\right.}{3b^2}\right.$$

$$b$$

$$\frac{2e(e\sin(c+dx))^{5/2}}{5bd}$$

$\downarrow$ 3181

$$e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(a^2-b^2\right)-ab\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{2a\left(3a^2-4b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{3\left(a^2-b^2\right)^2\left(-\frac{be\int\frac{}{\sqrt{e\sin(c+dx)}\left(b^2\sin\right.}}{}\right.}{}\right.}{}\right.$$

$$b$$

$$\frac{2e(e\sin(c+dx))^{5/2}}{5bd}$$

$\downarrow$ 266

$$e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(a^2-b^2\right)-ab\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{2a\left(3a^2-4b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{3\left(a^2-b^2\right)^2\left(-\frac{2be\int\frac{1}{b^2e^4\sin^4(c+dx)+\left(\right.}}{}\right.}{}\right.}{}\right.$$

$$b$$

$$\frac{2e(e\sin(c+dx))^{5/2}}{5bd}$$

$\downarrow$ 756

$$\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(a^2-b^2\right)-ab\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{2a\left(3a^2-4b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{3\left(a^2-b^2\right)^2\left(-\frac{2be\left(-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\cdots}}{2\cdots}\cdots\right)}{\cdots}\cdots\right)}{\cdots}\cdots\right)}{\cdots}\cdots\right)}{\cdots}$$

$$\frac{2e(e\sin(c+dx))^{5/2}}{5bd}$$

$\downarrow$ 218

$$e^2 \left( \frac{2e\sqrt{e\sin(c+dx)}\left(3\left(a^2-b^2\right)-ab\cos(c+dx)\right)}{3b^2d} - \frac{e^2 \left( \frac{2a\left(3a^2-4b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}} - \frac{3\left(a^2-b^2\right)^2 \left( - \frac{2be\left( - \frac{\int \frac{1}{\sqrt{b^2-a^2}e-be^2 \cdots}}{2 \cdots} \cdots \right.}{\cdots} \cdots \right.}{\cdots} \cdots \right.}{\cdots} \cdots \right.}{b \cdots}$$

$$\frac{2e(e\sin(c+dx))^{5/2}}{5bd}$$

$$\Big\downarrow \ 221$$

$$3\left(a^2-b^2\right)^2 \left( -\frac{a \int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-\right.}}}{2\sqrt{b^2-}} \right.$$

$$e^2\left(\frac{2a\left(3a^2-4b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}} - \right.$$

$$e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(a^2-b^2\right)-ab\cos(c+dx)\right)}{3b^2d} - \right.$$

$$b$$

$$\frac{2e(e\sin(c+dx))^{5/2}}{5bd}$$

$\downarrow$ 3042

$$e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(a^2-b^2\right)-ab\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{2a\left(3a^2-4b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{3\left(a^2-b^2\right)^2\left(-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-\cdots}\right.}}{2\sqrt{b^2-\cdots}}\right.}{\cdots}\right.}{\cdots}\right.$$

$$\overline{\qquad\qquad b\qquad}$$

$$\frac{2e(e\sin(c+dx))^{5/2}}{5bd}$$

$\downarrow$ 3286

$$e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(a^2-b^2\right)-ab\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{2a\left(3a^2-4b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{3\left(a^2-b^2\right)^2\left(-\frac{a\sqrt{\sin(c+dx)}\int\frac{}{\sqrt{\sin(c+d}}}{2\sqrt{b^2-a^2}}\right.}{}\right.}{}\right.$$

$$\frac{2e(e\sin(c+dx))^{5/2}}{5bd}$$

$\downarrow$ 3042

$$3\left(a^2-b^2\right)^2 \left( -\frac{a\sqrt{\sin(c+dx)}\int \frac{}{\sqrt{\sin(c+c}}}{2\sqrt{b^2-a^2}} \right.$$

$$e^2 \left( \frac{2a\left(3a^2-4b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}} - \right.$$

$$e^2 \left( \frac{2e\sqrt{e\sin(c+dx)}\left(3\left(a^2-b^2\right)-ab\cos(c+dx)\right)}{3b^2d} - \right.$$

$$\frac{2e(e\sin(c+dx))^{5/2}}{5bd}$$

$\downarrow$ 3284

$$e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(a^2-b^2\right)-ab\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{2a\left(3a^2-4b^2\right)\sqrt{\sin(c+dx)}\,\mathrm{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{3\left(a^2-b^2\right)^2\left(-\frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c}}{\sqrt[4]{b^2-}}\right.}{2\sqrt{b}e^{3/2}\left(b^2-a\right.}\right.}{}\right.}{}\right.}{}\right.$$

$$\frac{2e(e\sin(c+dx))^{5/2}}{5bd}$$

input
```
Int[(e*Sin[c + d*x])^(7/2)/(a + b*Cos[c + d*x]),x]
```

output
```
(-2*e*(e*Sin[c + d*x])^(5/2))/(5*b*d) + (e^2*((2*e*(3*(a^2 - b^2) - a*b*Co
s[c + d*x])*Sqrt[e*Sin[c + d*x]])/(3*b^2*d) - (e^2*((2*a*(3*a^2 - 4*b^2)*E
llipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(b*d*Sqrt[e*Sin[c + d*
x]]) - (3*(a^2 - b^2)^2*((-2*b*e*(-1/2*ArcTan[(Sqrt[b]*Sqrt[e]*Sin[c + d*x
])/(-a^2 + b^2)^(1/4)]/(Sqrt[b]*(-a^2 + b^2)^(3/4)*e^(3/2)) - ArcTanh[(Sqr
t[b]*Sqrt[e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/(2*Sqrt[b]*(-a^2 + b^2)^(3/
4)*e^(3/2))))/d + (a*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 +
d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(Sqrt[-a^2 + b^2]*(b - Sqrt[-a^2 + b^2])*d*
Sqrt[e*Sin[c + d*x]]) - (a*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - P
i/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(Sqrt[-a^2 + b^2]*(b + Sqrt[-a^2 + b^
2])*d*Sqrt[e*Sin[c + d*x]])))/b))/(3*b^2)))/b
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3174
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Simp[g^2*((p - 1)/(b*(m + p)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; F
reeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
 0] && IntegersQ[2*m, 2*p]
```

rule 3181
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[-a/(2*q)   Int[1/(
Sqrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Simp[b*(g/f)   Subst[
Int[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - S
imp[a/(2*q)   Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3344
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Simp[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Si
n[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^
2*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1
, 0] && IntegerQ[2*m]
```

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

## Maple [A] (verified)

Time = 3.51 (sec) , antiderivative size = 773, normalized size of antiderivative = 1.63

| method | result |
|---|---|
| default | $-2eb\left(-\frac{\sqrt{e\sin(dx+c)}\,e^2\left(\cos(dx+c)^2b^2+5a^2-6b^2\right)}{5b^4}+\frac{e^4\left(a^4-2a^2b^2+b^4\right)\left(\frac{e^2\left(a^2-b^2\right)}{b^2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\sin(dx+c)+\left(\frac{e^2\left(a^2-b^2\right)}{b^2}\right)^{\frac{1}{4}}\sqrt{e\sin(dx+}}{e\sin(dx+c)-\left(\frac{e^2\left(a^2-b^2\right)}{b^2}\right)^{\frac{1}{4}}\sqrt{e\sin(dx+}}\right.\right.}{8b^4(}\right.$ |

input
```
int((e*sin(d*x+c))^(7/2)/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)
```

output
```
(-2*e*b*(-1/5/b^4*(e*sin(d*x+c))^(1/2)*e^2*(cos(d*x+c)^2*b^2+5*a^2-6*b^2)+
1/8*e^4*(a^4-2*a^2*b^2+b^4)/b^4*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2
)*2^(1/2)*(ln((e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)
*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4
)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arctan(2^(1/2
)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2)/(e^2*
(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)))+(cos(d*x+c)^2*e*sin(d*x+c))
^(1/2)*e^4*a*(1/3/b^4/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*(3*(1-sin(d*x+c))^
(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/
2),1/2*2^(1/2))*a^2-4*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+
c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^2-2*b^2*cos(d*x+c)^
2*sin(d*x+c))+(a^4-2*a^2*b^2+b^4)/b^4*(-1/2/b/(-a^2+b^2)^(1/2)*(1-sin(d*x+
c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+
c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^
2+b^2)^(1/2)/b),1/2*2^(1/2))+1/2/b/(-a^2+b^2)^(1/2)*(1-sin(d*x+c))^(1/2)*(
2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(
1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2
)/b),1/2*2^(1/2))))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

## Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{7/2}}{a+b\cos(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))^(7/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{7/2}}{a+b\cos(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))**(7/2)/(a+b*cos(d*x+c)),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(e\sin(c+dx))^{7/2}}{a+b\cos(c+dx)}\,dx = \int \frac{(e\sin(dx+c))^{\frac{7}{2}}}{b\cos(dx+c)+a}\,dx$$

input
```
integrate((e*sin(d*x+c))^(7/2)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((e*sin(d*x + c))^(7/2)/(b*cos(d*x + c) + a), x)
```

## Giac [F]

$$\int \frac{(e \sin(c+dx))^{7/2}}{a+b\cos(c+dx)}\,dx = \int \frac{(e \sin(dx+c))^{\frac{7}{2}}}{b\cos(dx+c)+a}\,dx$$

input
```
integrate((e*sin(d*x+c))^(7/2)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*sin(d*x + c))^(7/2)/(b*cos(d*x + c) + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(e \sin(c+dx))^{7/2}}{a+b\cos(c+dx)}\,dx = \int \frac{(e\,\sin(c+d\,x))^{7/2}}{a+b\,\cos(c+d\,x)}\,dx$$

input
```
int((e*sin(c + d*x))^(7/2)/(a + b*cos(c + d*x)),x)
```

output
```
int((e*sin(c + d*x))^(7/2)/(a + b*cos(c + d*x)), x)
```

## Reduce [F]

$$\int \frac{(e \sin(c+dx))^{7/2}}{a+b\cos(c+dx)}\,dx = \sqrt{e}\left(\int \frac{\sqrt{\sin(dx+c)}\,\sin(dx+c)^3}{\cos(dx+c)\,b+a}dx\right)e^3$$

input
```
int((e*sin(d*x+c))^(7/2)/(a+b*cos(d*x+c)),x)
```

output
```
sqrt(e)*int((sqrt(sin(c + d*x))*sin(c + d*x)**3)/(cos(c + d*x)*b + a),x)*e
**3
```

# 3.61 $\int \frac{(e\sin(c+dx))^{5/2}}{a+b\cos(c+dx)}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 399

$$\begin{aligned}\int \frac{(e\sin(c+dx))^{5/2}}{a+b\cos(c+dx)}\,dx =& -\frac{(-a^2+b^2)^{3/4}\,e^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{5/2}d} \\ &+\frac{(-a^2+b^2)^{3/4}\,e^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{5/2}d} \\ &-\frac{a(a^2-b^2)\,e^3\operatorname{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b^3\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} \\ &-\frac{a(a^2-b^2)\,e^3\operatorname{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b^3\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} \\ &+\frac{2ae^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{b^2d\sqrt{\sin(c+dx)}}-\frac{2e(e\sin(c+dx))^{3/2}}{3bd}\end{aligned}$$

output
```
-(-a^2+b^2)^(3/4)*e^(5/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(
1/4)/e^(1/2))/b^(5/2)/d+(-a^2+b^2)^(3/4)*e^(5/2)*arctanh(b^(1/2)*(e*sin(d*
x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(5/2)/d+a*(a^2-b^2)*e^3*EllipticPi
(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1
/2)/b^3/(b-(-a^2+b^2)^(1/2))/d/(e*sin(d*x+c))^(1/2)+a*(a^2-b^2)*e^3*Ellipt
icPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c
)^(1/2)/b^3/(b+(-a^2+b^2)^(1/2))/d/(e*sin(d*x+c))^(1/2)-2*a*e^2*EllipticE(
cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/b^2/d/sin(d*x+c)^(
1/2)-2/3*e*(e*sin(d*x+c))^(3/2)/b/d
```

## Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 16.60 (sec) , antiderivative size = 757, normalized size of antiderivative = 1.90

$$\int \frac{(e\sin(c+dx))^{5/2}}{a+b\cos(c+dx)}\,dx = -\frac{2\csc(c+dx)(e\sin(c+dx))^{5/2}}{3bd}$$

$$+\frac{(e\sin(c+dx))^{5/2}\left(\frac{a\cos^2(c+dx)\left(3\sqrt{2}a\left(a^2-b^2\right)^{3/4}\left(2\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-2\arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-\log\left(\sqrt{a^2-b^2}-\cdots\right.\right.\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(e*Sin[c + d*x])^(5/2)/(a + b*Cos[c + d*x]),x]
```

output
```
(-2*Csc[c + d*x]*(e*Sin[c + d*x])^(5/2))/(3*b*d) + ((e*Sin[c + d*x])^(5/2)
*((a*Cos[c + d*x]^2*(3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*
Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqr
t[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2
]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]] + Log[Sqr
t[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Si
n[c + d*x]]) + 8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*
Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2))*(a + b*Sqrt[1 - Sin[c +
d*x]^2]))/(12*b^(3/2)*(-a^2 + b^2)*(a + b*Cos[c + d*x])*(1 - Sin[c + d*x]^
2)) + (2*b*Cos[c + d*x]*(((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[
Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Si
n[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*
(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]] + Log[Sqrt[-a^2
+ b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c
 + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)) + (a*AppellF1[3/4, 1/2, 1, 7/4, Si
n[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2))/(3*(a
^2 - b^2)))*(a + b*Sqrt[1 - Sin[c + d*x]^2]))/((a + b*Cos[c + d*x])*Sqrt[1
 - Sin[c + d*x]^2])))/(b*d*Sin[c + d*x]^(5/2))
```

## Rubi [A] (warning: unable to verify)

Time = 1.74 (sec) , antiderivative size = 390, normalized size of antiderivative = 0.98, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.720$, Rules used = {3042, 3174, 25, 3042, 3346, 3042, 3121, 3042, 3119, 3180, 266, 827, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sin(c+dx))^{5/2}}{a+b\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3174}$$

$$-\frac{e^2\int -\frac{(b+a\cos(c+dx))\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx}{b}-\frac{2e(e\sin(c+dx))^{3/2}}{3bd}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{e^2\int \frac{(b+a\cos(c+dx))\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx}{b}-\frac{2e(e\sin(c+dx))^{3/2}}{3bd}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{e^2\int \frac{\sqrt{-e\cos\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{2e(e\sin(c+dx))^{3/2}}{3bd}$$

$$\Big\downarrow \text{ 3346}$$

$$\frac{e^2\left(\frac{a\int \sqrt{e\sin(c+dx)}dx}{b}-\frac{(a^2-b^2)\int \frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx}{b}\right)}{b}-\frac{2e(e\sin(c+dx))^{3/2}}{3bd}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{e^2\left(\frac{a\int \sqrt{e\sin(c+dx)}dx}{b}-\frac{(a^2-b^2)\int \frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{b}\right)}{b}-\frac{2e(e\sin(c+dx))^{3/2}}{3bd}$$

$$\Big\downarrow \text{ 3121}$$

$$\frac{e^2\left(\frac{a\sqrt{e\sin(c+dx)}\int \sqrt{\sin(c+dx)}dx}{b\sqrt{\sin(c+dx)}}-\frac{(a^2-b^2)\int \frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{b}\right)}{b}-\frac{2e(e\sin(c+dx))^{3/2}}{3bd}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{e^2\left(\frac{a\sqrt{e\sin(c+dx)}\int \sqrt{\sin(c+dx)}dx}{b\sqrt{\sin(c+dx)}}-\frac{(a^2-b^2)\int \frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{b}\right)}{b}-\frac{2e(e\sin(c+dx))^{3/2}}{3bd}$$

$$\Big\downarrow \text{ 3119}$$

$$\frac{e^2\left(\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{(a^2-b^2)\int \frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{b}\right)}{b}-\frac{2e(e\sin(c+dx))^{3/2}}{3bd}$$

$$\Big\downarrow \text{ 3180}$$

$$\frac{e^2\left(\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{(a^2-b^2)\left(-\frac{be\int\frac{\sqrt{e\sin(c+dx)}}{b^2\sin^2(c+dx)e^2+\left(a^2-b^2\right)e^2}d(e\sin(c+dx))}{d}-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}d}{2b}\right.}{b}\right.}{b}$$

$$\frac{2e(e\sin(c+dx))^{3/2}}{3bd}$$

$\downarrow$ 266

$$\frac{e^2\left(\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{(a^2-b^2)\left(-\frac{2be\int\frac{e^2\sin^2(c+dx)}{b^2e^4\sin^4(c+dx)+\left(a^2-b^2\right)e^2}d\sqrt{e\sin(c+dx)}}{d}-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}}{2b}\right.}{b}\right.}{b}$$

$$\frac{2e(e\sin(c+dx))^{3/2}}{3bd}$$

$\downarrow$ 827

$$\frac{e^2\left(\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{(a^2-b^2)\left(-\frac{2be\left(\frac{\int\frac{1}{be^2\sin^2(c+dx)+\sqrt{b^2-a^2}e}d\sqrt{e\sin(c+dx)}}{2b}-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2b}\right.}{d}\right.}{b}\right.}{b}$$

$$\frac{2e(e\sin(c+dx))^{3/2}}{3bd}$$

$\downarrow$ 218

$$e^2\left(\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{(a^2-b^2)\left(-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2b}\right)}{d}-\frac{ae\int\frac{}{\sqrt{e\sin(}}\cdots}{}\right)}{b}\cdots}{b}\right.$$

$$\frac{2e(e\sin(c+dx))^{3/2}}{3bd}$$

$\downarrow$ 221

$$e^2\left(\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{(a^2-b^2)\left(-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b}-\cdots\right)}{b}\cdots}{b}\right.$$

$$\frac{2e(e\sin(c+dx))^{3/2}}{3bd}$$

$\downarrow$ 3042

$$e^2\left(\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{(a^2-b^2)\left(-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b}-\right.}{b}\right.$$

$$\frac{2e(e\sin(c+dx))^{3/2}}{3bd}$$

$\big\downarrow$ 3286

$$e^2\left(\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{(a^2-b^2)\left(-\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b\sqrt{e\sin(c+dx)}}+\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c\right.}}{2b\sqrt{e\sin(c+dx)}}\right.}{b}\right.$$

$$\frac{2e(e\sin(c+dx))^{3/2}}{3bd}$$

$\big\downarrow$ 3042

$$e^2\left(\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{(a^2-b^2)\left(-\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b\sqrt{e\sin(c+dx)}}+\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c\cdots\right.}}{2b\sqrt{e\sin(c+dx)}}\right.}{b}\right.}{b}$$

$$\frac{2e(e\sin(c+dx))^{3/2}}{3bd}$$

$\big\downarrow$ 3284

$$e^2\left(\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{(a^2-b^2)\left(-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{d}+\frac{ae\sqrt{\sin(c+dx)}\,\text{EllipticPi}\cdots}{bd\left(b-\sqrt{b^2\cdots}\right.}\right.}{b}\right.}{b}$$

$$\frac{2e(e\sin(c+dx))^{3/2}}{3bd}$$

input
```
Int[(e*Sin[c + d*x])^(5/2)/(a + b*Cos[c + d*x]),x]
```

output
```
(-2*e*(e*Sin[c + d*x])^(3/2))/(3*b*d) + (e^2*((2*a*EllipticE[(c - Pi/2 + d
*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(b*d*Sqrt[Sin[c + d*x]]) - ((a^2 - b^2)*((
-2*b*e*(ArcTan[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/(2*b^(3/
2)*(-a^2 + b^2)^(1/4)*Sqrt[e]) - ArcTanh[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(-
a^2 + b^2)^(1/4)]/(2*b^(3/2)*(-a^2 + b^2)^(1/4)*Sqrt[e])))/d + (a*e*Ellipt
icPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x
]])/(b*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]]) + (a*e*EllipticPi[(2
*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(b*
(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]])))/b))/b
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3174
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Simp[g^2*((p - 1)/(b*(m + p)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; F
reeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
 0] && IntegersQ[2*m, 2*p]
```

rule 3180
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[a*(g/(2*b))   Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Simp[a*(g/(2*b))   In
t[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Simp[b*(g/f)   Su
bst[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

## Maple [A] (verified)

Time = 2.87 (sec) , antiderivative size = 639, normalized size of antiderivative = 1.60

| method | result |
|---|---|
| default | $-2eb\left(\frac{(e\sin(dx+c))^{\frac{3}{2}}}{3b^2}-\frac{e^2\left(a^2-b^2\right)\sqrt{2}\left(\ln\left(\frac{e\sin(dx+c)-\left(\frac{e^2\left(a^2-b^2\right)}{b^2}\right)^{\frac{1}{4}}\sqrt{e\sin(dx+c)}\,\sqrt{2}+\sqrt{\frac{e^2\left(a^2-b^2\right)}{b^2}}}{e\sin(dx+c)+\left(\frac{e^2\left(a^2-b^2\right)}{b^2}\right)^{\frac{1}{4}}\sqrt{e\sin(dx+c)}\,\sqrt{2}+\sqrt{\frac{e^2\left(a^2-b^2\right)}{b^2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\sin(dx+c)}}{\left(\frac{e^2\left(a^2-b^2\right)}{b^2}\right)^{\frac{1}{4}}}+\cdots\right.\right.}{8b^4\left(\frac{e^2\left(a^2-b^2\right)}{b^2}\right)^{\frac{1}{4}}}\cdots$ |

input
```
int((e*sin(d*x+c))^(5/2)/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)
```

output
```
(-2*e*b*(1/3*(e*sin(d*x+c))^(3/2)/b^2-1/8*e^2*(a^2-b^2)/b^4/(e^2*(a^2-b^2)
/b^2)^(1/4)*2^(1/2)*(ln((e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x
+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)+(e^2*(a^2-b^2)
/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arc
tan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(
1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)))+(cos(d*x+c)^2*e*s
in(d*x+c))^(1/2)*e^3*a*(-1/b^2*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)
*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*(2*EllipticE((1-sin(d*
x+c))^(1/2),1/2*2^(1/2))-EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2)))-(a^2
-b^2)/b^2*(-1/2/b^2*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)
^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi
((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-1/2/b^2*(1-sin
(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin
(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1
+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2)))))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

## Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{5/2}}{a+b\cos(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))^(5/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{5/2}}{a+b\cos(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))**(5/2)/(a+b*cos(d*x+c)),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(e\sin(c+dx))^{5/2}}{a+b\cos(c+dx)}\,dx = \int \frac{(e\sin(dx+c))^{\frac{5}{2}}}{b\cos(dx+c)+a}\,dx$$

input
```
integrate((e*sin(d*x+c))^(5/2)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((e*sin(d*x + c))^(5/2)/(b*cos(d*x + c) + a), x)
```

## Giac [F]

$$\int \frac{(e\sin(c+dx))^{5/2}}{a+b\cos(c+dx)}\,dx = \int \frac{(e\sin(dx+c))^{\frac{5}{2}}}{b\cos(dx+c)+a}\,dx$$

input
```
integrate((e*sin(d*x+c))^(5/2)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*sin(d*x + c))^(5/2)/(b*cos(d*x + c) + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{5/2}}{a+b\cos(c+dx)}\,dx = \int \frac{(e\,\sin(c+d\,x))^{5/2}}{a+b\,\cos(c+d\,x)}\,dx$$

input
```
int((e*sin(c + d*x))^(5/2)/(a + b*cos(c + d*x)),x)
```

output
```
int((e*sin(c + d*x))^(5/2)/(a + b*cos(c + d*x)), x)
```

## Reduce [F]

$$\int \frac{(e\sin(c+dx))^{5/2}}{a+b\cos(c+dx)}\,dx = \sqrt{e}\left(\int \frac{\sqrt{\sin(dx+c)}\,\sin(dx+c)^2}{\cos(dx+c)\,b+a}dx\right)e^2$$

input
```
int((e*sin(d*x+c))^(5/2)/(a+b*cos(d*x+c)),x)
```

output
```
sqrt(e)*int((sqrt(sin(c + d*x))*sin(c + d*x)**2)/(cos(c + d*x)*b + a),x)*e
**2
```

# 3.62 $\int \frac{(e\sin(c+dx))^{3/2}}{a+b\cos(c+dx)}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 410

$$\begin{aligned}\int \frac{(e\sin(c+dx))^{3/2}}{a+b\cos(c+dx)}\,dx &= \frac{\sqrt[4]{-a^2+b^2}\,e^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{3/2}d} \\ &+ \frac{\sqrt[4]{-a^2+b^2}\,e^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{3/2}d} \\ &+ \frac{2ae^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b^2d\sqrt{e\sin(c+dx)}} \\ &- \frac{a(a^2-b^2)\,e^2\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b^2\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}} \\ &- \frac{a(a^2-b^2)\,e^2\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b^2\left(a^2-b\left(b+\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}} \\ &- \frac{2e\sqrt{e\sin(c+dx)}}{bd}\end{aligned}$$

output
```
(-a^2+b^2)^(1/4)*e^(3/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1
/4)/e^(1/2))/b^(3/2)/d+(-a^2+b^2)^(1/4)*e^(3/2)*arctanh(b^(1/2)*(e*sin(d*x
+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(3/2)/d+2*a*e^2*InverseJacobiAM(1/2
*c-1/4*Pi+1/2*d*x,2^(1/2))*sin(d*x+c)^(1/2)/b^2/d/(e*sin(d*x+c))^(1/2)+a*(
a^2-b^2)*e^2*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2))
,2^(1/2))*sin(d*x+c)^(1/2)/b^2/(a^2-b*(b-(-a^2+b^2)^(1/2)))/d/(e*sin(d*x+c
))^(1/2)+a*(a^2-b^2)*e^2*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2
+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^2/(a^2-b*(b+(-a^2+b^2)^(1/2)))/d/
(e*sin(d*x+c))^(1/2)-2*e*(e*sin(d*x+c))^(1/2)/b/d
```

## Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 7.15 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.06

$$\int \frac{(e\sin(c+dx))^{3/2}}{a+b\cos(c+dx)}\,dx =$$
$$-\frac{\left(\frac{1}{20}-\frac{i}{20}\right)\cos(c+dx)\left(a+b\sqrt{\cos^2(c+dx)}\right)(e\sin(c+dx))^{3/2}\left(-5(a^2-b^2)\left(2\sqrt[4]{-a^2+b^2}\arctan\left(1-\right.\right.\right.}{}$$

input
```
Integrate[(e*Sin[c + d*x])^(3/2)/(a + b*Cos[c + d*x]),x]
```

output
```
((-1/20 + I/20)*Cos[c + d*x]*(a + b*Sqrt[Cos[c + d*x]^2])*(e*Sin[c + d*x])
^(3/2)*(-5*(a^2 - b^2)*(2*(-a^2 + b^2)^(1/4)*ArcTan[1 - ((1 + I)*Sqrt[b]*S
qrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*(-a^2 + b^2)^(1/4)*ArcTan[1 + (
(1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] + (-a^2 + b^2)^(1/
4)*Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c +
d*x]] + I*b*Sin[c + d*x]] - (-a^2 + b^2)^(1/4)*Log[Sqrt[-a^2 + b^2] + (1 +
 I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]] + (4
 + 4*I)*Sqrt[b]*Sqrt[Sin[c + d*x]]) + (4 + 4*I)*a*b^(3/2)*AppellF1[5/4, 1/
2, 1, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]
^(5/2)))/(b^(3/2)*(-a^2 + b^2)*d*Sqrt[Cos[c + d*x]^2]*(a + b*Cos[c + d*x])
*Sin[c + d*x]^(3/2))
```

## Rubi [A] (warning: unable to verify)

Time = 1.72 (sec) , antiderivative size = 407, normalized size of antiderivative = 0.99, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.720, Rules used = {3042, 3174, 25, 3042, 3346, 3042, 3121, 3042, 3120, 3181, 266, 756, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sin(c+dx))^{3/2}}{a+b\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3174}$$

$$-\frac{e^2\int -\frac{b+a\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{b}-\frac{2e\sqrt{e\sin(c+dx)}}{bd}$$

$$\Big\downarrow \text{25}$$

$$\frac{e^2\int \frac{b+a\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{b}-\frac{2e\sqrt{e\sin(c+dx)}}{bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e^2\int \frac{b-a\sin\left(c+dx-\frac{\pi}{2}\right)}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{b}-\frac{2e\sqrt{e\sin(c+dx)}}{bd}$$

$$\Big\downarrow \text{3346}$$

$$\frac{e^2\left(\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}}dx}{b}-\frac{(a^2-b^2)\int \frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{b}\right)}{b}-\frac{2e\sqrt{e\sin(c+dx)}}{bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e^2\left(\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}}dx}{b}-\frac{(a^2-b^2)\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{b}\right)}{b}-\frac{2e\sqrt{e\sin(c+dx)}}{bd}$$

$$\Big\downarrow \text{3121}$$

$$\frac{e^2\left(\frac{a\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}}dx}{b\sqrt{e\sin(c+dx)}}-\frac{(a^2-b^2)\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{b}\right)}{b}-\frac{2e\sqrt{e\sin(c+dx)}}{bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e^2\left(\frac{a\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}}dx}{b\sqrt{e\sin(c+dx)}}-\frac{(a^2-b^2)\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{b}\right)}{b}-\frac{2e\sqrt{e\sin(c+dx)}}{bd}$$

$$\Big\downarrow \text{3120}$$

$$\frac{e^2\left(\frac{2a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{(a^2-b^2)\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{b}\right)}{b}-\frac{2e\sqrt{e\sin(c+dx)}}{bd}$$

$$\Big\downarrow \text{3181}$$

$$\frac{e^2\left(\frac{2a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{(a^2-b^2)\left(-\frac{be\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b^2\sin^2(c+dx)e^2+\left(a^2-b^2\right)e^2\right)}d(e\sin(c+dx))}{d}-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b\cdots}\right.}}{2\sqrt{b\cdots}}\cdots\right)}{b}\cdots\right)}{b}\cdots-\frac{2e\sqrt{e\sin(c+dx)}}{bd}$$

$$\Big\downarrow \text{266}$$

$$\frac{e^2\left(\frac{2a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{(a^2-b^2)\left(-\frac{2be\int\frac{1}{b^2e^4\sin^4(c+dx)+\left(a^2-b^2\right)e^2}d\sqrt{e\sin(c+dx)}}{d}-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+\cdots\right.}}{2\sqrt{b^2-a^2}}\cdots\right)}{b}\cdots\right)}{b}\cdots-\frac{2e\sqrt{e\sin(c+dx)}}{bd}$$

$$\Big\downarrow \text{756}$$

$$\frac{e^2\left(\frac{2a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{(a^2-b^2)\left(-\frac{2be\left(-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2e\sqrt{b^2-a^2}}-\frac{\int\frac{1}{be^2\sin^2(c+dx)+\sqrt{b^2-a^2}e}d\sqrt{e\sin\cdots}}{2e\sqrt{b^2-a^2}}\cdots\right)}{d}\cdots\right)}{b}\cdots\right)}{\frac{2e\sqrt{e\sin(c+dx)}}{bd}}$$

$\downarrow$ 218

$$\frac{e^2\left(\frac{2a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{(a^2-b^2)\left(-\frac{2be\left(-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2e\sqrt{b^2-a^2}}-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}\right)}{d}-\frac{a\int\cdots}{\cdots}\cdots\right)}{b}\cdots\right)}{\frac{2e\sqrt{e\sin(c+dx)}}{bd}}$$

$\downarrow$ 221

$$e^2\left(\frac{2a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{(a^2-b^2)\left(-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}}{2\sqrt{b^2-a^2}}\right.}{b}\right.$$

$$\frac{\phantom{x}}{b}$$

$$\frac{2e\sqrt{e\sin(c+dx)}}{bd}$$

$$\Big\downarrow \text{3042}$$

$$e^2\left(\frac{2a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{(a^2-b^2)\left(-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}}{2\sqrt{b^2-a^2}}\right.}{b}\right.$$

$$\frac{\phantom{x}}{b}$$

$$\frac{2e\sqrt{e\sin(c+dx)}}{bd}$$

$$\Big\downarrow \text{3286}$$

$$e^2\left(\frac{2a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{(a^2-b^2)\left(-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}}-\frac{a\sqrt{\sin(c+dx)}\int\frac{}{\sqrt{\sin(c+dx)}\left(b\sin\right.}}{2\sqrt{b^2-a^2}\sqrt{e\sin}}\right.}{b}\right.$$

$$\frac{}{b}$$

$$\frac{2e\sqrt{e\sin(c+dx)}}{bd}$$

$\downarrow$ 3042

$$e^2\left(\frac{2a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{(a^2-b^2)\left(-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}}-\frac{a\sqrt{\sin(c+dx)}\int\frac{}{\sqrt{\sin(c+dx)}\left(b\sin\right.}}{2\sqrt{b^2-a^2}\sqrt{e\sin}}\right.}{b}\right.$$

$$\frac{}{b}$$

$$\frac{2e\sqrt{e\sin(c+dx)}}{bd}$$

$\downarrow$ 3284

$$\frac{e^2\left(\frac{2a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{(a^2-b^2)\left(-\frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}\right)}{d}+\frac{a\sqrt{\sin(c+dx)}\,\text{El}}{d\sqrt{b^2-}}\right.}{b}}{b}$$

$$\frac{2e\sqrt{e\sin(c+dx)}}{bd}$$

input
```
Int[(e*Sin[c + d*x])^(3/2)/(a + b*Cos[c + d*x]),x]
```

output
```
(-2*e*Sqrt[e*Sin[c + d*x]])/(b*d) + (e^2*((2*a*EllipticF[(c - Pi/2 + d*x)/
2, 2]*Sqrt[Sin[c + d*x]])/(b*d*Sqrt[e*Sin[c + d*x]]) - ((a^2 - b^2)*((-2*b
*e*(-1/2*ArcTan[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/(Sqrt[b
]*(-a^2 + b^2)^(3/4)*e^(3/2)) - ArcTanh[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(-a
^2 + b^2)^(1/4)]/(2*Sqrt[b]*(-a^2 + b^2)^(3/4)*e^(3/2))))/d + (a*EllipticP
i[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])
/(Sqrt[-a^2 + b^2]*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]]) - (a*Ell
ipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c +
d*x]])/(Sqrt[-a^2 + b^2]*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]])))/
b))/b
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3174
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Simp[g^2*((p - 1)/(b*(m + p)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; F
reeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
 0] && IntegersQ[2*m, 2*p]
```

rule 3181
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[-a/(2*q)   Int[1/(
Sqrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Simp[b*(g/f)   Subst[
Int[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - S
imp[a/(2*q)   Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

## Maple [A] (verified)

Time = 1.57 (sec) , antiderivative size = 655, normalized size of antiderivative = 1.60

| method | result |
|---|---|
| default | $-2eb\left(\frac{\sqrt{e\sin(dx+c)}}{b^2}-\frac{e^2\left(a^2-b^2\right)\left(\frac{e^2\left(a^2-b^2\right)}{b^2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\sin(dx+c)+\left(\frac{e^2\left(a^2-b^2\right)}{b^2}\right)^{\frac{1}{4}}\sqrt{e\sin(dx+c)}\,\sqrt{2}+\sqrt{\frac{e^2\left(a^2-b^2\right)}{b^2}}}{e\sin(dx+c)-\left(\frac{e^2\left(a^2-b^2\right)}{b^2}\right)^{\frac{1}{4}}\sqrt{e\sin(dx+c)}\,\sqrt{2}+\sqrt{\frac{e^2\left(a^2-b^2\right)}{b^2}}}\right)+2\arctan\left(\frac{\sqrt{2}\ldots}{\left(\frac{e\ldots}{\ldots}\right.}\right.\right.}{8b^2\left(a^2e^2-b^2e^2\right)}\right.$ |

input
```
int((e*sin(d*x+c))^(3/2)/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)
```

output
```
(-2*e*b*(1/b^2*(e*sin(d*x+c))^(1/2)-1/8*e^2*(a^2-b^2)/b^2*(e^2*(a^2-b^2)/b
^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*(ln((e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^
(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c
)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^
2)^(1/2)))+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)
+1)+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)))+(
cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*a*e^2*(-1/b^2*(1-sin(d*x+c))^(1/2)*(2+2*s
in(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*Ellipt
icF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-(a^2-b^2)/b^2*(-1/2/b/(-a^2+b^2)^(1/
2)*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c
)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(
1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/2/b/(-a^2+b^2)^(1/2)*(1-sin(d
*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d
*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(
-a^2+b^2)^(1/2)/b),1/2*2^(1/2))))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

## Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{3/2}}{a+b\cos(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))^(3/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

## Sympy [F]

$$\int \frac{(e\sin(c+dx))^{3/2}}{a+b\cos(c+dx)}\,dx = \int \frac{(e\sin(c+dx))^{\frac{3}{2}}}{a+b\cos(c+dx)}\,dx$$

input
```
integrate((e*sin(d*x+c))**(3/2)/(a+b*cos(d*x+c)),x)
```

output
```
Integral((e*sin(c + d*x))**(3/2)/(a + b*cos(c + d*x)), x)
```

## Maxima [F]

$$\int \frac{(e\sin(c+dx))^{3/2}}{a+b\cos(c+dx)}\,dx = \int \frac{(e\sin(dx+c))^{\frac{3}{2}}}{b\cos(dx+c)+a}\,dx$$

input
```
integrate((e*sin(d*x+c))^(3/2)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((e*sin(d*x + c))^(3/2)/(b*cos(d*x + c) + a), x)
```

## Giac [F]

$$\int \frac{(e\sin(c+dx))^{3/2}}{a+b\cos(c+dx)}\,dx = \int \frac{(e\sin(dx+c))^{\frac{3}{2}}}{b\cos(dx+c)+a}\,dx$$

input
```
integrate((e*sin(d*x+c))^(3/2)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*sin(d*x + c))^(3/2)/(b*cos(d*x + c) + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{3/2}}{a+b\cos(c+dx)}\,dx = \int \frac{(e\,\sin(c+d\,x))^{3/2}}{a+b\,\cos(c+d\,x)}\,dx$$

input
```
int((e*sin(c + d*x))^(3/2)/(a + b*cos(c + d*x)),x)
```

output
```
int((e*sin(c + d*x))^(3/2)/(a + b*cos(c + d*x)), x)
```

## Reduce [F]

$$\int \frac{(e\sin(c+dx))^{3/2}}{a+b\cos(c+dx)}\,dx = \sqrt{e}\left(\int \frac{\sqrt{\sin(dx+c)}\,\sin(dx+c)}{\cos(dx+c)\,b+a}dx\right)e$$

input
```
int((e*sin(d*x+c))^(3/2)/(a+b*cos(d*x+c)),x)
```

output
```
sqrt(e)*int((sqrt(sin(c + d*x))*sin(c + d*x))/(cos(c + d*x)*b + a),x)*e
```

# 3.63 $\int \frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 302

$$\int \frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx = -\frac{\sqrt{e}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{\sqrt{b}\sqrt[4]{-a^2+b^2}d}+\frac{\sqrt{e}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{\sqrt{b}\sqrt[4]{-a^2+b^2}d} + \frac{ae\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} + \frac{ae\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}$$

output
```
-e^(1/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(
1/2)/(-a^2+b^2)^(1/4)/d+e^(1/2)*arctanh(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2
+b^2)^(1/4)/e^(1/2))/b^(1/2)/(-a^2+b^2)^(1/4)/d-a*e*EllipticPi(cos(1/2*c+1
/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b/(b-(-a
^2+b^2)^(1/2))/d/(e*sin(d*x+c))^(1/2)-a*e*EllipticPi(cos(1/2*c+1/4*Pi+1/2*
d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b/(b+(-a^2+b^2)^(1
/2))/d/(e*sin(d*x+c))^(1/2)
```

## Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 3.24 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx$$

$$= \frac{2\cos(c+dx)\left(a+b\sqrt{\cos^2(c+dx)}\right)\sqrt{e\sin(c+dx)}\left(\frac{\left(\frac{1}{8}+\frac{i}{8}\right)\left(2\arctan\left(1-\frac{(1+i)\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}}\right)-2\arctan\left(1+\frac{(1+i)\sqrt{b}\sqrt{s}}{\sqrt[4]{-a^2}}\right.\right.}{}\right.}{}$$

input

```
Integrate[Sqrt[e*Sin[c + d*x]]/(a + b*Cos[c + d*x]),x]
```

output

```
(2*Cos[c + d*x]*(a + b*Sqrt[Cos[c + d*x]^2])*Sqrt[e*Sin[c + d*x]]*(((1/8 +
 I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4
)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)]
 - Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c +
d*x]] + I*b*Sin[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 +
 b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)
^(1/4)) + (a*AppellF1[3/4, 1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^
2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2))/(3*(a^2 - b^2))))/(d*Sqrt[Cos[c + d*x
]^2]*(a + b*Cos[c + d*x])*Sqrt[Sin[c + d*x]])
```

## Rubi [A] (warning: unable to verify)

Time = 1.08 (sec) , antiderivative size = 297, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3180, 266, 827, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{e \cos \left(c+dx-\frac{\pi}{2}\right)}}{a-b \sin \left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \textbf{3180}$$

$$-\frac{be\int \frac{\sqrt{e\sin(c+dx)}}{b^2\sin^2(c+dx)e^2+(a^2-b^2)e^2}d(e\sin(c+dx))}{d}-\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b}$$

$$\Big\downarrow \textbf{266}$$

$$-\frac{2be\int \frac{e^2\sin^2(c+dx)}{b^2e^4\sin^4(c+dx)+(a^2-b^2)e^2}d\sqrt{e\sin(c+dx)}}{d}-\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b}$$

$$\Big\downarrow \textbf{827}$$

$$-\frac{2be\left(\frac{\int \frac{1}{be^2\sin^2(c+dx)+\sqrt{b^2-a^2}e}d\sqrt{e\sin(c+dx)}}{2b}-\frac{\int \frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2b}\right)}{d}-\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b}$$

$$\Big\downarrow \textbf{218}$$

$$-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\int \frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2b}\right)}{d}-\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b}$$

$$\Big\downarrow \textbf{221}$$

$$-\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b}-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b}-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{d}$$

$$\Big\downarrow \text{3286}$$

$$-\frac{ae\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b\sqrt{e\sin(c+dx)}}+\frac{ae\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b\sqrt{e\sin(c+dx)}}-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{ae\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b\sqrt{e\sin(c+dx)}}+\frac{ae\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b\sqrt{e\sin(c+dx)}}-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{d}$$

$$\Big\downarrow \text{3284}$$

$$-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{d}+\frac{ae\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\sin(c+dx)}}+\frac{ae\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\left(\sqrt{b^2-a^2}+b\right)\sqrt{e\sin(c+dx)}}$$

input
```
Int[Sqrt[e*Sin[c + d*x]]/(a + b*Cos[c + d*x]),x]
```

output
```
(-2*b*e*(ArcTan[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/(2*b^(3
/2)*(-a^2 + b^2)^(1/4)*Sqrt[e]) - ArcTanh[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(
-a^2 + b^2)^(1/4)]/(2*b^(3/2)*(-a^2 + b^2)^(1/4)*Sqrt[e])))/d + (a*e*Ellip
ticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*
x]])/(b*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]]) + (a*e*EllipticPi[(
2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(b
*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]])
```

### Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3180
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[a*(g/(2*b))   Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Simp[a*(g/(2*b))   In
t[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Simp[b*(g/f)   Su
bst[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

## Maple [A] (verified)

Time = 1.67 (sec) , antiderivative size = 496, normalized size of antiderivative = 1.64

| method | result |
|---|---|
| default | $-\frac{e\sqrt{2}\left(\ln\left(\frac{e\sin(dx+c)-\left(\frac{e^2(a^2-b^2)}{b^2}\right)^{\frac{1}{4}}\sqrt{e\sin(dx+c)}\,\sqrt{2}+\sqrt{\frac{e^2(a^2-b^2)}{b^2}}}{e\sin(dx+c)+\left(\frac{e^2(a^2-b^2)}{b^2}\right)^{\frac{1}{4}}\sqrt{e\sin(dx+c)}\,\sqrt{2}+\sqrt{\frac{e^2(a^2-b^2)}{b^2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\sin(dx+c)}}{\left(\frac{e^2(a^2-b^2)}{b^2}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\sin(dx+c)}}{\left(\frac{e^2(a^2-b^2)}{b^2}\right)^{\frac{1}{4}}}\right.\right.}{4b\left(\frac{e^2(a^2-b^2)}{b^2}\right)^{\frac{1}{4}}}$ |

input
```
int((e*sin(d*x+c))^(1/2)/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)
```

output
```
(-1/4/b*e/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*(ln((e*sin(d*x+c)-(e^2*(a^2-b^
2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*s
in(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2
-b^2)/b^2)^(1/2)))+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c
))^(1/2)+1)+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2
)-1))+1/2*a*e*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)
/b*(EllipticPi((1-sin(d*x+c))^(1/2),-b/(-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*
(-a^2+b^2)^(1/2)-EllipticPi((1-sin(d*x+c))^(1/2),b/(b+(-a^2+b^2)^(1/2)),1/
2*2^(1/2))*(-a^2+b^2)^(1/2)+EllipticPi((1-sin(d*x+c))^(1/2),-b/(-b+(-a^2+b
^2)^(1/2)),1/2*2^(1/2))*b+EllipticPi((1-sin(d*x+c))^(1/2),b/(b+(-a^2+b^2)^
(1/2)),1/2*2^(1/2))*b)/(-b+(-a^2+b^2)^(1/2))/(b+(-a^2+b^2)^(1/2))/cos(d*x+
c)/(e*sin(d*x+c))^(1/2))/d
```

## Fricas [F]

$$\int \frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx=\int \frac{\sqrt{e\sin\left(dx+c\right)}}{b\cos\left(dx+c\right)+a}\,dx$$

input
```
integrate((e*sin(d*x+c))^(1/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

output
```
integral(sqrt(e*sin(d*x + c))/(b*cos(d*x + c) + a), x)
```

## Sympy [F]

$$\int \frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx = \int \frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx$$

input
```
integrate((e*sin(d*x+c))**(1/2)/(a+b*cos(d*x+c)),x)
```

output
```
Integral(sqrt(e*sin(c + d*x))/(a + b*cos(c + d*x)), x)
```

## Maxima [F]

$$\int \frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx = \int \frac{\sqrt{e\sin(dx+c)}}{b\cos(dx+c)+a}\,dx$$

input
```
integrate((e*sin(d*x+c))^(1/2)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*sin(d*x + c))/(b*cos(d*x + c) + a), x)
```

## Giac [F]

$$\int \frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx = \int \frac{\sqrt{e\sin(dx+c)}}{b\cos(dx+c)+a}\,dx$$

input
```
integrate((e*sin(d*x+c))^(1/2)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sqrt(e*sin(d*x + c))/(b*cos(d*x + c) + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx=\int \frac{\sqrt{e\,\sin\left(c+d\,x\right)}}{a+b\,\cos\left(c+d\,x\right)}\,dx$$

input
```
int((e*sin(c + d*x))^(1/2)/(a + b*cos(c + d*x)),x)
```

output
```
int((e*sin(c + d*x))^(1/2)/(a + b*cos(c + d*x)), x)
```

## Reduce [F]

$$\int \frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx=\sqrt{e}\left(\int \frac{\sqrt{\sin\left(dx+c\right)}}{\cos\left(dx+c\right)b+a}dx\right)$$

input
```
int((e*sin(d*x+c))^(1/2)/(a+b*cos(d*x+c)),x)
```

output
```
sqrt(e)*int(sqrt(sin(c + d*x))/(cos(c + d*x)*b + a),x)
```

# 3.64 $\int \frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 307

$$\int \frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx = \frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{(-a^2+b^2)^{3/4}\,d\sqrt{e}} + \frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{(-a^2+b^2)^{3/4}\,d\sqrt{e}} + \frac{a\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}} + \frac{a\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{\left(a^2-b\left(b+\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}}$$

output

```
b^(1/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a^
2+b^2)^(3/4)/d/e^(1/2)+b^(1/2)*arctanh(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+
b^2)^(1/4)/e^(1/2))/(-a^2+b^2)^(3/4)/d/e^(1/2)-a*EllipticPi(cos(1/2*c+1/4*
Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b*(b-(
-a^2+b^2)^(1/2)))/d/(e*sin(d*x+c))^(1/2)-a*EllipticPi(cos(1/2*c+1/4*Pi+1/2
*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b*(b+(-a^2+b
^2)^(1/2)))/d/(e*sin(d*x+c))^(1/2)
```

## Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 3.64 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.85

$$\int \frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx$$

$$= \frac{10(a+b)\,\text{Appel}}{de(a+b\cos(c+dx))\left(5(a+b)\,\text{AppellF1}\left(\frac{1}{4},-\frac{1}{2},1,\frac{5}{4},-\tan^2\left(\frac{1}{2}(c+dx)\right),\frac{(-a+b)\tan^2\left(\frac{1}{2}(c+dx)\right)}{a+b}\right)+2\left(-2(\right.\right.}$$

input
```
Integrate[1/((a + b*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]]),x]
```

output
```
(10*(a + b)*AppellF1[1/4, -1/2, 1, 5/4, -Tan[(c + d*x)/2]^2, ((-a + b)*Tan
[(c + d*x)/2]^2)/(a + b)]*Sqrt[e*Sin[c + d*x]])/(d*e*(a + b*Cos[c + d*x])*
(5*(a + b)*AppellF1[1/4, -1/2, 1, 5/4, -Tan[(c + d*x)/2]^2, ((-a + b)*Tan[
(c + d*x)/2]^2)/(a + b)] + 2*(-2*(a - b)*AppellF1[5/4, -1/2, 2, 9/4, -Tan[
(c + d*x)/2]^2, ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*AppellF1[
5/4, 1/2, 1, 9/4, -Tan[(c + d*x)/2]^2, ((-a + b)*Tan[(c + d*x)/2]^2)/(a +
b)])*Tan[(c + d*x)/2]^2))
```

## Rubi [A] (warning: unable to verify)

Time = 1.18 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3181, 266, 756, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \ 3181$$

$$-\frac{be\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b^2\sin^2(c+dx)e^2+\left(a^2-b^2\right)e^2\right)}d(e\sin(c+dx))}{d}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}}$$

$$\Big\downarrow \ 266$$

$$-\frac{2be\int \frac{1}{b^2e^4\sin^4(c+dx)+\left(a^2-b^2\right)e^2}d\sqrt{e\sin(c+dx)}}{d}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}}$$

$$\Big\downarrow \ 756$$

$$-\frac{2be\left(-\frac{\int \frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2e\sqrt{b^2-a^2}}-\frac{\int \frac{1}{be^2\sin^2(c+dx)+\sqrt{b^2-a^2}e}d\sqrt{e\sin(c+dx)}}{2e\sqrt{b^2-a^2}}\right)}{d}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}}$$

$$\Big\downarrow \ 218$$

$$-\frac{2be\left(-\frac{\int \frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2e\sqrt{b^2-a^2}}-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}\right)}{d}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}}$$

$$\Big\downarrow \ 221$$

$$-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}}-\frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}\right)}{d}$$

$$\Big\downarrow \ 3042$$

$$-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}}-\frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}(b^2-a^2)^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}(b^2-a^2)^{3/4}}\right)}{d}$$

$$\Big\downarrow \ 3286$$

$$-\frac{a\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}}-\frac{a\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}}-\frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}(b^2-a^2)^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}(b^2-a^2)^{3/4}}\right)}{d}$$

$$\Big\downarrow \ 3042$$

$$-\frac{a\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}}-\frac{a\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}}-\frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}(b^2-a^2)^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}(b^2-a^2)^{3/4}}\right)}{d}$$

$$\Big\downarrow \ 3284$$

$$-\frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}(b^2-a^2)^{3/4}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}(b^2-a^2)^{3/4}}\right)}{d}+\frac{a\sqrt{\sin(c+dx)}\operatorname{EllipticPi}\left(\frac{2b}{b-\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{b^2-a^2}\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\sin(c+dx)}}-\frac{a\sqrt{\sin(c+dx)}\operatorname{EllipticPi}\left(\frac{2b}{b+\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{b^2-a^2}\left(\sqrt{b^2-a^2}+b\right)\sqrt{e\sin(c+dx)}}$$

input
```
Int[1/((a + b*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]]),x]
```

output
```
(-2*b*e*(-1/2*ArcTan[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/(S
qrt[b]*(-a^2 + b^2)^(3/4)*e^(3/2)) - ArcTanh[(Sqrt[b]*Sqrt[e]*Sin[c + d*x]
)/(-a^2 + b^2)^(1/4)]/(2*Sqrt[b]*(-a^2 + b^2)^(3/4)*e^(3/2))))/d + (a*Elli
pticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d
*x]])/(Sqrt[-a^2 + b^2]*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]]) - (
a*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin
[c + d*x]])/(Sqrt[-a^2 + b^2]*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]
])
```

## Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3181
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[-a/(2*q)   Int[1/(
Sqrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Simp[b*(g/f)   Subst[
Int[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - S
imp[a/(2*q)   Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

## Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 519, normalized size of antiderivative = 1.69

| method | result |
|---|---|
| default | $-\frac{b e \left(\frac{e^{2}\left(a^{2}-b^{2}\right)}{b^{2}}\right)^{\frac{1}{4}} \sqrt{2}\left(\ln\left(\frac{e \sin(dx+c)+\left(\frac{e^{2}\left(a^{2}-b^{2}\right)}{b^{2}}\right)^{\frac{1}{4}} \sqrt{e \sin(dx+c)}\,\sqrt{2}+\sqrt{\frac{e^{2}\left(a^{2}-b^{2}\right)}{b^{2}}}}{e \sin(dx+c)-\left(\frac{e^{2}\left(a^{2}-b^{2}\right)}{b^{2}}\right)^{\frac{1}{4}} \sqrt{e \sin(dx+c)}\,\sqrt{2}+\sqrt{\frac{e^{2}\left(a^{2}-b^{2}\right)}{b^{2}}}}\right)+2 \arctan\left(\frac{\sqrt{2}\,\sqrt{e \sin(dx+c)}}{\left(\frac{e^{2}\left(a^{2}-b^{2}\right)}{b^{2}}\right)^{\frac{1}{4}}}+1\right)+2 \arctan\left(\right.}{4\left(a^{2} e^{2}-b^{2} e^{2}\right)}$ |

input
```
int(1/(a+cos(d*x+c)*b)/(e*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-1/4*b*e*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*(ln((e*sin(d
*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2
)/b^2)^(1/2))/(e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)
*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^
(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(
e*sin(d*x+c))^(1/2)-1))+1/2*a*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*
sin(d*x+c)^(1/2)*(EllipticPi((1-sin(d*x+c))^(1/2),-b/(-b+(-a^2+b^2)^(1/2))
,1/2*2^(1/2))*(-a^2+b^2)^(1/2)+EllipticPi((1-sin(d*x+c))^(1/2),-b/(-b+(-a^
2+b^2)^(1/2)),1/2*2^(1/2))*b+EllipticPi((1-sin(d*x+c))^(1/2),b/(b+(-a^2+b^
2)^(1/2)),1/2*2^(1/2))*(-a^2+b^2)^(1/2)-EllipticPi((1-sin(d*x+c))^(1/2),b/
(b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*b)/(-a^2+b^2)^(1/2)/(-b+(-a^2+b^2)^(1/2)
)/(b+(-a^2+b^2)^(1/2))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

## Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

## Sympy [F]

$$\int \frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx = \int \frac{1}{\sqrt{e\sin(c+dx)}\,(a+b\cos(c+dx))}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(e*sin(c + d*x))*(a + b*cos(c + d*x))), x)
```

## Maxima [F]

$$\int \frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx = \int \frac{1}{(b\cos(dx+c)+a)\sqrt{e\sin(dx+c)}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*cos(d*x + c) + a)*sqrt(e*sin(d*x + c))), x)
```

## Giac [F]

$$\int \frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx = \int \frac{1}{(b\cos(dx+c)+a)\sqrt{e\sin(dx+c)}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c) + a)*sqrt(e*sin(d*x + c))), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx = \int \frac{1}{\sqrt{e\,\sin\left(c+d\,x\right)}\,\left(a+b\,\cos\left(c+d\,x\right)\right)}\,dx$$

input
```
int(1/((e*sin(c + d*x))^(1/2)*(a + b*cos(c + d*x))),x)
```

output
```
int(1/((e*sin(c + d*x))^(1/2)*(a + b*cos(c + d*x))), x)
```

## Reduce [F]

$$\int \frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx = \frac{\sqrt{e}\left(\int \frac{\sqrt{\sin(dx+c)}}{\cos(dx+c)\sin(dx+c)b+\sin(dx+c)a}dx\right)}{e}$$

input
```
int(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(1/2),x)
```

output
```
(sqrt(e)*int(sqrt(sin(c + d*x))/(cos(c + d*x)*sin(c + d*x)*b + sin(c + d*x
)*a),x))/e
```

# 3.65 $\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 426

$$\begin{aligned}\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}\,dx &= -\frac{b^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{(-a^2+b^2)^{5/4}\,de^{3/2}}\\
&+\frac{b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{(-a^2+b^2)^{5/4}\,de^{3/2}}+\frac{2(b-a\cos(c+dx))}{(a^2-b^2)\,de\sqrt{e\sin(c+dx)}}\\
&-\frac{ab\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{(a^2-b^2)\left(b-\sqrt{-a^2+b^2}\right)de\sqrt{e\sin(c+dx)}}\\
&-\frac{ab\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{(a^2-b^2)\left(b+\sqrt{-a^2+b^2}\right)de\sqrt{e\sin(c+dx)}}\\
&-\frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{(a^2-b^2)\,de^2\sqrt{\sin(c+dx)}}\end{aligned}$$

output
```
-b^(3/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a
^2+b^2)^(5/4)/d/e^(3/2)+b^(3/2)*arctanh(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2
+b^2)^(1/4)/e^(1/2))/(-a^2+b^2)^(5/4)/d/e^(3/2)+2*(b-a*cos(d*x+c))/(a^2-b^
2)/d/e/(e*sin(d*x+c))^(1/2)+a*b*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(
b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)/(b-(-a^2+b^2)^(1/2
))/d/e/(e*sin(d*x+c))^(1/2)+a*b*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(
b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)/(b+(-a^2+b^2)^(1/2
))/d/e/(e*sin(d*x+c))^(1/2)+2*a*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2
))*(e*sin(d*x+c))^(1/2)/(a^2-b^2)/d/e^2/sin(d*x+c)^(1/2)
```

## Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 15.98 (sec) , antiderivative size = 791, normalized size of antiderivative = 1.86

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}\,dx = -\frac{2(-b+a\cos(c+dx))\sin(c+dx)}{(a^2-b^2)\,d(e\sin(c+dx))^{3/2}}$$

$$-\frac{\sin^{\frac{3}{2}}(c+dx)\left(\frac{a\cos^2(c+dx)\left(3\sqrt{2}a\left(a^2-b^2\right)^{3/4}\left(2\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-2\arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-\log\left(\sqrt{a^2-b^2}-\sqrt{2}\sqrt{b}\right.\right.\right.}{}\right.}{}$$

input
```
Integrate[1/((a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2)),x]
```

output
```
(-2*(-b + a*Cos[c + d*x])*Sin[c + d*x])/((a^2 - b^2)*d*(e*Sin[c + d*x])^(3
/2)) - (Sin[c + d*x]^(3/2)*((a*Cos[c + d*x]^2*(3*Sqrt[2]*a*(a^2 - b^2)^(3/
4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] -
 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Lo
g[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] +
 b*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)
*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]]) + 8*b^(5/2)*AppellF1[3/4, -1/2, 1,
7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2)
)*(a + b*Sqrt[1 - Sin[c + d*x]^2]))/(12*Sqrt[b]*(-a^2 + b^2)*(a + b*Cos[c
+ d*x])*(1 - Sin[c + d*x]^2)) + (2*(a^2 + b^2)*Cos[c + d*x]*(((1/8 + I/8)*
(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2
*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log
[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]]
+ I*b*Sin[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^
(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)
) + (a*AppellF1[3/4, 1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a
^2 + b^2)]*Sin[c + d*x]^(3/2))/(3*(a^2 - b^2)))*(a + b*Sqrt[1 - Sin[c + d*
x]^2]))/((a + b*Cos[c + d*x])*Sqrt[1 - Sin[c + d*x]^2])))/((a - b)*(a + b)
*d*(e*Sin[c + d*x])^(3/2))
```

## Rubi [A] (warning: unable to verify)

Time = 1.87 (sec) , antiderivative size = 405, normalized size of antiderivative = 0.95, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.720$, Rules used = {3042, 3175, 27, 3042, 3346, 3042, 3121, 3042, 3119, 3180, 266, 827, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{ 3175}$$

$$\frac{2(b-a\cos(c+dx))}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{2\int \frac{\left(a^2+b\cos(c+dx)a+b^2\right)\sqrt{e\sin(c+dx)}}{2(a+b\cos(c+dx))}dx}{e^2\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{2(b-a\cos(c+dx))}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{\int \frac{\left(a^2+b\cos(c+dx)a+b^2\right)\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx}{e^2\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2(b-a\cos(c+dx))}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{\int \frac{\sqrt{-e\cos\left(c+dx+\frac{\pi}{2}\right)}\left(a^2+b\sin\left(c+dx+\frac{\pi}{2}\right)a+b^2\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{e^2\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3346}$$

$$\frac{2(b-a\cos(c+dx))}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{b^2\int \frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx+a\int \sqrt{e\sin(c+dx)}dx}{e^2\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2(b-a\cos(c+dx))}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{b^2\int \frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx+a\int \sqrt{e\sin(c+dx)}dx}{e^2\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3121}$$

$$\frac{2(b-a\cos(c+dx))}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{b^2\int \frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx+\frac{a\sqrt{e\sin(c+dx)}\int \sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}}{e^2\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2(b-a\cos(c+dx))}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{b^2\int \frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx+\frac{a\sqrt{e\sin(c+dx)}\int \sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}}{e^2\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2(b-a\cos(c+dx))}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{b^2\int \frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx+\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}}{e^2\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3180}$$

$$\frac{2(b-a\cos(c+dx))}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{b^2\left(-\frac{be\int\frac{\sqrt{e\sin(c+dx)}}{b^2\sin^2(c+dx)e^2+\left(a^2-b^2\right)e^2}d(e\sin(c+dx))}{d}-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}}{2b}\right.}{e^2\left(a^2-b^2\right)}$$

$\downarrow$ 266

$$\frac{2(b-a\cos(c+dx))}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{b^2\left(-\frac{2be\int\frac{e^2\sin^2(c+dx)}{b^2e^4\sin^4(c+dx)+\left(a^2-b^2\right)e^2}d\sqrt{e\sin(c+dx)}}{d}-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right.}}{2b}\right.}{e^2\left(a^2-b^2\right)}$$

$\downarrow$ 827

$$\frac{2(b-a\cos(c+dx))}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{b^2\left(-\frac{2be\left(\frac{\int\frac{1}{be^2\sin^2(c+dx)+\sqrt{b^2-a^2}e}d\sqrt{e\sin(c+dx)}}{2b}-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2b}\right)}{d}-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}\right.}{e^2\left(a^2-b^2\right)}$$

$\downarrow$ 218

$$\frac{2(b-a\cos(c+dx))}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{b^2\left(-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2b}\right)}{d}-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int\frac{}{\sqrt{e\sin(}}}{}\right.}{e^2\left(a^2-b^2\right)}$$

$\downarrow$ 221

$$\frac{2(b-a\cos(c+dx))}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-$$

$$\frac{b^2\left(-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b}-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\text{arctanh}\left(\cdots\right.}{2b^{3/2}\sqrt{e}\cdots}\cdots\right.}{d}\cdots\right.}{e^2\left(a^2-b^2\right)\cdots}$$

$$\Big\downarrow \; 3042$$

$$\frac{2(b-a\cos(c+dx))}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-$$

$$\frac{b^2\left(-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b}-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\text{arctanh}\left(\cdots\right.}{2b^{3/2}\sqrt{e}\cdots}\cdots\right.}{d}\cdots\right.}{e^2\left(a^2-b^2\right)\cdots}$$

$$\Big\downarrow \; 3286$$

$$\frac{2(b-a\cos(c+dx))}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-$$

$$\frac{b^2\left(-\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b\sqrt{e\sin(c+dx)}}+\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b\sqrt{e\sin(c+dx)}}-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(\cdots}{\sqrt[4]{b^2-\cdots}}\right.}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2\cdots}}\cdots\right.}{\cdots}\cdots\right.}{e^2\left(a^2-b^2\right)\cdots}$$

$$\Big\downarrow \; 3042$$

$$\frac{2(b-a\cos(c+dx))}{de\,(a^2-b^2)\sqrt{e\sin(c+dx)}}-$$

$$\frac{b^2\left(-\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b\sqrt{e\sin(c+dx)}}+\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b\sqrt{e\sin(c+dx)}}-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(\cdots}{\sqrt[4]{b^2-\cdots}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2\cdots}}\cdots\right.}{\cdots}\cdots\right.}{e^2\,(a^2-b^2)}$$

$\Big\downarrow$ 3284

$$\frac{2(b-a\cos(c+dx))}{de\,(a^2-b^2)\sqrt{e\sin(c+dx)}}-$$

$$\frac{b^2\left(-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{d}+\frac{ae\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\sin(c+dx)}}+\frac{ae\sqrt{\sin(c\cdots}}{\cdots}\cdots\right.}{e^2\,(a^2-b^2)}$$

input
```
Int[1/((a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2)),x]
```

output
```
(2*(b - a*Cos[c + d*x]))/((a^2 - b^2)*d*e*Sqrt[e*Sin[c + d*x]]) - ((2*a*El
lipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(d*Sqrt[Sin[c + d*x]]
) + b^2*((-2*b*e*(ArcTan[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)
]/(2*b^(3/2)*(-a^2 + b^2)^(1/4)*Sqrt[e]) - ArcTanh[(Sqrt[b]*Sqrt[e]*Sin[c
+ d*x])/(-a^2 + b^2)^(1/4)]/(2*b^(3/2)*(-a^2 + b^2)^(1/4)*Sqrt[e])))/d + (
a*e*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[S
in[c + d*x]])/(b*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]]) + (a*e*Ell
ipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c +
d*x]])/(b*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]])))/((a^2 - b^2)*e^
2)
```

**Defintions of rubi rules used**

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3175
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Simp[1/(g^2*
(a^2 - b^2)*(p + 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*
(a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; F
reeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ
[2*m, 2*p]
```

rule 3180
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[a*(g/(2*b))   Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Simp[a*(g/(2*b))   In
t[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Simp[b*(g/f)   Su
bst[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 773 vs. $2(374) = 748$.

Time = 1.97 (sec) , antiderivative size = 774, normalized size of antiderivative = 1.82

| method | result |
|---|---|
| default | $-be\left(-\frac{\sqrt{2}\left(\ln\left(\frac{e\sin(dx+c)-\left(\frac{e^2(a^2-b^2)}{b^2}\right)^{\frac{1}{4}}\sqrt{e\sin(dx+c)}\,\sqrt{2}+\sqrt{\frac{e^2(a^2-b^2)}{b^2}}}{e\sin(dx+c)+\left(\frac{e^2(a^2-b^2)}{b^2}\right)^{\frac{1}{4}}\sqrt{e\sin(dx+c)}\,\sqrt{2}+\sqrt{\frac{e^2(a^2-b^2)}{b^2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\sin(dx+c)}}{\left(\frac{e^2(a^2-b^2)}{b^2}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\sin(dx}{\left(\frac{e^2(a^2-b^2)}{b^2}\right.}\right.\right.}{4e^2(a-b)(a+b)\left(\frac{e^2(a^2-b^2)}{b^2}\right)^{\frac{1}{4}}}\right.$ |

input
```
int(1/(a+cos(d*x+c)*b)/(e*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-b*e*(-1/4/e^2/(a-b)/(a+b)/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*(ln((e*sin(d
*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2
)/b^2)^(1/2))/(e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)
*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^
(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(
e*sin(d*x+c))^(1/2)-1))-2/e^2/(a^2-b^2)/(e*sin(d*x+c))^(1/2))-1/2*(4*(1-si
n(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticE((1-sin(d
*x+c))^(1/2),1/2*2^(1/2))*a^2-2*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2
)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2+(1-sin(
d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticPi((1-sin(d*
x+c))^(1/2),-b/(-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b+(1-si
n(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticPi((1-sin(
d*x+c))^(1/2),-b/(-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*b^2-(1-sin(d*x+c))^(1/
2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticPi((1-sin(d*x+c))^(1/2)
,b/(b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b+(1-sin(d*x+c))^(1/
2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticPi((1-sin(d*x+c))^(1/2)
,b/(b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*b^2-4*a^2*cos(d*x+c)^2)*a/e/(b+(-a^2+
b^2)^(1/2))/(-b+(-a^2+b^2)^(1/2))/(a+b)/(a-b)/cos(d*x+c)/(e*sin(d*x+c))^(1
/2))/d
```

## Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

## Sympy [F]

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}\,dx = \int \frac{1}{(e\sin{(c+dx)})^{\frac{3}{2}}\,(a+b\cos{(c+dx)})}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))**(3/2),x)
```

output
```
Integral(1/((e*sin(c + d*x))**(3/2)*(a + b*cos(c + d*x))), x)
```

## Maxima [F]

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\cos{(dx+c)}+a)\,(e\sin{(dx+c)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*cos(d*x + c) + a)*(e*sin(d*x + c))^(3/2)), x)
```

## Giac [F]

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\cos(dx+c)+a)\,(e\sin(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c) + a)*(e*sin(d*x + c))^(3/2)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}\,dx = \int \frac{1}{(e\,\sin(c+d\,x))^{3/2}\,(a+b\,\cos(c+d\,x))}\,dx$$

input
```
int(1/((e*sin(c + d*x))^(3/2)*(a + b*cos(c + d*x))),x)
```

output
```
int(1/((e*sin(c + d*x))^(3/2)*(a + b*cos(c + d*x))), x)
```

## Reduce [F]

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}\,dx = \frac{\sqrt{e}\left(\int \frac{\sqrt{\sin(dx+c)}}{\cos(dx+c)\sin(dx+c)^2 b+\sin(dx+c)^2 a}dx\right)}{e^2}$$

input
```
int(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(3/2),x)
```

output
```
(sqrt(e)*int(sqrt(sin(c + d*x))/(cos(c + d*x)*sin(c + d*x)**2*b + sin(c +
d*x)**2*a),x))/e**2
```

# 3.66 $\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 447

$$\begin{aligned}
\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}\,dx &= \frac{b^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{(-a^2+b^2)^{7/4}\,de^{5/2}} \\
&+ \frac{b^{5/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{(-a^2+b^2)^{7/4}\,de^{5/2}} + \frac{2(b-a\cos(c+dx))}{3(a^2-b^2)\,de(e\sin(c+dx))^{3/2}} \\
&+ \frac{2a\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3(a^2-b^2)\,de^2\sqrt{e\sin(c+dx)}} \\
&- \frac{ab^2\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{(a^2-b^2)\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)de^2\sqrt{e\sin(c+dx)}} \\
&- \frac{ab^2\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{(a^2-b^2)\left(a^2-b\left(b+\sqrt{-a^2+b^2}\right)\right)de^2\sqrt{e\sin(c+dx)}}
\end{aligned}$$

output
```
b^(5/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a^
2+b^2)^(7/4)/d/e^(5/2)+b^(5/2)*arctanh(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+
b^2)^(1/4)/e^(1/2))/(-a^2+b^2)^(7/4)/d/e^(5/2)+2/3*(b-a*cos(d*x+c))/(a^2-b
^2)/d/e/(e*sin(d*x+c))^(3/2)+2/3*a*InverseJacobiAM(1/2*c-1/4*Pi+1/2*d*x,2^
(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)/d/e^2/(e*sin(d*x+c))^(1/2)+a*b^2*Ellipti
cPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)
^(1/2)/(a^2-b^2)/(a^2-b*(b-(-a^2+b^2)^(1/2)))/d/e^2/(e*sin(d*x+c))^(1/2)+a
*b^2*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2)
)*sin(d*x+c)^(1/2)/(a^2-b^2)/(a^2-b*(b+(-a^2+b^2)^(1/2)))/d/e^2/(e*sin(d*x
+c))^(1/2)
```

## Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 13.67 (sec) , antiderivative size = 1192, normalized size of antiderivative = 2.67

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[1/((a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(5/2)),x]
```

output
```
(-2*(-b + a*Cos[c + d*x])*Sin[c + d*x])/(3*(a^2 - b^2)*d*(e*Sin[c + d*x])^
(5/2)) + (Sin[c + d*x]^(5/2)*((2*a*b*Cos[c + d*x]^2*(a + b*Sqrt[1 - Sin[c
+ d*x]^2])*((a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 -
b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2
)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Si
n[c + d*x]] + b*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2
 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a
^2 - b^2)^(3/4)) + (5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*
x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Sin[c + d*x]]*Sqrt[1 - Sin[c
 + d*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (
b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, S
in[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5
/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)])*Sin[c
 + d*x]^2)*(a^2 + b^2*(-1 + Sin[c + d*x]^2)))))/((a + b*Cos[c + d*x])*(1 -
 Sin[c + d*x]^2)) + (2*(a^2 - 3*b^2)*Cos[c + d*x]*(a + b*Sqrt[1 - Sin[c +
d*x]^2])*(((-1/8 + I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c
+ d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c +
d*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2
+ b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]] - Log[Sqrt[-a^2 + b^2]
 + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + ...
```

## Rubi [A] (warning: unable to verify)

Time = 1.88 (sec) , antiderivative size = 429, normalized size of antiderivative = 0.96, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.720$, Rules used = {3042, 3175, 27, 3042, 3346, 3042, 3121, 3042, 3120, 3181, 266, 756, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{ 3175}$$

$$\frac{2(b-a\cos(c+dx))}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}-\frac{2\int -\frac{a^2+b\cos(c+dx)a-3b^2}{2(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{3e^2\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{a^2+b\cos(c+dx)a-3b^2}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{3e^2\left(a^2-b^2\right)}+\frac{2(b-a\cos(c+dx))}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{a^2-b\sin\left(c+dx-\frac{\pi}{2}\right)a-3b^2}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{3e^2\left(a^2-b^2\right)}+\frac{2(b-a\cos(c+dx))}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 3346}$$

$$\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}}dx-3b^2\int \frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{3e^2\left(a^2-b^2\right)}+\frac{2(b-a\cos(c+dx))}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}}dx-3b^2\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{3e^2\left(a^2-b^2\right)}+\frac{2(b-a\cos(c+dx))}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 3121}$$

$$\frac{\frac{a\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}}dx}{\sqrt{e\sin(c+dx)}}-3b^2\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{3e^2\left(a^2-b^2\right)}+\frac{2(b-a\cos(c+dx))}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{a\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}}dx}{\sqrt{e\sin(c+dx)}}-3b^2\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{3e^2\left(a^2-b^2\right)}+\frac{2(b-a\cos(c+dx))}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 3120}$$

$$\frac{\frac{2a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-3b^2\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{3e^2\left(a^2-b^2\right)}+\frac{2(b-a\cos(c+dx))}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}$$

$\Big\downarrow$ 3181

$$\frac{\frac{2a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-3b^2\left(-\frac{be\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b^2\sin^2(c+dx)e^2+\left(a^2-b^2\right)e^2\right)}d(e\sin(c+dx))}{d}-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}\right.}}{2\sqrt{b^2-a^2}}\right.}{3e^2\left(a^2-b^2\right)}$$

$$\frac{2(b-a\cos(c+dx))}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}$$

$\Big\downarrow$ 266

$$\frac{\frac{2a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-3b^2\left(-\frac{2be\int\frac{1}{b^2e^4\sin^4(c+dx)+\left(a^2-b^2\right)e^2}d\sqrt{e\sin(c+dx)}}{d}-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}}{2\sqrt{b^2-a^2}}\right.}{3e^2\left(a^2-b^2\right)}$$

$$\frac{2(b-a\cos(c+dx))}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}$$

$\Big\downarrow$ 756

$$\frac{\frac{2a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-3b^2\left(-\frac{2be\left(-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2e\sqrt{b^2-a^2}}-\frac{\int\frac{1}{be^2\sin^2(c+dx)+\sqrt{b^2-a^2}e}d\sqrt{e\sin(c+dx)}}{2e\sqrt{b^2-a^2}}\right)}{d}\right.}{3e^2\left(a^2-b^2\right)}$$

$$\frac{2(b-a\cos(c+dx))}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}$$

$\Big\downarrow$ 218

$$\frac{\frac{2a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-3b^2\left(-\frac{2be\left(-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2e\sqrt{b^2-a^2}}-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}\right)}{d}-\frac{a\int\frac{}{\sqrt{e\sin}}}{}\right.}{3e^2\left(a^2-b^2\right)}$$

$$\frac{2(b-a\cos(c+dx))}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}$$

$\Big\downarrow$ 221

$$\frac{\frac{2a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-3b^2\left(-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}}-\right.}{3e^2\left(a^2-b^2\right)}$$

$$\frac{2(b-a\cos(c+dx))}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-3b^2\left(-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}}-\right.}{3e^2\left(a^2-b^2\right)}$$

$$\frac{2(b-a\cos(c+dx))}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3286}$$

$$\frac{\frac{2a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-3b^2\left(-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}}-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(\right.}}{2\sqrt{b^2-a^2}\sqrt{e\sin(c}}\right.}{3e^2\left(a^2-b^2\right)}$$

$$\frac{2(b-a\cos(c+dx))}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-3b^2\left(-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}}-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(\right.}}{2\sqrt{b^2-a^2}\sqrt{e\sin(c-}}\right.}{3e^2\left(a^2-b^2\right)}$$

$$\frac{2(b-a\cos(c+dx))}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}$$

$\big\downarrow$ 3284

$$\frac{\frac{2a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-3b^2\left(-\frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}\right)}{d}+\frac{a\sqrt{\sin(c+dx)}\,\text{Ellipt}}{d\sqrt{b^2-a^2}\left(b\right.}\right.}{3e^2\left(a^2-b^2\right)}$$

$$\frac{2(b-a\cos(c+dx))}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}$$

input
```
Int[1/((a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(5/2)),x]
```

output
```
(2*(b - a*Cos[c + d*x]))/(3*(a^2 - b^2)*d*e*(e*Sin[c + d*x])^(3/2)) + ((2*
a*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(d*Sqrt[e*Sin[c + d
*x]]) - 3*b^2*((-2*b*e*(-1/2*ArcTan[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(-a^2 +
 b^2)^(1/4)]/(Sqrt[b]*(-a^2 + b^2)^(3/4)*e^(3/2)) - ArcTanh[(Sqrt[b]*Sqrt[
e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/(2*Sqrt[b]*(-a^2 + b^2)^(3/4)*e^(3/2)
)))/d + (a*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]
*Sqrt[Sin[c + d*x]])/(Sqrt[-a^2 + b^2]*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin
[c + d*x]]) - (a*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)
/2, 2]*Sqrt[Sin[c + d*x]])/(Sqrt[-a^2 + b^2]*(b + Sqrt[-a^2 + b^2])*d*Sqrt
[e*Sin[c + d*x]])))/(3*(a^2 - b^2)*e^2)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3175
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Simp[1/(g^2*
(a^2 - b^2)*(p + 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*
(a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; F
reeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ
[2*m, 2*p]
```

rule 3181
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[-a/(2*q)   Int[1/(
Sqrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Simp[b*(g/f)   Subst[
Int[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - S
imp[a/(2*q)   Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

## Maple [A] (warning: unable to verify)

Time = 2.03 (sec) , antiderivative size = 711, normalized size of antiderivative = 1.59

| method | result |
|---|---|
| default | $-2eb\left(-\frac{b^{2}\left(\frac{e^{2}\left(a^{2}-b^{2}\right)}{b^{2}}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\sin(dx+c)+\left(\frac{e^{2}\left(a^{2}-b^{2}\right)}{b^{2}}\right)^{\frac{1}{4}}\sqrt{e\sin(dx+c)}\,\sqrt{2}+\sqrt{\frac{e^{2}\left(a^{2}-b^{2}\right)}{b^{2}}}}{e\sin(dx+c)-\left(\frac{e^{2}\left(a^{2}-b^{2}\right)}{b^{2}}\right)^{\frac{1}{4}}\sqrt{e\sin(dx+c)}\,\sqrt{2}+\sqrt{\frac{e^{2}\left(a^{2}-b^{2}\right)}{b^{2}}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\sin(dx+c)}}{\left(\frac{e^{2}\left(a^{2}-b^{2}\right)}{b^{2}}\right)^{\frac{1}{4}}}+1\right)+2\ldots}{8e^{2}(a-b)(a+b)\left(a^{2}e^{2}-b^{2}e^{2}\right)}\right.$ |

input
```
int(1/(a+cos(d*x+c)*b)/(e*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(-2*e*b*(-1/8/e^2/(a-b)/(a+b)*b^2*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e
^2)*2^(1/2)*(ln((e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/
2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1
/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arctan(2^(1
/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2)/(e^
2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1))-1/3/e^2/(a^2-b^2)/(e*sin(d
*x+c))^(3/2))+(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*a/e^2*(1/3/(a^2-b^2)/(cos(
d*x+c)^2*e*sin(d*x+c))^(1/2)/(cos(d*x+c)^2-1)*((1-sin(d*x+c))^(1/2)*(2+2*s
in(d*x+c))^(1/2)*sin(d*x+c)^(5/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/
2))+2*cos(d*x+c)^2*sin(d*x+c))-1/(a-b)/(a+b)*b^2*(-1/2/b/(-a^2+b^2)^(1/2)*
(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2
*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2
),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/2/b/(-a^2+b^2)^(1/2)*(1-sin(d*x+
c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+
c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^
2+b^2)^(1/2)/b),1/2*2^(1/2)))))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

## Fricas [F]

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\cos(dx+c)+a)\,(e\sin(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(e*sin(d*x + c))/((b*e^3*cos(d*x + c)^3 + a*e^3*cos(d*x + c)
^2 - b*e^3*cos(d*x + c) - a*e^3)*sin(d*x + c)), x)
```

## Sympy [F]

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}\,dx = \int \frac{1}{(e\sin(c+dx))^{\frac{5}{2}}\,(a+b\cos(c+dx))}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))**(5/2),x)
```

output
```
Integral(1/((e*sin(c + d*x))**(5/2)*(a + b*cos(c + d*x))), x)
```

## Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

## Giac [F]

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\cos(dx+c)+a)\,(e\sin(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c) + a)*(e*sin(d*x + c))^(5/2)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}\,dx = \int \frac{1}{(e\sin(c+dx))^{5/2}\,(a+b\cos(c+dx))}\,dx$$

input
```
int(1/((e*sin(c + d*x))^(5/2)*(a + b*cos(c + d*x))),x)
```

output
```
int(1/((e*sin(c + d*x))^(5/2)*(a + b*cos(c + d*x))), x)
```

## Reduce [F]

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}\,dx = \frac{\sqrt{e}\left(\int \frac{\sqrt{\sin(dx+c)}}{\cos(dx+c)\sin(dx+c)^3 b+\sin(dx+c)^3 a}dx\right)}{e^3}$$

input
```
int(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(5/2),x)
```

output
```
(sqrt(e)*int(sqrt(sin(c + d*x))/(cos(c + d*x)*sin(c + d*x)**3*b + sin(c +
d*x)**3*a),x))/e**3
```

# 3.67 $\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 501

$$\begin{aligned}
&\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}\,dx = \\
&-\frac{b^{7/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{(-a^2+b^2)^{9/4}\,de^{7/2}}+\frac{b^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{(-a^2+b^2)^{9/4}\,de^{7/2}} \\
&+\frac{2(b-a\cos(c+dx))}{5(a^2-b^2)\,de(e\sin(c+dx))^{5/2}}-\frac{2(5b^3+a(3a^2-8b^2)\cos(c+dx))}{5(a^2-b^2)^2\,de^3\sqrt{e\sin(c+dx)}} \\
&+\frac{ab^3\operatorname{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{(a^2-b^2)^2\left(b-\sqrt{-a^2+b^2}\right)de^3\sqrt{e\sin(c+dx)}} \\
&+\frac{ab^3\operatorname{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{(a^2-b^2)^2\left(b+\sqrt{-a^2+b^2}\right)de^3\sqrt{e\sin(c+dx)}} \\
&-\frac{2a(3a^2-8b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5(a^2-b^2)^2\,de^4\sqrt{\sin(c+dx)}}
\end{aligned}$$

output
```
-b^(7/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a
^2+b^2)^(9/4)/d/e^(7/2)+b^(7/2)*arctanh(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2
+b^2)^(1/4)/e^(1/2))/(-a^2+b^2)^(9/4)/d/e^(7/2)+2/5*(b-a*cos(d*x+c))/(a^2-
b^2)/d/e/(e*sin(d*x+c))^(5/2)-2/5*(5*b^3+a*(3*a^2-8*b^2)*cos(d*x+c))/(a^2-
b^2)^2/d/e^3/(e*sin(d*x+c))^(1/2)-a*b^3*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*
x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^2/(b-(-a^2
+b^2)^(1/2))/d/e^3/(e*sin(d*x+c))^(1/2)-a*b^3*EllipticPi(cos(1/2*c+1/4*Pi+
1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^2/(b
+(-a^2+b^2)^(1/2))/d/e^3/(e*sin(d*x+c))^(1/2)+2/5*a*(3*a^2-8*b^2)*Elliptic
E(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^2/d/e^
4/sin(d*x+c)^(1/2)
```

## Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 7.65 (sec) , antiderivative size = 881, normalized size of antiderivative = 1.76

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}\,dx = \frac{\left(-\frac{2(5b^3+3a^3\cos(c+dx)-8ab^2\cos(c+dx))\csc(c+dx)}{5(a^2-b^2)^2}-\frac{2(-b+a\cos(c+dx))\csc^{3}\cdots}{5(a^2-b^2)}\cdots\right.}{d(e\sin(c+dx))^{7/2}}$$

$$-\frac{\sin^{\frac{7}{2}}(c+dx)\left(\frac{(3a^3b-8ab^3)\cos^2(c+dx)\left(3\sqrt{2}a(a^2-b^2)^{3/4}\left(2\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-2\arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-\log\left(\sqrt{a^2}\cdots\right.\right.\right.}{\cdots}\cdots\right.}{\cdots}$$

input
```
Integrate[1/((a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(7/2)),x]
```

output
```
(((-2*(5*b^3 + 3*a^3*Cos[c + d*x] - 8*a*b^2*Cos[c + d*x])*Csc[c + d*x])/(5
*(a^2 - b^2)^2) - (2*(-b + a*Cos[c + d*x])*Csc[c + d*x]^3)/(5*(a^2 - b^2))
)*Sin[c + d*x]^4)/(d*(e*Sin[c + d*x])^(7/2)) - (Sin[c + d*x]^(7/2)*(((3*a^
3*b - 8*a*b^3)*Cos[c + d*x]^2*(3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 -
 (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (S
qrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2
] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]]
 + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*
x]] + b*Sin[c + d*x]]) + 8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin[c + d*x
]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2))*(a + b*Sqrt[1
- Sin[c + d*x]^2]))/(12*b^(3/2)*(-a^2 + b^2)*(a + b*Cos[c + d*x])*(1 - Sin
[c + d*x]^2)) + (2*(3*a^4 - 8*a^2*b^2 - 5*b^4)*Cos[c + d*x]*(((1/8 + I/8)*
(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2
*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log
[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]]
+ I*b*Sin[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^
(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)
) + (a*AppellF1[3/4, 1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a
^2 + b^2)]*Sin[c + d*x]^(3/2))/(3*(a^2 - b^2)))*(a + b*Sqrt[1 - Sin[c + d*
x]^2]))/((a + b*Cos[c + d*x])*Sqrt[1 - Sin[c + d*x]^2])))/(5*(a - b)^2*...
```

## Rubi [A] (warning: unable to verify)

Time = 2.45 (sec) , antiderivative size = 494, normalized size of antiderivative = 0.99, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.840$, Rules used = {3042, 3175, 27, 3042, 3345, 27, 3042, 3346, 3042, 3121, 3042, 3119, 3180, 266, 827, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(e\sin(c+dx))^{7/2}(a+b\cos(c+dx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{7/2}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3175}$$

$$\frac{2(b-a\cos(c+dx))}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}}-\frac{2\int -\frac{3a^2+3b\cos(c+dx)a-5b^2}{2(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}dx}{5e^2\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{3a^2+3b\cos(c+dx)a-5b^2}{(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}dx}{5e^2\left(a^2-b^2\right)}+\frac{2(b-a\cos(c+dx))}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{3a^2-3b\sin\left(c+dx-\frac{\pi}{2}\right)a-5b^2}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{5e^2\left(a^2-b^2\right)}+\frac{2(b-a\cos(c+dx))}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \text{ 3345}$$

$$\frac{-\frac{2\int \frac{\left(3a^4-8b^2a^2+b\left(3a^2-8b^2\right)\cos(c+dx)a-5b^4\right)\sqrt{e\sin(c+dx)}}{2(a+b\cos(c+dx))}dx}{e^2(a^2-b^2)}-\frac{2\left(a\left(3a^2-8b^2\right)\cos(c+dx)+5b^3\right)}{de(a^2-b^2)\sqrt{e\sin(c+dx)}}}{5e^2\left(a^2-b^2\right)}+\frac{2(b-a\cos(c+dx))}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{-\frac{\int \frac{\left(3a^4-8b^2a^2+b\left(3a^2-8b^2\right)\cos(c+dx)a-5b^4\right)\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx}{e^2(a^2-b^2)}-\frac{2\left(a\left(3a^2-8b^2\right)\cos(c+dx)+5b^3\right)}{de(a^2-b^2)\sqrt{e\sin(c+dx)}}}{5e^2\left(a^2-b^2\right)}+\frac{2(b-a\cos(c+dx))}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{-\frac{\int \frac{\sqrt{-e\cos\left(c+dx+\frac{\pi}{2}\right)}\left(3a^4-8b^2a^2+b\left(3a^2-8b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a-5b^4\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{e^2(a^2-b^2)}-\frac{2\left(a\left(3a^2-8b^2\right)\cos(c+dx)+5b^3\right)}{de(a^2-b^2)\sqrt{e\sin(c+dx)}}}{5e^2\left(a^2-b^2\right)}+\frac{2(b-a\cos(c+dx))}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \text{ 3346}$$

$$\frac{-\frac{a\left(3a^2-8b^2\right)\int\sqrt{e\sin(c+dx)}dx-5b^4\int\frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx}{e^2(a^2-b^2)}-\frac{2\left(a\left(3a^2-8b^2\right)\cos(c+dx)+5b^3\right)}{de(a^2-b^2)\sqrt{e\sin(c+dx)}}}{5e^2\left(a^2-b^2\right)}+\frac{2(b-a\cos(c+dx))}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{-\frac{a\left(3a^2-8b^2\right)\int \sqrt{e\sin(c+dx)}dx-5b^4\int \frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{e^2\left(a^2-b^2\right)}-\frac{2\left(a\left(3a^2-8b^2\right)\cos(c+dx)+5b^3\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}}{5e^2\left(a^2-b^2\right)}+\frac{2(b-a\cos(c+dx))}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \; 3121$$

$$\frac{-\frac{\frac{a\left(3a^2-8b^2\right)\sqrt{e\sin(c+dx)}\int \sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}-5b^4\int \frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{e^2\left(a^2-b^2\right)}-\frac{2\left(a\left(3a^2-8b^2\right)\cos(c+dx)+5b^3\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}}{5e^2\left(a^2-b^2\right)}+\frac{2(b-a\cos(c+dx))}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{\frac{a\left(3a^2-8b^2\right)\sqrt{e\sin(c+dx)}\int \sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}-5b^4\int \frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{e^2\left(a^2-b^2\right)}-\frac{2\left(a\left(3a^2-8b^2\right)\cos(c+dx)+5b^3\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}}{5e^2\left(a^2-b^2\right)}+\frac{2(b-a\cos(c+dx))}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \; 3119$$

$$\frac{-\frac{\frac{2a\left(3a^2-8b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-5b^4\int \frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{e^2\left(a^2-b^2\right)}-\frac{2\left(a\left(3a^2-8b^2\right)\cos(c+dx)+5b^3\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}}{5e^2\left(a^2-b^2\right)}+\frac{2(b-a\cos(c+dx))}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \; 3180$$

$$-\frac{\frac{2a\left(3a^2-8b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-5b^4\left(-\frac{be\int \frac{\sqrt{e\sin(c+dx)}}{b^2\sin^2(c+dx)e^2+\left(a^2-b^2\right)e^2}d(e\sin(c+dx))}{d}-\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\right.}{e^2\left(a^2-b^2\right)}$$

$$5e^2\left(a^2-b^2\right)$$

$$\frac{2(b-a\cos(c+dx))}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \; 266$$

$$-\frac{\frac{2a\left(3a^2-8b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-5b^4\left(-\frac{2be\int\frac{e^2\sin^2(c+dx)}{b^2e^4\sin^4(c+dx)+\left(a^2-b^2\right)e^2}d\sqrt{e\sin(c+dx)}}{d}-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}\right.}{e^2(a^2-b^2)}}{5e^2\left(a^2-b^2\right)}$$

$$\frac{2(b-a\cos(c+dx))}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}}$$

$\downarrow$ 827

$$-\frac{\frac{2a\left(3a^2-8b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-5b^4\left(-\frac{2be\left(\frac{\int\frac{1}{be^2\sin^2(c+dx)+\sqrt{b^2-a^2}e}d\sqrt{e\sin(c+dx)}}{2b}-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2b}\right)}{d}-\right.}{e^2(a^2-b^2)}}{5e^2\left(a^2-b^2\right)}$$

$$\frac{2(b-a\cos(c+dx))}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}}$$

$\downarrow$ 218

$$-\frac{\frac{2a\left(3a^2-8b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-5b^4\left(-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2b}\right)}{d}-\frac{ae\int\frac{}{\sqrt{e\sin(c+d}}}{}\right.}{e^2(a^2-b^2)}}{5e^2\left(a^2-b^2\right)}$$

$$\frac{2(b-a\cos(c+dx))}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}}$$

$\downarrow$ 221

$$-\frac{\frac{2a\left(3a^2-8b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-5b^4\left(-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b}-\cdots\right.}{e^2(a^2-b^2)}}{5e^2\left(a^2-b^2\right)}$$

$$\frac{2(b-a\cos(c+dx))}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}}$$

$\downarrow$ 3042

$$-\frac{\frac{2a\left(3a^2-8b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-5b^4\left(-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b}-\cdots\right.}{e^2(a^2-b^2)}}{5e^2\left(a^2-b^2\right)}$$

$$\frac{2(b-a\cos(c+dx))}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}}$$

$\downarrow$ 3286

$$-\frac{\frac{2a\left(3a^2-8b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-5b^4\left(-\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b\sqrt{e\sin(c+dx)}}+\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c+dx\cdots\right.}}{2b\sqrt{e\sin(c+dx)}}\cdots\right.}{e^2(a^2-b^2)}}{5e^2\left(a^2-b^2\right)}$$

$$\frac{2(b-a\cos(c+dx))}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}}$$

$\downarrow$ 3042

$$-\frac{\frac{2a\left(3a^2-8b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-5b^4\left(-\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b\sqrt{e\sin(c+dx)}}+\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c+dx\right.}}{2b\sqrt{e\sin(c+dx)}}\right.}{e^2(a^2-b^2)}$$

$$\overline{5e^2\left(a^2-b^2\right)}$$

$$\frac{2(b-a\cos(c+dx))}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}}$$

$\downarrow$ 3284

$$-\frac{\frac{2a\left(3a^2-8b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-5b^4\left(-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{d}+\frac{ae\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{}{b}\right.}{bd\left(b-\sqrt{b^2-a^2}\right.}\right.}{e^2(a^2-b^2)}$$

$$\overline{5e^2\left(a^2-b^2\right)}$$

$$\frac{2(b-a\cos(c+dx))}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}}$$

input
```
Int[1/((a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(7/2)),x]
```

output
```
(2*(b - a*Cos[c + d*x]))/(5*(a^2 - b^2)*d*e*(e*Sin[c + d*x])^(5/2)) + ((-2
*(5*b^3 + a*(3*a^2 - 8*b^2)*Cos[c + d*x]))/((a^2 - b^2)*d*e*Sqrt[e*Sin[c +
 d*x]]) - ((2*a*(3*a^2 - 8*b^2)*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Si
n[c + d*x]])/(d*Sqrt[Sin[c + d*x]]) - 5*b^4*((-2*b*e*(ArcTan[(Sqrt[b]*Sqrt
[e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/(2*b^(3/2)*(-a^2 + b^2)^(1/4)*Sqrt[e
]) - ArcTanh[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/(2*b^(3/2)
*(-a^2 + b^2)^(1/4)*Sqrt[e])))/d + (a*e*EllipticPi[(2*b)/(b - Sqrt[-a^2 +
b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(b*(b - Sqrt[-a^2 + b^2]
)*d*Sqrt[e*Sin[c + d*x]]) + (a*e*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]),
(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(b*(b + Sqrt[-a^2 + b^2])*d*Sqr
t[e*Sin[c + d*x]])))/((a^2 - b^2)*e^2))/(5*(a^2 - b^2)*e^2)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3175
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Simp[1/(g^2*
(a^2 - b^2)*(p + 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*
(a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; F
reeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ
[2*m, 2*p]
```

rule 3180
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[a*(g/(2*b))   Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Simp[a*(g/(2*b))   In
t[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Simp[b*(g/f)   Su
bst[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3345

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Simp[1/(g^2*(a^2 - b^2)*(p +
 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
 2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[p, -1] && IntegerQ[2*m]
```

rule 3346

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1006 vs. $2(445) = 890$.

Time = 2.45 (sec) , antiderivative size = 1007, normalized size of antiderivative = 2.01

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1007 |

input

```
int(1/(a+cos(d*x+c)*b)/(e*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
(-2*e*b*(1/8*b^2/e^4/(a-b)^2/(a+b)^2/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*(ln
((e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2
*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+
c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arctan(2^(1/2)/(e^2*(a^2-b
^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2
)^(1/4)*(e*sin(d*x+c))^(1/2)-1))+1/e^4/(a-b)^2/(a+b)^2*b^2/(e*sin(d*x+c))^
(1/2)-1/5/e^2/(a+b)/(a-b)/(e*sin(d*x+c))^(5/2))+1/10/e^3*(5*(-a^2+b^2)^(1/
2)*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(7/2)*EllipticPi
((1-sin(d*x+c))^(1/2),-b/(-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*b^3-5*(-a^2+b^
2)^(1/2)*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(7/2)*Elli
pticPi((1-sin(d*x+c))^(1/2),b/(b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*b^3-12*(1-
sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(7/2)*EllipticE((1-sin
(d*x+c))^(1/2),1/2*2^(1/2))*a^4+32*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(
1/2)*sin(d*x+c)^(7/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2*b^2+
6*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(7/2)*EllipticF((
1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^4-16*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+
c))^(1/2)*sin(d*x+c)^(7/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2
*b^2+5*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(7/2)*Ellipt
icPi((1-sin(d*x+c))^(1/2),-b/(-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*b^4+5*(1-s
in(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(7/2)*EllipticPi((1-...
```

## Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
Timed out
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))**(7/2),x)
```

output
```
Timed out
```

## Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
Timed out
```

## Giac [F]

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}\,dx = \int \frac{1}{(b\cos(dx+c)+a)(e\sin(dx+c))^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c) + a)*(e*sin(d*x + c))^(7/2)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}\,dx = \int \frac{1}{\left(e\,\sin\left(c+d\,x\right)\right)^{7/2}\,\left(a+b\,\cos\left(c+d\,x\right)\right)}\,dx$$

input
```
int(1/((e*sin(c + d*x))^(7/2)*(a + b*cos(c + d*x))),x)
```

output
```
int(1/((e*sin(c + d*x))^(7/2)*(a + b*cos(c + d*x))), x)
```

## Reduce [F]

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}\,dx = \frac{\sqrt{e}\left(\int \frac{\sqrt{\sin(dx+c)}}{\cos(dx+c)\sin(dx+c)^4 b+\sin(dx+c)^4 a}dx\right)}{e^4}$$

input
```
int(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(7/2),x)
```

output
```
(sqrt(e)*int(sqrt(sin(c + d*x))/(cos(c + d*x)*sin(c + d*x)**4*b + sin(c +
d*x)**4*a),x))/e**4
```

# 3.68 $\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^2}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 557

$$
\begin{aligned}
\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^2}\,dx &= \frac{9a(-a^2+b^2)^{5/4}\,e^{11/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{11/2}d} \\
&+ \frac{9a(-a^2+b^2)^{5/4}\,e^{11/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{11/2}d} \\
&- \frac{3(21a^4-28a^2b^2+5b^4)\,e^6\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{7b^6d\sqrt{e\sin(c+dx)}} \\
&+ \frac{9a^2(a^2-b^2)^2\,e^6\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2b^6\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}} \\
&+ \frac{9a^2(a^2-b^2)^2\,e^6\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2b^6\left(a^2-b\left(b+\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}} \\
&+ \frac{3e^5(21a(a^2-b^2)-b(7a^2-5b^2)\cos(c+dx))\sqrt{e\sin(c+dx)}}{7b^5d} \\
&- \frac{9e^3(7a-5b\cos(c+dx))(e\sin(c+dx))^{5/2}}{35b^3d} + \frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))}
\end{aligned}
$$

output
```
9/2*a*(-a^2+b^2)^(5/4)*e^(11/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+
b^2)^(1/4)/e^(1/2))/b^(11/2)/d+9/2*a*(-a^2+b^2)^(5/4)*e^(11/2)*arctanh(b^(
1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(11/2)/d-3/7*(21*a^4
-28*a^2*b^2+5*b^4)*e^6*InverseJacobiAM(1/2*c-1/4*Pi+1/2*d*x,2^(1/2))*sin(d
*x+c)^(1/2)/b^6/d/(e*sin(d*x+c))^(1/2)-9/2*a^2*(a^2-b^2)^2*e^6*EllipticPi(
cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/
2)/b^6/(a^2-b*(b-(-a^2+b^2)^(1/2)))/d/(e*sin(d*x+c))^(1/2)-9/2*a^2*(a^2-b^
2)^2*e^6*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(
1/2))*sin(d*x+c)^(1/2)/b^6/(a^2-b*(b+(-a^2+b^2)^(1/2)))/d/(e*sin(d*x+c))^(
1/2)+3/7*e^5*(21*a*(a^2-b^2)-b*(7*a^2-5*b^2)*cos(d*x+c))*(e*sin(d*x+c))^(1
/2)/b^5/d-9/35*e^3*(7*a-5*b*cos(d*x+c))*(e*sin(d*x+c))^(5/2)/b^3/d+e*(e*si
n(d*x+c))^(9/2)/b/d/(a+b*cos(d*x+c))
```

## Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 19.39 (sec) , antiderivative size = 2029, normalized size of antiderivative = 3.64

$$\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[(e*Sin[c + d*x])^(11/2)/(a + b*Cos[c + d*x])^2,x]
```

output
```
((((-28*a^2 + 17*b^2)*Cos[c + d*x])/(14*b^4) + (-a^2 + b^2)^2/(b^5*(a + b*
Cos[c + d*x])) + (2*a*Cos[2*(c + d*x)])/(5*b^3) - Cos[3*(c + d*x)]/(14*b^2
))*Csc[c + d*x]^5*(e*Sin[c + d*x])^(11/2))/d - ((e*Sin[c + d*x])^(11/2)*((
2*(35*a^4 - 126*a^2*b^2 + 75*b^4)*Cos[c + d*x]^2*(a + b*Sqrt[1 - Sin[c + d
*x]^2])*((a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2
)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(
1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c
 + d*x]] + b*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 -
b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2
- b^2)^(3/4)) + (5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^
2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Sin[c + d*x]]*Sqrt[1 - Sin[c +
d*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2
*Sin[c + d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Sin[
c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4,
 1/2, 1, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)])*Sin[c +
d*x]^2)*(a^2 + b^2*(-1 + Sin[c + d*x]^2)))))/((a + b*Cos[c + d*x])*(1 - Si
n[c + d*x]^2)) + (2*(70*a^3*b - 93*a*b^3)*Cos[c + d*x]*(a + b*Sqrt[1 - Sin
[c + d*x]^2])*(((-1/8 + I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[S
in[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin
[c + d*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b...
```

## Rubi [A] (warning: unable to verify)

Time = 2.68 (sec) , antiderivative size = 556, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.960$, Rules used = {3042, 3172, 25, 3042, 3344, 27, 3042, 3344, 27, 3042, 3346, 3042, 3121, 3042, 3120, 3181, 266, 756, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{11/2}}{\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3172}$$

$$\frac{9e^2\int -\frac{\cos(c+dx)(e\sin(c+dx))^{7/2}}{a+b\cos(c+dx)}dx}{2b}+\frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))}$$

$$\Big\downarrow 25$$

$$\frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))}-\frac{9e^2\int \frac{\cos(c+dx)(e\sin(c+dx))^{7/2}}{a+b\cos(c+dx)}dx}{2b}$$

$$\Big\downarrow 3042$$

$$\frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))}-\frac{9e^2\int \frac{\left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2b}$$

$$\Big\downarrow 3344$$

$$\frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))}-\frac{9e^2\left(\frac{2e^2\int -\frac{\left(2ab+\left(7a^2-5b^2\right)\cos(c+dx)\right)(e\sin(c+dx))^{3/2}}{2(a+b\cos(c+dx))}dx}{7b^2}+\frac{2e(e\sin(c+dx))^{5/2}(7a-5b\cos(c+dx))}{35b^2d}\right)}{2b}$$

$$\Big\downarrow 27$$

$$\frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))}-\frac{9e^2\left(\frac{2e(e\sin(c+dx))^{5/2}(7a-5b\cos(c+dx))}{35b^2d}-\frac{e^2\int \frac{\left(2ab+\left(7a^2-5b^2\right)\cos(c+dx)\right)(e\sin(c+dx))^{3/2}}{a+b\cos(c+dx)}dx}{7b^2}\right)}{2b}$$

$$\Big\downarrow 3042$$

$$\frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))}-\frac{9e^2\left(\frac{2e(e\sin(c+dx))^{5/2}(7a-5b\cos(c+dx))}{35b^2d}-\frac{e^2\int \frac{\left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(2ab+\left(7a^2-5b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{7b^2}\right)}{2b}$$

$$\Big\downarrow 3344$$

$$\frac{\frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))}-\frac{9e^2\left(\frac{2e(e\sin(c+dx))^{5/2}(7a-5b\cos(c+dx))}{35b^2d}-\frac{e^2\left(\frac{2e^2\int -\frac{2ab\left(7a^2-8b^2\right)+\left(21a^4-28b^2a^2+5b^4\right)\cos(c+dx)}{2(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{3b^2}+\frac{2e\sqrt{e\sin(c+dx)}\left(21a\left(a^2-b^2\right)-b\left(7a^2-\right.\right.}{3b^2d}\right.}{7b^2}\right.}{2b}$$

$\downarrow$ 27

$$\frac{\frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))}-\frac{9e^2\left(\frac{2e(e\sin(c+dx))^{5/2}(7a-5b\cos(c+dx))}{35b^2d}-\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(21a\left(a^2-b^2\right)-b\left(7a^2-5b^2\right)\cos(c+dx)\right)}{3b^2d}-\frac{e^2\int\frac{2ab\left(7a^2-8b^2\right)+\left(21a^4-28b^2a^2+5b^4\right.}{(a+b\cos(c+dx))\sqrt{e\sin(c+d}}}{3b^2}\right.}{7b^2}\right.}{2b}$$

$\downarrow$ 3042

$$\frac{\frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))}-\frac{9e^2\left(\frac{2e(e\sin(c+dx))^{5/2}(7a-5b\cos(c+dx))}{35b^2d}-\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(21a\left(a^2-b^2\right)-b\left(7a^2-5b^2\right)\cos(c+dx)\right)}{3b^2d}-\frac{e^2\int\frac{2ab\left(7a^2-8b^2\right)-\left(21a^4-28b^2a^2+5b^4\right.}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+\right.\right.}}{3b^2}\right.}{7b^2}\right.}{2b}$$

$\downarrow$ 3346

$$\frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))} - \frac{9e^2\left(\frac{2e(e\sin(c+dx))^{5/2}(7a-5b\cos(c+dx))}{35b^2d} - \frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(21a\left(a^2-b^2\right)-b\left(7a^2-5b^2\right)\cos(c+dx)\right)}{3b^2d} - \frac{e^2\left(\frac{\left(21a^4-28a^2b^2+5b^4\right)\int\frac{1}{\sqrt{e\sin(c+dx)}}}{b}\right.}{}\right.}{7b^2}\right.}{2b}$$

↓ 3042

$$\frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))} - \frac{9e^2\left(\frac{2e(e\sin(c+dx))^{5/2}(7a-5b\cos(c+dx))}{35b^2d} - \frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(21a\left(a^2-b^2\right)-b\left(7a^2-5b^2\right)\cos(c+dx)\right)}{3b^2d} - \frac{e^2\left(\frac{\left(21a^4-28a^2b^2+5b^4\right)\int\frac{1}{\sqrt{e\sin(c+dx)}}}{b}\right.}{}\right.}{7b^2}\right.}{2b}$$

↓ 3121

$$\frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))} - \frac{9e^2\left(\frac{2e(e\sin(c+dx))^{5/2}(7a-5b\cos(c+dx))}{35b^2d} - \frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(21a\left(a^2-b^2\right)-b\left(7a^2-5b^2\right)\cos(c+dx)\right)}{3b^2d} - \frac{e^2\left(\frac{\left(21a^4-28a^2b^2+5b^4\right)\sqrt{\sin(c+dx)}\int -\ldots}{b\sqrt{e\sin(c+dx)}}\ldots\right.}{\ldots}\right.}{7b^2}\ldots\right.}{2b}$$

$\downarrow$ 3042

$$\frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))} - \frac{9e^2\left(\frac{2e(e\sin(c+dx))^{5/2}(7a-5b\cos(c+dx))}{35b^2d} - \frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(21a\left(a^2-b^2\right)-b\left(7a^2-5b^2\right)\cos(c+dx)\right)}{3b^2d} - \frac{e^2\left(\frac{\left(21a^4-28a^2b^2+5b^4\right)\sqrt{\sin(c+dx)}\int -\ldots}{b\sqrt{e\sin(c+dx)}}\ldots\right.}{\ldots}\right.}{7b^2}\ldots\right.}{2b}$$

$\downarrow$ 3120

$$\frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))} -$$

$$9e^2\left(\frac{2e(e\sin(c+dx))^{5/2}(7a-5b\cos(c+dx))}{35b^2d} - \frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(21a\left(a^2-b^2\right)-b\left(7a^2-5b^2\right)\cos(c+dx)\right)}{3b^2d} - \frac{e^2\left(\frac{2\left(21a^4-28a^2b^2+5b^4\right)\sqrt{\sin(c+dx)}\,\text{El}}{bd\sqrt{e\sin(c+dx}}\right.}{}\right.}{7b^2}\right.$$

$$2b$$

$\downarrow$ 3181

$$\frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))} -$$

$$9e^2\left(\frac{2e(e\sin(c+dx))^{5/2}(7a-5b\cos(c+dx))}{35b^2d} - \frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(21a\left(a^2-b^2\right)-b\left(7a^2-5b^2\right)\cos(c+dx)\right)}{3b^2d} - \frac{e^2\left(\frac{2\left(21a^4-28a^2b^2+5b^4\right)\sqrt{\sin(c+dx)}\,\text{El}}{bd\sqrt{e\sin(c+dx}}\right.}{}\right.}{}\right.$$

$\downarrow$ 266

$$\frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))} -$$

$$9e^2\left(\frac{2e(e\sin(c+dx))^{5/2}(7a-5b\cos(c+dx))}{35b^2d} - \frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(21a\left(a^2-b^2\right)-b\left(7a^2-5b^2\right)\cos(c+dx)\right)}{3b^2d} - \frac{e^2\left(\frac{2\left(21a^4-28a^2b^2+5b^4\right)\sqrt{\sin(c+dx)}\,\mathrm{El}}{bd\sqrt{e\sin(c+dx}}\right.}{}\right.}{}\right.$$

↓ 756

$$\frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))} -$$

$$9e^2\left(\frac{2e(e\sin(c+dx))^{5/2}(7a-5b\cos(c+dx))}{35b^2d} - \frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(21a\left(a^2-b^2\right)-b\left(7a^2-5b^2\right)\cos(c+dx)\right)}{3b^2d} - \frac{e^2\left(\frac{2\left(21a^4-28a^2b^2+5b^4\right)\sqrt{\sin(c+dx)}\,\mathrm{El}\ldots}{bd\sqrt{e\sin(c+da\ldots}}\right.}{\ldots}\right.}{\ldots}\right.$$

↓ 218

$$\frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))} -$$

$$9e^2\left(\frac{2e(e\sin(c+dx))^{5/2}(7a-5b\cos(c+dx))}{35b^2d} - \frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(21a\left(a^2-b^2\right)-b\left(7a^2-5b^2\right)\cos(c+dx)\right)}{3b^2d} - \frac{e^2\left(\frac{2\left(21a^4-28a^2b^2+5b^4\right)\sqrt{\sin(c+dx)}\,\mathrm{El}}{bd\sqrt{e\sin(c+dx}}\right.}{}\right.}{}\right.$$

↓ 221

$$\frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))} -$$

$$9e^2\left(\frac{2e(e\sin(c+dx))^{5/2}(7a-5b\cos(c+dx))}{35b^2d} - \frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(21a\left(a^2-b^2\right)-b\left(7a^2-5b^2\right)\cos(c+dx)\right)}{3b^2d} - \frac{e^2\left(\frac{2\left(21a^4-28a^2b^2+5b^4\right)\sqrt{\sin(c+dx)}\,\mathrm{El}}{bd\sqrt{e\sin(c+dx}}\right.}{}\right.}{}\right.$$

↓ 3042

$$\frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))}-$$

$$e^2\left(\frac{2\left(21a^4-28a^2b^2+5b^4\right)\sqrt{\sin(c+dx)}\,\mathrm{El}}{bd\sqrt{e\sin(c+dx}}\right.$$

$$e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(21a\left(a^2-b^2\right)-b\left(7a^2-5b^2\right)\cos(c+dx)\right)}{3b^2d}-\frac{\quad}{\quad}\right.$$

$$9e^2\left(\frac{2e(e\sin(c+dx))^{5/2}(7a-5b\cos(c+dx))}{35b^2d}-\frac{\quad}{\quad}\right.$$

↓ 3286

$$\frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))} -$$

$$9e^2\left(\frac{2e(e\sin(c+dx))^{5/2}(7a-5b\cos(c+dx))}{35b^2d} - \frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(21a\left(a^2-b^2\right)-b\left(7a^2-5b^2\right)\cos(c+dx)\right)}{3b^2d} - \frac{e^2\left(\frac{2\left(21a^4-28a^2b^2+5b^4\right)\sqrt{\sin(c+dx)}\,\mathrm{El}}{bd\sqrt{e\sin(c+dx}}\right.}{}\right.}{}\right.$$

↓ 3042

$$\frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))} -$$

$$9e^2\left(\frac{2e(e\sin(c+dx))^{5/2}(7a-5b\cos(c+dx))}{35b^2d} - \frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(21a\left(a^2-b^2\right)-b\left(7a^2-5b^2\right)\cos(c+dx)\right)}{3b^2d} - \frac{e^2\left(\frac{2\left(21a^4-28a^2b^2+5b^4\right)\sqrt{\sin(c+dx)}\,\mathrm{El}}{bd\sqrt{e\sin(c+da}}\right.}{}\right.}{}\right.$$

↓ 3284

$$\frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))} -$$

$$e^2 \left( \frac{2\left(21a^4-28a^2b^2+5b^4\right)\sqrt{\sin(c+dx)}\,\text{El}\cdots}{bd\sqrt{e\sin(c+dx}\cdots} \right.$$

$$e^2 \left( \frac{2e\sqrt{e\sin(c+dx)}\left(21a\left(a^2-b^2\right)-b\left(7a^2-5b^2\right)\cos(c+dx)\right)}{3b^2d} - \frac{\cdots}{\cdots} \right.$$

$$9e^2 \left( \frac{2e(e\sin(c+dx))^{5/2}(7a-5b\cos(c+dx))}{35b^2d} - \frac{\cdots}{\cdots} \right.$$

input
```
Int[(e*Sin[c + d*x])^(11/2)/(a + b*Cos[c + d*x])^2,x]
```

output
```
(e*(e*Sin[c + d*x])^(9/2))/(b*d*(a + b*Cos[c + d*x])) - (9*e^2*((2*e*(7*a
- 5*b*Cos[c + d*x])*(e*Sin[c + d*x])^(5/2))/(35*b^2*d) - (e^2*((2*e*(21*a*
(a^2 - b^2) - b*(7*a^2 - 5*b^2)*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]])/(3*b^2
*d) - (e^2*((2*(21*a^4 - 28*a^2*b^2 + 5*b^4)*EllipticF[(c - Pi/2 + d*x)/2,
 2]*Sqrt[Sin[c + d*x]])/(b*d*Sqrt[e*Sin[c + d*x]]) - (21*a*(a^2 - b^2)^2*(
(-2*b*e*(-1/2*ArcTan[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/(S
qrt[b]*(-a^2 + b^2)^(3/4)*e^(3/2)) - ArcTanh[(Sqrt[b]*Sqrt[e]*Sin[c + d*x]
)/(-a^2 + b^2)^(1/4)]/(2*Sqrt[b]*(-a^2 + b^2)^(3/4)*e^(3/2))))/d + (a*Elli
pticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d
*x]])/(Sqrt[-a^2 + b^2]*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]]) - (
a*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin
[c + d*x]])/(Sqrt[-a^2 + b^2]*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]
])))/b))/(3*b^2)))/(7*b^2)))/(2*b)
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3172
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Simp[g^2*((p - 1)/(b*(m + 1)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Fre
eQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && I
ntegersQ[2*m, 2*p]
```

rule 3181
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[-a/(2*q)   Int[1/(
Sqrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Simp[b*(g/f)   Subst[
Int[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - S
imp[a/(2*q)   Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3344
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Simp[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Si
n[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^
2*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1
, 0] && IntegerQ[2*m]
```

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

## Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1658 vs. 2(488) = 976.

Time = 17.01 (sec) , antiderivative size = 1659, normalized size of antiderivative = 2.98

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1659 |

input
```
int((e*sin(d*x+c))^(11/2)/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)
```

output
```
(-4*e^3*a*b*(-1/5/b^6*(e*sin(d*x+c))^(1/2)*e^2*(cos(d*x+c)^2*b^2+10*a^2-11
*b^2)+e^4/b^6*((-1/4*a^4+1/2*a^2*b^2-1/4*b^4)*(e*sin(d*x+c))^(1/2)/(-b^2*c
os(d*x+c)^2*e^2+a^2*e^2)+9/32*(a^4-2*a^2*b^2+b^4)*(e^2*(a^2-b^2)/b^2)^(1/4
)/(a^2*e^2-b^2*e^2)*2^(1/2)*(ln((e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e
*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)-(e^2*(
a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)
))+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*ar
ctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1))))+(cos(d*x
+c)^2*e*sin(d*x+c))^(1/2)*e^6*(1/7/b^6/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*(
-2*b^4*cos(d*x+c)^4*sin(d*x+c)+35*a^4*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c)
)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-49*a^
2*b^2*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*Ellipti
cF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))+11*b^4*(1-sin(d*x+c))^(1/2)*(2+2*sin(
d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))
-14*a^2*b^2*cos(d*x+c)^2*sin(d*x+c)+10*b^4*cos(d*x+c)^2*sin(d*x+c))+1/b^6*
(7*a^6-15*a^4*b^2+9*a^2*b^4-b^6)*(-1/2/b/(-a^2+b^2)^(1/2)*(1-sin(d*x+c))^(
1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(
1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2
)^(1/2)/b),1/2*2^(1/2))+1/2/b/(-a^2+b^2)^(1/2)*(1-sin(d*x+c))^(1/2)*(2+2*s
in(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+...
```

## Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))^(11/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
Timed out
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))**(11/2)/(a+b*cos(d*x+c))**2,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^{\frac{11}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

input
```
integrate((e*sin(d*x+c))^(11/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((e*sin(d*x + c))^(11/2)/(b*cos(d*x + c) + a)^2, x)
```

## Giac [F]

$$\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^{\frac{11}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

input
```
integrate((e*sin(d*x+c))^(11/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*sin(d*x + c))^(11/2)/(b*cos(d*x + c) + a)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\left(e\,\sin\left(c+d\,x\right)\right)^{11/2}}{\left(a+b\,\cos\left(c+d\,x\right)\right)^2}\,dx$$

input
```
int((e*sin(c + d*x))^(11/2)/(a + b*cos(c + d*x))^2,x)
```

output
```
int((e*sin(c + d*x))^(11/2)/(a + b*cos(c + d*x))^2, x)
```

## Reduce [F]

$$\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^2}\,dx = \sqrt{e}\left(\int \frac{\sqrt{\sin\left(dx+c\right)}\,\sin\left(dx+c\right)^5}{\cos\left(dx+c\right)^2 b^2+2\cos\left(dx+c\right)ab+a^2}dx\right)e^5$$

input
```
int((e*sin(d*x+c))^(11/2)/(a+b*cos(d*x+c))^2,x)
```

output
```
sqrt(e)*int((sqrt(sin(c + d*x))*sin(c + d*x)**5)/(cos(c + d*x)**2*b**2 + 2
*cos(c + d*x)*a*b + a**2),x)*e**5
```

# 3.69 $\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^2}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 473

$$\begin{aligned}
\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^2}\,dx &= -\frac{7a(-a^2+b^2)^{3/4}\,e^{9/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{9/2}d} \\
&+ \frac{7a(-a^2+b^2)^{3/4}\,e^{9/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{9/2}d} \\
&- \frac{7a^2(a^2-b^2)\,e^5\,\operatorname{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2b^5\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&- \frac{7a^2(a^2-b^2)\,e^5\,\operatorname{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2b^5\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&+ \frac{7(5a^2-3b^2)\,e^4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{e\sin(c+dx)}}{5b^4d\sqrt{\sin(c+dx)}} \\
&- \frac{7e^3(5a-3b\cos(c+dx))(e\sin(c+dx))^{3/2}}{15b^3d} + \frac{e(e\sin(c+dx))^{7/2}}{bd(a+b\cos(c+dx))}
\end{aligned}$$

output
```
-7/2*a*(-a^2+b^2)^(3/4)*e^(9/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+
b^2)^(1/4)/e^(1/2))/b^(9/2)/d+7/2*a*(-a^2+b^2)^(3/4)*e^(9/2)*arctanh(b^(1/
2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(9/2)/d+7/2*a^2*(a^2-b
^2)*e^5*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1
/2))*sin(d*x+c)^(1/2)/b^5/(b-(-a^2+b^2)^(1/2))/d/(e*sin(d*x+c))^(1/2)+7/2*
a^2*(a^2-b^2)*e^5*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(
1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^5/(b+(-a^2+b^2)^(1/2))/d/(e*sin(d*x+c))^
(1/2)-7/5*(5*a^2-3*b^2)*e^4*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(
e*sin(d*x+c))^(1/2)/b^4/d/sin(d*x+c)^(1/2)-7/15*e^3*(5*a-3*b*cos(d*x+c))*(
e*sin(d*x+c))^(3/2)/b^3/d+e*(e*sin(d*x+c))^(7/2)/b/d/(a+b*cos(d*x+c))
```

## Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 17.29 (sec) , antiderivative size = 835, normalized size of antiderivative = 1.77

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^2}\,dx = \frac{7(e\sin(c+dx))^{9/2}\Bigg(\frac{(5a^2-3b^2)\cos^2(c+dx)\Big(3\sqrt{2}a(a^2-b^2)^{3/4}\Big(2\arctan\Big(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\Big)-\cdots}{\cdots}}{\cdots} + \frac{\csc^4(c+dx)(e\sin(c+dx))^{9/2}\Big(-\frac{4a\sin(c+dx)}{3b^3}+\frac{-a^2\sin(c+dx)+b^2\sin(c+dx)}{b^3(a+b\cos(c+dx))}+\frac{\sin(2(c+dx))}{5b^2}\Big)}{d}$$

input
```
Integrate[(e*Sin[c + d*x])^(9/2)/(a + b*Cos[c + d*x])^2,x]
```

output
```
(7*(e*Sin[c + d*x])^(9/2)*(((5*a^2 - 3*b^2)*Cos[c + d*x]^2*(3*Sqrt[2]*a*(a
^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 -
b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2
)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Si
n[c + d*x]] + b*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2
 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]]) + 8*b^(5/2)*AppellF1[3
/4, -1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c
 + d*x]^(3/2))*(a + b*Sqrt[1 - Sin[c + d*x]^2]))/(12*b^(3/2)*(-a^2 + b^2)*
(a + b*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) + (4*a*b*Cos[c + d*x]*(((1/8 +
I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)
] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)]
- Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d
*x]] + I*b*Sin[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 +
b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^
(1/4)) + (a*AppellF1[3/4, 1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2
)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2))/(3*(a^2 - b^2)))*(a + b*Sqrt[1 - Sin[c
 + d*x]^2]))/((a + b*Cos[c + d*x])*Sqrt[1 - Sin[c + d*x]^2])))/(10*b^3*d*S
in[c + d*x]^(9/2)) + (Csc[c + d*x]^4*(e*Sin[c + d*x])^(9/2)*((-4*a*Sin[c +
 d*x])/(3*b^3) + (-(a^2*Sin[c + d*x]) + b^2*Sin[c + d*x])/(b^3*(a + b*Cos[
c + d*x])) + Sin[2*(c + d*x)]/(5*b^2)))/d
```

## Rubi [A] (warning: unable to verify)

Time = 2.10 (sec) , antiderivative size = 460, normalized size of antiderivative = 0.97, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.840$, Rules used = {3042, 3172, 25, 3042, 3344, 27, 3042, 3346, 3042, 3121, 3042, 3119, 3180, 266, 827, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^2}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{9/2}}{\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \; 3172$$

$$\frac{7e^2\int -\frac{\cos(c+dx)(e\sin(c+dx))^{5/2}}{a+b\cos(c+dx)}dx}{2b}+\frac{e(e\sin(c+dx))^{7/2}}{bd(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{e(e\sin(c+dx))^{7/2}}{bd(a+b\cos(c+dx))}-\frac{7e^2\int \frac{\cos(c+dx)(e\sin(c+dx))^{5/2}}{a+b\cos(c+dx)}dx}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e(e\sin(c+dx))^{7/2}}{bd(a+b\cos(c+dx))}-\frac{7e^2\int \frac{\left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2b}$$

$$\Big\downarrow \text{3344}$$

$$\frac{e(e\sin(c+dx))^{7/2}}{bd(a+b\cos(c+dx))}-\frac{7e^2\left(\frac{2e^2\int -\frac{\left(2ab+\left(5a^2-3b^2\right)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{2(a+b\cos(c+dx))}dx}{5b^2}+\frac{2e(e\sin(c+dx))^{3/2}(5a-3b\cos(c+dx))}{15b^2d}\right)}{2b}$$

$$\Big\downarrow \text{27}$$

$$\frac{e(e\sin(c+dx))^{7/2}}{bd(a+b\cos(c+dx))}-\frac{7e^2\left(\frac{2e(e\sin(c+dx))^{3/2}(5a-3b\cos(c+dx))}{15b^2d}-\frac{e^2\int \frac{\left(2ab+\left(5a^2-3b^2\right)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx}{5b^2}\right)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e(e\sin(c+dx))^{7/2}}{bd(a+b\cos(c+dx))}-\frac{7e^2\left(\frac{2e(e\sin(c+dx))^{3/2}(5a-3b\cos(c+dx))}{15b^2d}-\frac{e^2\int \frac{\sqrt{-e\cos\left(c+dx+\frac{\pi}{2}\right)}\left(2ab+\left(5a^2-3b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5b^2}\right)}{2b}$$

$$\Big\downarrow \text{3346}$$

$$\frac{\frac{e(e\sin(c+dx))^{7/2}}{bd(a+b\cos(c+dx))}-7e^2\left(\frac{2e(e\sin(c+dx))^{3/2}(5a-3b\cos(c+dx))}{15b^2d}-\frac{e^2\left(\frac{\left(5a^2-3b^2\right)\int\sqrt{e\sin(c+dx)}dx}{b}-\frac{5a\left(a^2-b^2\right)\int\frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx}{b}\right)}{5b^2}\right)}{2b}$$

$\Big\downarrow$ 3042

$$\frac{\frac{e(e\sin(c+dx))^{7/2}}{bd(a+b\cos(c+dx))}-7e^2\left(\frac{2e(e\sin(c+dx))^{3/2}(5a-3b\cos(c+dx))}{15b^2d}-\frac{e^2\left(\frac{\left(5a^2-3b^2\right)\int\sqrt{e\sin(c+dx)}dx}{b}-\frac{5a\left(a^2-b^2\right)\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{b}\right)}{5b^2}\right)}{2b}$$

$\Big\downarrow$ 3121

$$\frac{\frac{e(e\sin(c+dx))^{7/2}}{bd(a+b\cos(c+dx))}-7e^2\left(\frac{2e(e\sin(c+dx))^{3/2}(5a-3b\cos(c+dx))}{15b^2d}-\frac{e^2\left(\frac{\left(5a^2-3b^2\right)\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{b\sqrt{\sin(c+dx)}}-\frac{5a\left(a^2-b^2\right)\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{b}\right)}{5b^2}\right)}{2b}$$

$\Big\downarrow$ 3042

$$\frac{e(e\sin(c+dx))^{7/2}}{bd(a+b\cos(c+dx))}-\frac{7e^2\left(\frac{2e(e\sin(c+dx))^{3/2}(5a-3b\cos(c+dx))}{15b^2d}-\frac{e^2\left(\frac{\left(5a^2-3b^2\right)\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{b\sqrt{\sin(c+dx)}}-\frac{5a\left(a^2-b^2\right)\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{b}\right)}{5b^2}\right)}{2b}$$

$$\Big\downarrow \; 3119$$

$$\frac{e(e\sin(c+dx))^{7/2}}{bd(a+b\cos(c+dx))}-\frac{7e^2\left(\frac{2e(e\sin(c+dx))^{3/2}(5a-3b\cos(c+dx))}{15b^2d}-\frac{e^2\left(\frac{2\left(5a^2-3b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{5a\left(a^2-b^2\right)\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{b}\right)}{5b^2}\right)}{2b}$$

$$\Big\downarrow \; 3180$$

$$\frac{e(e\sin(c+dx))^{7/2}}{bd(a+b\cos(c+dx))}-\frac{7e^2\left(\frac{2e(e\sin(c+dx))^{3/2}(5a-3b\cos(c+dx))}{15b^2d}-\frac{e^2\left(\frac{2\left(5a^2-3b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{5a\left(a^2-b^2\right)\left(-\frac{be\int\frac{\sqrt{e\sin(c+dx)}}{b^2\sin^2(c+dx)e^2+\left(a^2-b^2\right)e^2}d}{d}\right.}{}\right.}{}\right.}{2b}$$

$$\Big\downarrow \; 266$$

$$\frac{e(e\sin(c+dx))^{7/2}}{bd(a+b\cos(c+dx))}-\frac{7e^2\left(\frac{2e(e\sin(c+dx))^{3/2}(5a-3b\cos(c+dx))}{15b^2d}-\frac{e^2\left(\frac{2\left(5a^2-3b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{5a\left(a^2-b^2\right)\left(-\frac{2be\int\frac{e^2\sin^2(c+dx)}{b^2e^4\sin^4(c+dx)+\left(a^2-b^2\right)e^2\cdots}}{d}\cdots\right)}{\cdots}\right)}{\cdots}\right)}{2b\cdots}$$

$\downarrow$ 827

$$\frac{e(e\sin(c+dx))^{7/2}}{bd(a+b\cos(c+dx))}-\frac{7e^2\left(\frac{2e(e\sin(c+dx))^{3/2}(5a-3b\cos(c+dx))}{15b^2d}-\frac{e^2\left(\frac{2\left(5a^2-3b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{5a\left(a^2-b^2\right)\left(-\frac{2be\left(\frac{\int\frac{1}{be^2\sin^2(c+dx)+\sqrt{b^2-a^2}e\cdots}d\cdots}{2b}\cdots\right)}{\cdots}\right)}{\cdots}\right)}{\cdots}\right)}{\cdots}$$

$\downarrow$ 218

$$\frac{e(e\sin(c+dx))^{7/2}}{bd(a+b\cos(c+dx))} - \frac{7e^2\left(\frac{2e(e\sin(c+dx))^{3/2}(5a-3b\cos(c+dx))}{15b^2d} - \frac{e^2\left(\frac{2(5a^2-3b^2)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}} - \frac{5a(a^2-b^2)\left(-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}} - \cdots\right)}{\cdots}\right)}{\cdots}\right)}{\cdots}\right)}{2b}$$

↓ 221

$$\frac{e(e\sin(c+dx))^{7/2}}{bd(a+b\cos(c+dx))} -$$

$$7e^2 \left( \frac{2e(e\sin(c+dx))^{3/2}(5a-3b\cos(c+dx))}{15b^2d} - \frac{e^2 \left( \frac{2\left(5a^2-3b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}} - \frac{5a\left(a^2-b^2\right)\left(-\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c \ldots\right.}}{2b}\right.}{\ldots}\right.}{\ldots}\right.$$

$$\ldots \; 2b$$

↓ 3042

$$\frac{e(e\sin(c+dx))^{7/2}}{bd(a+b\cos(c+dx))}-\frac{7e^2\left(\frac{2e(e\sin(c+dx))^{3/2}(5a-3b\cos(c+dx))}{15b^2d}-\frac{e^2\left(\frac{2\left(5a^2-3b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{5a\left(a^2-b^2\right)\left(-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c\right.}}{2b}\right.}{}\right)}{}\right)}{2b}$$

$\downarrow$ 3286

$$\frac{e(e\sin(c+dx))^{7/2}}{bd(a+b\cos(c+dx))}-$$

$$7e^2\left(\frac{2e(e\sin(c+dx))^{3/2}(5a-3b\cos(c+dx))}{15b^2d}-\frac{e^2\left(\frac{2\left(5a^2-3b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{5a\left(a^2-b^2\right)\left(-\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-\right.}}{2b\sqrt{e\sin(c+dx)}}\right.}{}\right)}{}\right.$$

$$2b$$

↓ 3042

$$\frac{e(e\sin(c+dx))^{7/2}}{bd(a+b\cos(c+dx))} -$$

$$7e^2 \left( \frac{2e(e\sin(c+dx))^{3/2}(5a-3b\cos(c+dx))}{15b^2d} - \frac{e^2 \left( \frac{2\left(5a^2-3b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}} - \frac{5a\left(a^2-b^2\right)\left( -\frac{ae\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-\right.}}}{2b\sqrt{e\sin(c+dx)}}\right.}{}\right.}{}\right.$$

$$2b$$

↓ 3284

$$\frac{e(e\sin(c+dx))^{7/2}}{bd(a+b\cos(c+dx))}-$$

$$7e^2\left(\frac{2e(e\sin(c+dx))^{3/2}(5a-3b\cos(c+dx))}{15b^2d}-\frac{e^2\left(\frac{2\left(5a^2-3b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{5a\left(a^2-b^2\right)\left(-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\cdots\right.}{d}\right.}{}\right.}{}\right.$$

$$2b$$

input
```
Int[(e*Sin[c + d*x])^(9/2)/(a + b*Cos[c + d*x])^2,x]
```

output
```
(e*(e*Sin[c + d*x])^(7/2))/(b*d*(a + b*Cos[c + d*x])) - (7*e^2*((2*e*(5*a
- 3*b*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2))/(15*b^2*d) - (e^2*((2*(5*a^2 -
 3*b^2)*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(b*d*Sqrt[S
in[c + d*x]]) - (5*a*(a^2 - b^2)*((-2*b*e*(ArcTan[(Sqrt[b]*Sqrt[e]*Sin[c +
 d*x])/(-a^2 + b^2)^(1/4)]/(2*b^(3/2)*(-a^2 + b^2)^(1/4)*Sqrt[e]) - ArcTan
h[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/(2*b^(3/2)*(-a^2 + b^
2)^(1/4)*Sqrt[e])))/d + (a*e*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c -
 Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(b*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*
Sin[c + d*x]]) + (a*e*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 +
 d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(b*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c +
 d*x]])))/b))/(5*b^2)))/(2*b)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3172
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Simp[g^2*((p - 1)/(b*(m + 1)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Fre
eQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && I
ntegersQ[2*m, 2*p]
```

rule 3180
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[a*(g/(2*b))   Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Simp[a*(g/(2*b))   In
t[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Simp[b*(g/f)   Su
bst[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3344
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Simp[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Si
n[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^
2*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1
, 0] && IntegerQ[2*m]
```

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

## Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1627 vs. $2(409) = 818$.

Time = 16.41 (sec) , antiderivative size = 1628, normalized size of antiderivative = 3.44

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1628 |

input
```
int((e*sin(d*x+c))^(9/2)/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)
```

output
```
(-4*e^3*a*b*(1/3*(e*sin(d*x+c))^(3/2)/b^4-e^2/b^4*((-1/4*a^2+1/4*b^2)*(e*s
in(d*x+c))^(3/2)/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)+1/8*(7/4*a^2-7/4*b^2)/b^2
/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*(ln((e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(
1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)
+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2
)^(1/2)))+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+
1)+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1))))+(
cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*e^5*(-1/5/b^4/(cos(d*x+c)^2*e*sin(d*x+c))
^(1/2)*(30*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*El
lipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2-16*(1-sin(d*x+c))^(1/2)*(2+2
*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(
1/2))*b^2-15*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*
EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2+8*(1-sin(d*x+c))^(1/2)*(2+
2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^
(1/2))*b^2+2*b^2*cos(d*x+c)^4-2*cos(d*x+c)^2*b^2)-(5*a^4-6*a^2*b^2+b^4)/b^
4*(-1/2/b^2*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(
cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(
d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-1/2/b^2*(1-sin(d*x+c))
^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))
^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^...
```

## Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))^(9/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
Timed out
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))**(9/2)/(a+b*cos(d*x+c))**2,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^{\frac{9}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

input
```
integrate((e*sin(d*x+c))^(9/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((e*sin(d*x + c))^(9/2)/(b*cos(d*x + c) + a)^2, x)
```

## Giac [F]

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^{\frac{9}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

input
```
integrate((e*sin(d*x+c))^(9/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*sin(d*x + c))^(9/2)/(b*cos(d*x + c) + a)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^2}\,dx=\int \frac{(e\,\sin\left(c+d\,x\right))^{9/2}}{\left(a+b\,\cos\left(c+d\,x\right)\right)^2}\,dx$$

input
```
int((e*sin(c + d*x))^(9/2)/(a + b*cos(c + d*x))^2,x)
```

output
```
int((e*sin(c + d*x))^(9/2)/(a + b*cos(c + d*x))^2, x)
```

## Reduce [F]

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^2}\,dx=\sqrt{e}\left(\int \frac{\sqrt{\sin\left(dx+c\right)}\,\sin\left(dx+c\right)^4}{\cos\left(dx+c\right)^2b^2+2\cos\left(dx+c\right)ab+a^2}dx\right)e^4$$

input
```
int((e*sin(d*x+c))^(9/2)/(a+b*cos(d*x+c))^2,x)
```

output
```
sqrt(e)*int((sqrt(sin(c + d*x))*sin(c + d*x)**4)/(cos(c + d*x)**2*b**2 + 2
*cos(c + d*x)*a*b + a**2),x)*e**4
```

# 3.70 $\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^2}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 487

$$\begin{aligned}
\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^2}\,dx &= \frac{5a\sqrt[4]{-a^2+b^2}\,e^{7/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{7/2}d} \\
&+ \frac{5a\sqrt[4]{-a^2+b^2}\,e^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{7/2}d} \\
&+ \frac{5(3a^2-b^2)\,e^4\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3b^4d\sqrt{e\sin(c+dx)}} \\
&- \frac{5a^2(a^2-b^2)\,e^4\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2b^4\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}} \\
&- \frac{5a^2(a^2-b^2)\,e^4\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2b^4\left(a^2-b\left(b+\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}} \\
&- \frac{5e^3(3a-b\cos(c+dx))\sqrt{e\sin(c+dx)}}{3b^3d} + \frac{e(e\sin(c+dx))^{5/2}}{bd(a+b\cos(c+dx))}
\end{aligned}$$

output
```
5/2*a*(-a^2+b^2)^(1/4)*e^(7/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b
^2)^(1/4)/e^(1/2))/b^(7/2)/d+5/2*a*(-a^2+b^2)^(1/4)*e^(7/2)*arctanh(b^(1/2
)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(7/2)/d+5/3*(3*a^2-b^2)
*e^4*InverseJacobiAM(1/2*c-1/4*Pi+1/2*d*x,2^(1/2))*sin(d*x+c)^(1/2)/b^4/d/
(e*sin(d*x+c))^(1/2)+5/2*a^2*(a^2-b^2)*e^4*EllipticPi(cos(1/2*c+1/4*Pi+1/2
*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^4/(a^2-b*(b-(-a
^2+b^2)^(1/2)))/d/(e*sin(d*x+c))^(1/2)+5/2*a^2*(a^2-b^2)*e^4*EllipticPi(co
s(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)
/b^4/(a^2-b*(b+(-a^2+b^2)^(1/2)))/d/(e*sin(d*x+c))^(1/2)-5/3*e^3*(3*a-b*co
s(d*x+c))*(e*sin(d*x+c))^(1/2)/b^3/d+e*(e*sin(d*x+c))^(5/2)/b/d/(a+b*cos(d
*x+c))
```

## Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 18.68 (sec) , antiderivative size = 1956, normalized size of antiderivative = 4.02

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(e*Sin[c + d*x])^(7/2)/(a + b*Cos[c + d*x])^2,x]
```

output
```
(((2*Cos[c + d*x])/(3*b^2) + (-a^2 + b^2)/(b^3*(a + b*Cos[c + d*x])))*Csc[
c + d*x]^3*(e*Sin[c + d*x])^(7/2))/d + ((e*Sin[c + d*x])^(7/2)*((2*(3*a^2
- 5*b^2)*Cos[c + d*x]^2*(a + b*Sqrt[1 - Sin[c + d*x]^2])*((a*(-2*ArcTan[1
- (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (
Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^
2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]
] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d
*x]] + b*Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)) + (5*b*(a^2
 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(
-a^2 + b^2)]*Sqrt[Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((-5*(a^2 - b^2)
*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 +
b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Sin[c + d*x]^2, (b^2*Sin[c +
d*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x
]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)])*Sin[c + d*x]^2)*(a^2 + b^2*(-1 +
Sin[c + d*x]^2)))))/((a + b*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) + (8*a*b*C
os[c + d*x]*(a + b*Sqrt[1 - Sin[c + d*x]^2])*(((-1/8 + I/8)*Sqrt[b]*(2*Arc
Tan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTa
n[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[
-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*
Sin[c + d*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/...
```

## Rubi [A] (warning: unable to verify)

Time = 2.20 (sec) , antiderivative size = 479, normalized size of antiderivative = 0.98, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.840$, Rules used = {3042, 3172, 25, 3042, 3344, 27, 3042, 3346, 3042, 3121, 3042, 3120, 3181, 266, 756, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{7/2}}{\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3172}$$

$$\frac{5e^2\int -\frac{\cos(c+dx)(e\sin(c+dx))^{3/2}}{a+b\cos(c+dx)}dx}{2b}+\frac{e(e\sin(c+dx))^{5/2}}{bd(a+b\cos(c+dx))}$$

$\Big\downarrow$ 25

$$\frac{e(e\sin(c+dx))^{5/2}}{bd(a+b\cos(c+dx))}-\frac{5e^2\int \frac{\cos(c+dx)(e\sin(c+dx))^{3/2}}{a+b\cos(c+dx)}dx}{2b}$$

$\Big\downarrow$ 3042

$$\frac{e(e\sin(c+dx))^{5/2}}{bd(a+b\cos(c+dx))}-\frac{5e^2\int \frac{\left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2b}$$

$\Big\downarrow$ 3344

$$\frac{e(e\sin(c+dx))^{5/2}}{bd(a+b\cos(c+dx))}-\frac{5e^2\left(\frac{2e^2\int -\frac{2ab+\left(3a^2-b^2\right)\cos(c+dx)}{2(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{3b^2}+\frac{2e\sqrt{e\sin(c+dx)}(3a-b\cos(c+dx))}{3b^2d}\right)}{2b}$$

$\Big\downarrow$ 27

$$\frac{e(e\sin(c+dx))^{5/2}}{bd(a+b\cos(c+dx))}-\frac{5e^2\left(\frac{2e\sqrt{e\sin(c+dx)}(3a-b\cos(c+dx))}{3b^2d}-\frac{e^2\int \frac{2ab+\left(3a^2-b^2\right)\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{3b^2}\right)}{2b}$$

$\Big\downarrow$ 3042

$$\frac{e(e\sin(c+dx))^{5/2}}{bd(a+b\cos(c+dx))}-\frac{5e^2\left(\frac{2e\sqrt{e\sin(c+dx)}(3a-b\cos(c+dx))}{3b^2d}-\frac{e^2\int \frac{2ab-\left(3a^2-b^2\right)\sin\left(c+dx-\frac{\pi}{2}\right)}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{3b^2}\right)}{2b}$$

$\Big\downarrow$ 3346

$$\frac{e(e\sin(c+dx))^{5/2}}{bd(a+b\cos(c+dx))}-\frac{5e^2\left(\frac{2e\sqrt{e\sin(c+dx)}(3a-b\cos(c+dx))}{3b^2d}-\frac{e^2\left(\frac{\left(3a^2-b^2\right)\int \frac{1}{\sqrt{e\sin(c+dx)}}dx}{b}-\frac{3a\left(a^2-b^2\right)\int \frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{b}\right)}{3b^2}\right)}{2b}$$

$\downarrow$ 3042

$$\frac{e(e\sin(c+dx))^{5/2}}{bd(a+b\cos(c+dx))}-\frac{5e^2\left(\frac{2e\sqrt{e\sin(c+dx)}(3a-b\cos(c+dx))}{3b^2d}-\frac{e^2\left(\frac{\left(3a^2-b^2\right)\int\frac{1}{\sqrt{e\sin(c+dx)}}dx}{b}-\frac{3a\left(a^2-b^2\right)\int\frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{b}\right)}{3b^2}\right)}{2b}$$

$\downarrow$ 3121

$$\frac{e(e\sin(c+dx))^{5/2}}{bd(a+b\cos(c+dx))}-\frac{5e^2\left(\frac{2e\sqrt{e\sin(c+dx)}(3a-b\cos(c+dx))}{3b^2d}-\frac{e^2\left(\frac{\left(3a^2-b^2\right)\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{b\sqrt{e\sin(c+dx)}}-\frac{3a\left(a^2-b^2\right)\int\frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{b}\right)}{3b^2}\right)}{2b}$$

$\downarrow$ 3042

$$\frac{e(e\sin(c+dx))^{5/2}}{bd(a+b\cos(c+dx))}-\frac{5e^2\left(\frac{2e\sqrt{e\sin(c+dx)}(3a-b\cos(c+dx))}{3b^2d}-\frac{e^2\left(\frac{\left(3a^2-b^2\right)\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{b\sqrt{e\sin(c+dx)}}-\frac{3a\left(a^2-b^2\right)\int\frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{b}\right)}{3b^2}\right)}{2b}$$

$\downarrow$ 3120

$$\frac{e(e\sin(c+dx))^{5/2}}{bd(a+b\cos(c+dx))}-\frac{5e^2\left(\frac{2e\sqrt{e\sin(c+dx)}(3a-b\cos(c+dx))}{3b^2d}-\frac{e^2\left(\frac{2\left(3a^2-b^2\right)\sqrt{\sin(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{3a\left(a^2-b^2\right)\int\frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right.}}{b}\right.}{3b^2}\right.}{2b}$$

$\downarrow$ 3181

$$\frac{e(e\sin(c+dx))^{5/2}}{bd(a+b\cos(c+dx))}-\frac{5e^2\left(\frac{2e\sqrt{e\sin(c+dx)}(3a-b\cos(c+dx))}{3b^2d}-\frac{e^2\left(\frac{2\left(3a^2-b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{3a\left(a^2-b^2\right)\left(-\frac{be\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b^2\sin^2(c+dx)e^2\right.}}{d}\right.}{}\right.}{}\right.}{2b}$$

$\downarrow$ 266

$$\frac{e(e\sin(c+dx))^{5/2}}{bd(a+b\cos(c+dx))}-\frac{5e^2\left(\frac{2e\sqrt{e\sin(c+dx)}(3a-b\cos(c+dx))}{3b^2d}-\frac{e^2\left(\frac{2\left(3a^2-b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{3a\left(a^2-b^2\right)\left(-\frac{2be\int\frac{1}{b^2e^4\sin^4(c+dx)+\left(a^2-b^2\right)e^2}}{d}\right.}{}\right.}{}\right.}{2b}$$

$\downarrow$ 756

$$\frac{e(e\sin(c+dx))^{5/2}}{bd(a+b\cos(c+dx))} - \frac{5e^2\left(\frac{2e\sqrt{e\sin(c+dx)}(3a-b\cos(c+dx))}{3b^2d} - \frac{e^2\left(\frac{2\left(3a^2-b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}} - \frac{3a\left(a^2-b^2\right)\left(-\frac{2be\left(-\frac{\int \frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)} \cdots}{2e\sqrt{b^2-a^2}} \cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

$\downarrow$ 218

$$\frac{e(e\sin(c+dx))^{5/2}}{bd(a+b\cos(c+dx))} -$$

$$5e^2\left(\frac{2e\sqrt{e\sin(c+dx)}(3a-b\cos(c+dx))}{3b^2d} - \frac{e^2\left(\frac{2\left(3a^2-b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}} - \frac{3a\left(a^2-b^2\right)\left(-\frac{2be\left(-\frac{\int \frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}}{2e\sqrt{b^2-a^2}}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

$$\overline{\qquad\qquad\qquad\qquad 2b \qquad}$$

↓ 221

$$\frac{e(e\sin(c+dx))^{5/2}}{bd(a+b\cos(c+dx))} -$$

$$5e^2\left(\frac{2e\sqrt{e\sin(c+dx)}(3a-b\cos(c+dx))}{3b^2d} - \frac{e^2\left(\frac{2\left(3a^2-b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}} - \frac{3a\left(a^2-b^2\right)\left(-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c\cdots\right.}}{2\sqrt{b^2-a^2}}\cdots\right.}{\cdots}\right.}{\cdots}\right.$$

$$\overline{\qquad\qquad\qquad 2b \cdots}$$

$\downarrow$ 3042

$$\frac{e(e\sin(c+dx))^{5/2}}{bd(a+b\cos(c+dx))} -$$

$$3a\left(a^2-b^2\right)\left(-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c\cdots\right.}}{2\sqrt{b^2-a^2}}\right.$$

$$e^2\left(\frac{2\left(3a^2-b^2\right)\sqrt{\sin(c+dx)}\,\mathrm{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\cdots\right.$$

$$5e^2\left(\frac{2e\sqrt{e\sin(c+dx)}(3a-b\cos(c+dx))}{3b^2d}-\cdots\right.$$

$$\overline{\qquad 2b \cdots}$$

↓ 3286

$$\frac{e(e\sin(c+dx))^{5/2}}{bd(a+b\cos(c+dx))} -$$

$$3a\left(a^2-b^2\right)\left( -\frac{a\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a\right.}}}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+}} \right.$$

$$e^2\left(\frac{2\left(3a^2-b^2\right)\sqrt{\sin(c+dx)}\,\mathrm{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}} - \right.$$

$$5e^2\left(\frac{2e\sqrt{e\sin(c+dx)}(3a-b\cos(c+dx))}{3b^2d} - \right.$$

$$2b$$

$\downarrow$ 3042

$$\frac{e(e\sin(c+dx))^{5/2}}{bd(a+b\cos(c+dx))}-$$

$$5e^2\left(\frac{2e\sqrt{e\sin(c+dx)}(3a-b\cos(c+dx))}{3b^2d}-\frac{e^2\left(\frac{2(3a^2-b^2)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{3a(a^2-b^2)\left(-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}\ldots\right.}}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+\ldots}}\right.}{\ldots}}{\ldots}\right.$$

$2b\ldots$

↓ 3284

$$\frac{e(e\sin(c+dx))^{5/2}}{bd(a+b\cos(c+dx))} - \frac{5e^2\left(\frac{2e\sqrt{e\sin(c+dx)}(3a-b\cos(c+dx))}{3b^2d} - \frac{e^2\left(\frac{2\left(3a^2-b^2\right)\sqrt{\sin(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}} - \frac{3a\left(a^2-b^2\right)\left(-\frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}} - \cdots\right)}{d\cdots}\cdots\right)}{\cdots}\right)}{\cdots}\right)}{2b\cdots}$$

input
```
Int[(e*Sin[c + d*x])^(7/2)/(a + b*Cos[c + d*x])^2,x]
```

output
```
(e*(e*Sin[c + d*x])^(5/2))/(b*d*(a + b*Cos[c + d*x])) - (5*e^2*((2*e*(3*a
- b*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]])/(3*b^2*d) - (e^2*((2*(3*a^2 - b^2)
*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(b*d*Sqrt[e*Sin[c +
d*x]]) - (3*a*(a^2 - b^2)*((-2*b*e*(-1/2*ArcTan[(Sqrt[b]*Sqrt[e]*Sin[c + d
*x])/(-a^2 + b^2)^(1/4)]/(Sqrt[b]*(-a^2 + b^2)^(3/4)*e^(3/2)) - ArcTanh[(S
qrt[b]*Sqrt[e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/(2*Sqrt[b]*(-a^2 + b^2)^(
3/4)*e^(3/2))))/d + (a*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2
+ d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(Sqrt[-a^2 + b^2]*(b - Sqrt[-a^2 + b^2])*
d*Sqrt[e*Sin[c + d*x]]) - (a*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c -
 Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(Sqrt[-a^2 + b^2]*(b + Sqrt[-a^2 +
b^2])*d*Sqrt[e*Sin[c + d*x]])))/b))/(3*b^2)))/(2*b)
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3172
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Simp[g^2*((p - 1)/(b*(m + 1)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Fre
eQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && I
ntegersQ[2*m, 2*p]
```

rule 3181
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[-a/(2*q)   Int[1/(
Sqrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Simp[b*(g/f)   Subst[
Int[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - S
imp[a/(2*q)   Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3344
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Simp[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Si
n[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^
2*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1
, 0] && IntegerQ[2*m]
```

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

## Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1500 vs. $2(422) = 844$.

Time = 16.47 (sec) , antiderivative size = 1501, normalized size of antiderivative = 3.08

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1501 |

input
```
int((e*sin(d*x+c))^(7/2)/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)
```

output
```
(-4*e^3*a*b*(1/b^4*(e*sin(d*x+c))^(1/2)-e^2/b^4*((-1/4*a^2+1/4*b^2)*(e*sin
(d*x+c))^(1/2)/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)+5/32*(a^2-b^2)*(e^2*(a^2-b^
2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*(ln((e*sin(d*x+c)+(e^2*(a^2-b^2)/b
^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d
*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2
)/b^2)^(1/2)))+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(
1/2)+1)+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)
)))+(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*e^4*(-1/3/b^4/(cos(d*x+c)^2*e*sin(d*
x+c))^(1/2)*(9*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2
)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2-4*(1-sin(d*x+c))^(1/2)*(
2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*
2^(1/2))*b^2-2*b^2*cos(d*x+c)^2*sin(d*x+c))-1/b^4*(5*a^4-6*a^2*b^2+b^4)*(-
1/2/b/(-a^2+b^2)^(1/2)*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x
+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*Ellipti
cPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/2/b/(-a^2
+b^2)^(1/2)*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(
cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(
d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2)))+2*a^2*(a^4-2*a^2*b^2+
b^4)/b^4*(1/2*b^2/e/a^2/(a^2-b^2)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-cos(
d*x+c)^2*b^2+a^2)+1/4/a^2/(a^2-b^2)*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c...
```

## Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))^(7/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
Timed out
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))**(7/2)/(a+b*cos(d*x+c))**2,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^{\frac{7}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

input
```
integrate((e*sin(d*x+c))^(7/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((e*sin(d*x + c))^(7/2)/(b*cos(d*x + c) + a)^2, x)
```

## Giac [F]

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^{\frac{7}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

input
```
integrate((e*sin(d*x+c))^(7/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*sin(d*x + c))^(7/2)/(b*cos(d*x + c) + a)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^2}\,dx=\int \frac{(e\ \sin\left(c+d\,x\right))^{7/2}}{\left(a+b\ \cos\left(c+d\,x\right)\right)^2}\,dx$$

input
```
int((e*sin(c + d*x))^(7/2)/(a + b*cos(c + d*x))^2,x)
```

output
```
int((e*sin(c + d*x))^(7/2)/(a + b*cos(c + d*x))^2, x)
```

## Reduce [F]

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^2}\,dx=\sqrt{e}\left(\int \frac{\sqrt{\sin\left(dx+c\right)}\ \sin\left(dx+c\right)^3}{\cos\left(dx+c\right)^2 b^2+2\cos\left(dx+c\right)ab+a^2}dx\right)e^3$$

input
```
int((e*sin(d*x+c))^(7/2)/(a+b*cos(d*x+c))^2,x)
```

output
```
sqrt(e)*int((sqrt(sin(c + d*x))*sin(c + d*x)**3)/(cos(c + d*x)**2*b**2 + 2
*cos(c + d*x)*a*b + a**2),x)*e**3
```

# 3.71 $\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^2}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 404

$$\begin{aligned}\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^2}\,dx &= -\frac{3ae^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{5/2}\sqrt[4]{-a^2+b^2}d} \\ &+ \frac{3ae^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{5/2}\sqrt[4]{-a^2+b^2}d} \\ &+ \frac{3a^2e^3\,\mathrm{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2b^3\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} \\ &+ \frac{3a^2e^3\,\mathrm{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2b^3\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} \\ &- \frac{3e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{b^2d\sqrt{\sin(c+dx)}} + \frac{e(e\sin(c+dx))^{3/2}}{bd(a+b\cos(c+dx))}\end{aligned}$$

output
```
-3/2*a*e^(5/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2
))/b^(5/2)/(-a^2+b^2)^(1/4)/d+3/2*a*e^(5/2)*arctanh(b^(1/2)*(e*sin(d*x+c))
^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(5/2)/(-a^2+b^2)^(1/4)/d-3/2*a^2*e^3*El
lipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d
*x+c)^(1/2)/b^3/(b-(-a^2+b^2)^(1/2))/d/(e*sin(d*x+c))^(1/2)-3/2*a^2*e^3*El
lipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d
*x+c)^(1/2)/b^3/(b+(-a^2+b^2)^(1/2))/d/(e*sin(d*x+c))^(1/2)+3*e^2*Elliptic
E(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/b^2/d/sin(d*x+c)
^(1/2)+e*(e*sin(d*x+c))^(3/2)/b/d/(a+b*cos(d*x+c))
```

## Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.20 (sec) , antiderivative size = 366, normalized size of antiderivative = 0.91

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^2}\,dx = \frac{(e\sin(c+dx))^{5/2}\left(8b^{3/2}\csc(c+dx) + \frac{\left(a+b\sqrt{\cos^2(c+dx)}\right)\left(3\sqrt{2}a(a^2-b^2)^{3/4}\left(2\arctan\left(\cdots\right.\right.\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(e*Sin[c + d*x])^(5/2)/(a + b*Cos[c + d*x])^2,x]
```

output
```
((e*Sin[c + d*x])^(5/2)*(8*b^(3/2)*Csc[c + d*x] + ((a + b*Sqrt[Cos[c + d*x
]^2])*(3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[S
in[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[
c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2
 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] +
 Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]]) +
 8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2
)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2)))/((a^2 - b^2)*Sin[c + d*x]^(5/2))))/(8
*b^(5/2)*d*(a + b*Cos[c + d*x]))
```

## Rubi [A] (warning: unable to verify)

Time = 1.66 (sec) , antiderivative size = 393, normalized size of antiderivative = 0.97, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.720, Rules used = {3042, 3172, 25, 3042, 3346, 3042, 3121, 3042, 3119, 3180, 266, 827, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2}}{\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3172}$$

$$\frac{3e^2\int -\frac{\cos(c+dx)\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx}{2b}+\frac{e(e\sin(c+dx))^{3/2}}{bd(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{e(e\sin(c+dx))^{3/2}}{bd(a+b\cos(c+dx))}-\frac{3e^2\int \frac{\cos(c+dx)\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e(e\sin(c+dx))^{3/2}}{bd(a+b\cos(c+dx))}-\frac{3e^2\int \frac{\sqrt{-e\cos\left(c+dx+\frac{\pi}{2}\right)}\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2b}$$

$$\Big\downarrow \text{3346}$$

$$\frac{e(e\sin(c+dx))^{3/2}}{bd(a+b\cos(c+dx))}-\frac{3e^2\left(\frac{\int \sqrt{e\sin(c+dx)}dx}{b}-\frac{a\int \frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx}{b}\right)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e(e\sin(c+dx))^{3/2}}{bd(a+b\cos(c+dx))}-\frac{3e^2\left(\frac{\int\sqrt{e\sin(c+dx)}dx}{b}-\frac{a\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{b}\right)}{2b}$$

$\downarrow$ 3121

$$\frac{e(e\sin(c+dx))^{3/2}}{bd(a+b\cos(c+dx))}-\frac{3e^2\left(\frac{\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{b\sqrt{\sin(c+dx)}}-\frac{a\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{b}\right)}{2b}$$

$\downarrow$ 3042

$$\frac{e(e\sin(c+dx))^{3/2}}{bd(a+b\cos(c+dx))}-\frac{3e^2\left(\frac{\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{b\sqrt{\sin(c+dx)}}-\frac{a\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{b}\right)}{2b}$$

$\downarrow$ 3119

$$\frac{e(e\sin(c+dx))^{3/2}}{bd(a+b\cos(c+dx))}-\frac{3e^2\left(\frac{2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{a\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{b}\right)}{2b}$$

$\downarrow$ 3180

$$\frac{e(e\sin(c+dx))^{3/2}}{bd(a+b\cos(c+dx))}-\frac{3e^2\left(\frac{2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{a\left(-\frac{be\int\frac{\sqrt{e\sin(c+dx)}}{b^2\sin^2(c+dx)e^2+\left(a^2-b^2\right)e^2}d(e\sin(c+dx))}{d}-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\cdots\right)}{b}\right)}{2b}$$

$\downarrow$ 266

$$\frac{3e^2\left(\frac{2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{\frac{e(e\sin(c+dx))^{3/2}}{bd(a+b\cos(c+dx))}-\frac{a\left(-\frac{2be\int\frac{e^2\sin^2(c+dx)}{b^2e^4\sin^4(c+dx)+\left(a^2-b^2\right)e^2}d\sqrt{e\sin(c+dx)}}{d}-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\cdots\right.}{b}}{\cdots}\right.}{2b}$$

$\downarrow$ 827

$$\frac{3e^2\left(\frac{2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{\frac{e(e\sin(c+dx))^{3/2}}{bd(a+b\cos(c+dx))}-\frac{a\left(-\frac{2be\left(\frac{\int\frac{1}{be^2\sin^2(c+dx)+\sqrt{b^2-a^2}e}d\sqrt{e\sin(c+dx)}}{2b}-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2b}\right)}{d}-\cdots\right.}{b}}{\cdots}\right.}{2b}$$

$\downarrow$ 218

$$\frac{3e^2\left(\frac{2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{\frac{e(e\sin(c+dx))^{3/2}}{bd(a+b\cos(c+dx))}-\frac{a\left(-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2b}\right)}{d}-\frac{ae\int\frac{\cdots}{\sqrt{e\sin(c+dx)}\left(\cdots\right.}}{\cdots}\right.}{b}}{\cdots}\right.}{2b}$$

$\downarrow$ 221

$$\frac{3e^2\left(\frac{2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{a\left(-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b}-\frac{2be\left(\cdots\right)}{\cdots}\cdots\right)}{b}\cdots\right)}{2b}\cdots \qquad \frac{e(e\sin(c+dx))^{3/2}}{bd(a+b\cos(c+dx))}-\cdots$$

$\downarrow$ 3042

$$\frac{3e^2\left(\frac{2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{a\left(-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b}-\frac{2be\left(\cdots\right)}{\cdots}\cdots\right)}{b}\cdots\right)}{2b}\cdots \qquad \frac{e(e\sin(c+dx))^{3/2}}{bd(a+b\cos(c+dx))}-\cdots$$

$\downarrow$ 3286

$$\frac{e(e\sin(c+dx))^{3/2}}{bd(a+b\cos(c+dx))} - \frac{3e^2\left(\frac{2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}} - \frac{a\left(-\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b\sqrt{e\sin(c+dx)}} + \frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{\right.}}}{2b\sqrt{e\sin(c+dx)}}\right.}{b}\right.}{2b}$$

$\downarrow$ 3042

$$\frac{e(e\sin(c+dx))^{3/2}}{bd(a+b\cos(c+dx))} - \frac{3e^2\left(\frac{2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}} - \frac{a\left(-\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b\sqrt{e\sin(c+dx)}} + \frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{\right.}}}{2b\sqrt{e\sin(c+dx)}}\right.}{b}\right.}{2b}$$

$\downarrow$ 3284

$$\frac{e(e\sin(c+dx))^{3/2}}{bd(a+b\cos(c+dx))} - \frac{3e^2\left(\frac{2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}} - \frac{a\left(-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{d} + \frac{ae\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2\cdots}{b-\sqrt{b\cdots}}\right.}{bd\left(b-\sqrt{b^2-a^2}\right)\sqrt{\cdots}}\cdots\right)}{b}\cdots\right)}{2b}$$

input
```
Int[(e*Sin[c + d*x])^(5/2)/(a + b*Cos[c + d*x])^2,x]
```

output
```
(e*(e*Sin[c + d*x])^(3/2))/(b*d*(a + b*Cos[c + d*x])) - (3*e^2*((2*Ellipti
cE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(b*d*Sqrt[Sin[c + d*x]]) -
 (a*((-2*b*e*(ArcTan[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/(2
*b^(3/2)*(-a^2 + b^2)^(1/4)*Sqrt[e]) - ArcTanh[(Sqrt[b]*Sqrt[e]*Sin[c + d*
x])/(-a^2 + b^2)^(1/4)]/(2*b^(3/2)*(-a^2 + b^2)^(1/4)*Sqrt[e])))/d + (a*e*
EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c
 + d*x]])/(b*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]]) + (a*e*Ellipti
cPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]
])/(b*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]])))/b))/(2*b)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3172
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Simp[g^2*((p - 1)/(b*(m + 1)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Fre
eQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && I
ntegersQ[2*m, 2*p]
```

rule 3180
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[a*(g/(2*b))   Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Simp[a*(g/(2*b))   In
t[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Simp[b*(g/f)   Su
bst[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1667 vs. $2(346) = 692$.

Time = 15.64 (sec) , antiderivative size = 1668, normalized size of antiderivative = 4.13

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1668 |

input
```
int((e*sin(d*x+c))^(5/2)/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)
```

output
```
(-2*e^3*a*b*(-1/2*(e*sin(d*x+c))^(3/2)/b^2/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)
+3/16/b^4/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*(ln((e*sin(d*x+c)-(e^2*(a^2-b^
2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*s
in(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2
-b^2)/b^2)^(1/2)))+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c
))^(1/2)+1)+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2
)-1)))+1/4*e^3*a^2*(3*(-a^2+b^2)^(1/2)*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c
))^(1/2)*sin(d*x+c)^(5/2)*EllipticPi((1-sin(d*x+c))^(1/2),-b/(-b+(-a^2+b^2
)^(1/2)),1/2*2^(1/2))*b^2-3*(-a^2+b^2)^(1/2)*(1-sin(d*x+c))^(1/2)*(2+2*sin
(d*x+c))^(1/2)*sin(d*x+c)^(5/2)*EllipticPi((1-sin(d*x+c))^(1/2),b/(b+(-a^2
+b^2)^(1/2)),1/2*2^(1/2))*b^2-12*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/
2)*sin(d*x+c)^(5/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^3+6*(1-s
in(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(5/2)*EllipticF((1-sin(
d*x+c))^(1/2),1/2*2^(1/2))*b^3+3*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/
2)*sin(d*x+c)^(5/2)*EllipticPi((1-sin(d*x+c))^(1/2),-b/(-b+(-a^2+b^2)^(1/2
)),1/2*2^(1/2))*b^3+3*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+
c)^(5/2)*EllipticPi((1-sin(d*x+c))^(1/2),b/(b+(-a^2+b^2)^(1/2)),1/2*2^(1/2
))*b^3+3*(-a^2+b^2)^(1/2)*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(
d*x+c)^(1/2)*EllipticPi((1-sin(d*x+c))^(1/2),-b/(-b+(-a^2+b^2)^(1/2)),1/2*
2^(1/2))*a^2-3*(-a^2+b^2)^(1/2)*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(...
```

## Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
Timed out
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))**(5/2)/(a+b*cos(d*x+c))**2,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^{\frac{5}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

input
```
integrate((e*sin(d*x+c))^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((e*sin(d*x + c))^(5/2)/(b*cos(d*x + c) + a)^2, x)
```

## Giac [F]

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^{\frac{5}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

input
```
integrate((e*sin(d*x+c))^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*sin(d*x + c))^(5/2)/(b*cos(d*x + c) + a)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(e\,\sin(c+d\,x))^{5/2}}{(a+b\,\cos(c+d\,x))^2}\,dx$$

input
```
int((e*sin(c + d*x))^(5/2)/(a + b*cos(c + d*x))^2,x)
```

output
```
int((e*sin(c + d*x))^(5/2)/(a + b*cos(c + d*x))^2, x)
```

## Reduce [F]

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^2}\,dx = \sqrt{e}\left(\int \frac{\sqrt{\sin(dx+c)}\,\sin(dx+c)^2}{\cos(dx+c)^2 b^2+2\cos(dx+c)\,ab+a^2}dx\right)e^2$$

input
```
int((e*sin(d*x+c))^(5/2)/(a+b*cos(d*x+c))^2,x)
```

output
```
sqrt(e)*int((sqrt(sin(c + d*x))*sin(c + d*x)**2)/(cos(c + d*x)**2*b**2 + 2
*cos(c + d*x)*a*b + a**2),x)*e**2
```

# 3.72 $\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^2}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 418

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^2}\,dx = \frac{ae^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{3/2}\left(-a^2+b^2\right)^{3/4}d} + \frac{ae^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{3/2}\left(-a^2+b^2\right)^{3/4}d} - \frac{e^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b^2d\sqrt{e\sin(c+dx)}} + \frac{a^2e^2\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2b^2\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}} + \frac{a^2e^2\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2b^2\left(a^2-b\left(b+\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}} + \frac{e\sqrt{e\sin(c+dx)}}{bd(a+b\cos(c+dx))}$$

output
```
1/2*a*e^(3/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2)
)/b^(3/2)/(-a^2+b^2)^(3/4)/d+1/2*a*e^(3/2)*arctanh(b^(1/2)*(e*sin(d*x+c))^
(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(3/2)/(-a^2+b^2)^(3/4)/d-e^2*InverseJaco
biAM(1/2*c-1/4*Pi+1/2*d*x,2^(1/2))*sin(d*x+c)^(1/2)/b^2/d/(e*sin(d*x+c))^(
1/2)-1/2*a^2*e^2*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1
/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^2/(a^2-b*(b-(-a^2+b^2)^(1/2)))/d/(e*sin(d
*x+c))^(1/2)-1/2*a^2*e^2*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2
+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^2/(a^2-b*(b+(-a^2+b^2)^(1/2)))/d/
(e*sin(d*x+c))^(1/2)+e*(e*sin(d*x+c))^(1/2)/b/d/(a+b*cos(d*x+c))
```

## Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.18 (sec) , antiderivative size = 614, normalized size of antiderivative = 1.47

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^2}\,dx = \frac{\csc(c+dx)(e\sin(c+dx))^{3/2}}{bd(a+b\cos(c+dx))}$$

$$-\frac{\cos^2(c+dx)(e\sin(c+dx))^{3/2}\left(a+b\sqrt{1-\sin^2(c+dx)}\right)\left(\frac{a\left(-2\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)+2\arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{\sin}}{\sqrt[4]{a^2-}}\right.\right.}{}\right.}{}$$

input
```
Integrate[(e*Sin[c + d*x])^(3/2)/(a + b*Cos[c + d*x])^2,x]
```

output
```
(Csc[c + d*x]*(e*Sin[c + d*x])^(3/2))/(b*d*(a + b*Cos[c + d*x])) - (Cos[c
+ d*x]^2*(e*Sin[c + d*x])^(3/2)*(a + b*Sqrt[1 - Sin[c + d*x]^2])*((a*(-2*A
rcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcT
an[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[
a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[
c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[S
in[c + d*x]] + b*Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)) + (
5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d
*x]^2)/(-a^2 + b^2)]*Sqrt[Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((-5*(a^
2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/
(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Sin[c + d*x]^2, (b^2*
Sin[c + d*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin
[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)])*Sin[c + d*x]^2)*(a^2 + b^
2*(-1 + Sin[c + d*x]^2)))))/(b*d*(a + b*Cos[c + d*x])*Sin[c + d*x]^(3/2)*(
1 - Sin[c + d*x]^2))
```

## Rubi [A] (warning: unable to verify)

Time = 1.70 (sec) , antiderivative size = 411, normalized size of antiderivative = 0.98, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.760$, Rules used = {3042, 3172, 25, 3042, 25, 3346, 3042, 3121, 3042, 3120, 3181, 266, 756, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^2}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}}{\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \; 3172$$

$$\frac{e^2\int -\frac{\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{2b}+\frac{e\sqrt{e\sin(c+dx)}}{bd(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 25$$

$$\frac{e\sqrt{e\sin(c+dx)}}{bd(a+b\cos(c+dx))}-\frac{e^2\int\frac{\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e\sqrt{e\sin(c+dx)}}{bd(a+b\cos(c+dx))}-\frac{e^2\int-\frac{\sin\left(c+dx-\frac{\pi}{2}\right)}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{2b}$$

$$\Big\downarrow \text{25}$$

$$\frac{e^2\int\frac{\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}{\sqrt{e\cos\left(\frac{1}{2}(2c-\pi)+dx\right)}\left(a-b\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)}dx}{2b}+\frac{e\sqrt{e\sin(c+dx)}}{bd(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3346}$$

$$\frac{e^2\left(\frac{a\int\frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{b}-\frac{\int\frac{1}{\sqrt{e\sin(c+dx)}}dx}{b}\right)}{2b}+\frac{e\sqrt{e\sin(c+dx)}}{bd(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e^2\left(\frac{a\int\frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{b}-\frac{\int\frac{1}{\sqrt{e\sin(c+dx)}}dx}{b}\right)}{2b}+\frac{e\sqrt{e\sin(c+dx)}}{bd(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3121}$$

$$\frac{e^2\left(\frac{a\int\frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{b}-\frac{\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{b\sqrt{e\sin(c+dx)}}\right)}{2b}+\frac{e\sqrt{e\sin(c+dx)}}{bd(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e^2\left(\frac{a\int\frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{b}-\frac{\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{b\sqrt{e\sin(c+dx)}}\right)}{2b}+\frac{e\sqrt{e\sin(c+dx)}}{bd(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3120}$$

$$\frac{e^2\left(\frac{a\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{b}-\frac{2\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}\right)}{2b}+\frac{e\sqrt{e\sin(c+dx)}}{bd(a+b\cos(c+dx))}$$

$\downarrow$ 3181

$$\frac{e^2\left(\frac{a\left(-\frac{be\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b^2\sin^2(c+dx)e^2+\left(a^2-b^2\right)e^2\right)}d(e\sin(c+dx))}{d}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{\ldots}\right.}}{2\sqrt{b^2-a^2}}\right.}{b}\right.}{2b}$$

$$\frac{e\sqrt{e\sin(c+dx)}}{bd(a+b\cos(c+dx))}$$

$\downarrow$ 266

$$\frac{e^2\left(\frac{a\left(-\frac{2be\int \frac{1}{b^2e^4\sin^4(c+dx)+\left(a^2-b^2\right)e^2}d\sqrt{e\sin(c+dx)}}{d}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}}\right)}{b}\right.}{2b}$$

$$\frac{e\sqrt{e\sin(c+dx)}}{bd(a+b\cos(c+dx))}$$

$\downarrow$ 756

$$\frac{e^2\left(\frac{a\left(-\frac{2be\left(-\frac{\int \frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2e\sqrt{b^2-a^2}}-\frac{\int \frac{1}{be^2\sin^2(c+dx)+\sqrt{b^2-a^2}e}d\sqrt{e\sin(c+dx)}}{2e\sqrt{b^2-a^2}}\right)}{d}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\ldots\right.}{b}\right.}{2b}$$

$$\frac{e\sqrt{e\sin(c+dx)}}{bd(a+b\cos(c+dx))}$$

$\downarrow$ 218

$$e^2\left(\frac{a\left(-\frac{2be\left(-\frac{\int \frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2e\sqrt{b^2-a^2}}-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}\right)}{d}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\ldots}}{\ldots}\right)}{b}\ldots}{2b}\ldots\right)$$

$$\frac{e\sqrt{e\sin(c+dx)}}{bd(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{221}$$

$$e^2\left(\frac{a\left(-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}}-\frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\ldots}{\sqrt[4]{\ldots}}\right)}{2\sqrt{b}e^{3/2}\left(b^2\ldots\right.}\ldots\right)}{d}\ldots\right)}{b}\ldots}{2b}\ldots\right)$$

$$\frac{e\sqrt{e\sin(c+dx)}}{bd(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$e^2 \left( \frac{a \left( -\frac{a \int \frac{1}{\sqrt{e \sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}} - \frac{a \int \frac{1}{\sqrt{e \sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}} - \frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{}}{\sqrt[4]{}}\right.}{2\sqrt{b}e^{3/2}\left(b^2\right.}\right.}{d} \right.}{b} \right.}{2b}$$

$$\frac{e\sqrt{e\sin(c+dx)}}{bd(a+b\cos(c+dx))}$$

$\downarrow$ 3286

$$e^2 \left( \frac{a \left( -\frac{a\sqrt{\sin(c+dx)} \int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}} - \frac{a\sqrt{\sin(c+dx)} \int \frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}} - \frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right.}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}\right.}{} \right.}{b} \right.}{2b}$$

$$\frac{e\sqrt{e\sin(c+dx)}}{bd(a+b\cos(c+dx))}$$

$\downarrow$ 3042

$$e^2\left(\frac{a\left(-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}}-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}}-\frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}\cdots\right.}{\cdots}\cdots\right.}{b}\cdots}{2b}\right.\cdots$$

$$\frac{e\sqrt{e\sin(c+dx)}}{bd(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3284}$$

$$e^2\left(\frac{a\left(-\frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}\right)}{d}+\frac{a\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{b^2-a^2}\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\sin(c+dx)}}-\frac{a\sqrt{\sin(c+dx)}\,\text{Ellip}\cdots}{d\sqrt{b^2-a^2}\cdots}\cdots\right.}{b}\cdots}{2b}\right.\cdots$$

$$\frac{e\sqrt{e\sin(c+dx)}}{bd(a+b\cos(c+dx))}$$

input
```
Int[(e*Sin[c + d*x])^(3/2)/(a + b*Cos[c + d*x])^2,x]
```

output
```
(e*Sqrt[e*Sin[c + d*x]])/(b*d*(a + b*Cos[c + d*x])) + (e^2*((-2*EllipticF[
(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(b*d*Sqrt[e*Sin[c + d*x]]) + (a
*((-2*b*e*(-1/2*ArcTan[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/
(Sqrt[b]*(-a^2 + b^2)^(3/4)*e^(3/2)) - ArcTanh[(Sqrt[b]*Sqrt[e]*Sin[c + d*
x])/(-a^2 + b^2)^(1/4)]/(2*Sqrt[b]*(-a^2 + b^2)^(3/4)*e^(3/2))))/d + (a*El
lipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c +
 d*x]])/(Sqrt[-a^2 + b^2]*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]]) -
 (a*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[S
in[c + d*x]])/(Sqrt[-a^2 + b^2]*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*
x]])))/b))/(2*b)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3172
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Simp[g^2*((p - 1)/(b*(m + 1)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Fre
eQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && I
ntegersQ[2*m, 2*p]
```

rule 3181
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[-a/(2*q)   Int[1/(
Sqrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Simp[b*(g/f)   Subst[
Int[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - S
imp[a/(2*q)   Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1370 vs. $2(359) = 718$.

Time = 2.32 (sec) , antiderivative size = 1371, normalized size of antiderivative = 3.28

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1371 |

input
```
int((e*sin(d*x+c))^(3/2)/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)
```

output

```
(-4*e^3*a*b*(-1/4/b^2*(e*sin(d*x+c))^(1/2)/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)
+1/32/b^2*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*(ln((e*sin(d
*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2
)/b^2)^(1/2))/(e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)
*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^
(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(
e*sin(d*x+c))^(1/2)-1)))+(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*e^2*(1/b^2*(1-s
in(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*s
in(d*x+c))^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))+1/b^2*(3*a^2-
b^2)*(-1/2/b/(-a^2+b^2)^(1/2)*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*
sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*
EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/2/
b/(-a^2+b^2)^(1/2)*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^
(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi(
(1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2)))-2*a^2*(a^2-b^2
)/b^2*(1/2*b^2/e/a^2/(a^2-b^2)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-cos(d*x
+c)^2*b^2+a^2)+1/4/a^2/(a^2-b^2)*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/
2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((1-sin(d*x
+c))^(1/2),1/2*2^(1/2))-5/8/(a^2-b^2)/b/(-a^2+b^2)^(1/2)*(1-sin(d*x+c))^(1
/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))...
```

## Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input

```
integrate((e*sin(d*x+c))^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

output

```
Timed out
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))**(3/2)/(a+b*cos(d*x+c))**2,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^{\frac{3}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

input
```
integrate((e*sin(d*x+c))^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((e*sin(d*x + c))^(3/2)/(b*cos(d*x + c) + a)^2, x)
```

## Giac [F]

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^{\frac{3}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

input
```
integrate((e*sin(d*x+c))^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*sin(d*x + c))^(3/2)/(b*cos(d*x + c) + a)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(e\,\sin(c+d\,x))^{3/2}}{(a+b\,\cos(c+d\,x))^2}\,dx$$

input
```
int((e*sin(c + d*x))^(3/2)/(a + b*cos(c + d*x))^2,x)
```

output
```
int((e*sin(c + d*x))^(3/2)/(a + b*cos(c + d*x))^2, x)
```

## Reduce [F]

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^2}\,dx = \frac{\sqrt{e}\,e\left(-\cos(dx+c)\left(\int \frac{\sqrt{\sin(dx+c)}\,\cos(dx+c)}{\cos(dx+c)\sin(dx+c)b+\sin(dx+c)a}dx\right)bd+2\sqrt{\sin(dx+c)}-\right.}{2bd\left(\cos(dx+c)\,b+a\right)}$$

input
```
int((e*sin(d*x+c))^(3/2)/(a+b*cos(d*x+c))^2,x)
```

output
```
(sqrt(e)*e*( - cos(c + d*x)*int((sqrt(sin(c + d*x))*cos(c + d*x))/(cos(c +
 d*x)*sin(c + d*x)*b + sin(c + d*x)*a),x)*b*d + 2*sqrt(sin(c + d*x)) - int
((sqrt(sin(c + d*x))*cos(c + d*x))/(cos(c + d*x)*sin(c + d*x)*b + sin(c +
d*x)*a),x)*a*d))/(2*b*d*(cos(c + d*x)*b + a))
```

# 3.73 $\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^2}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 438

$$\begin{aligned}
&\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^2}\,dx \\
&= \frac{a\sqrt{e}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2\sqrt{b}\,(-a^2+b^2)^{5/4}\,d} - \frac{a\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2\sqrt{b}\,(-a^2+b^2)^{5/4}\,d} \\
&\quad + \frac{a^2e\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2b\,(a^2-b^2)\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{a^2e\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2b\,(a^2-b^2)\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{e\sin(c+dx)}}{(a^2-b^2)\,d\sqrt{\sin(c+dx)}} - \frac{b(e\sin(c+dx))^{3/2}}{(a^2-b^2)\,de(a+b\cos(c+dx))}
\end{aligned}$$

output
```
1/2*a*e^(1/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2)
)/b^(1/2)/(-a^2+b^2)^(5/4)/d-1/2*a*e^(1/2)*arctanh(b^(1/2)*(e*sin(d*x+c))^
(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(1/2)/(-a^2+b^2)^(5/4)/d-1/2*a^2*e*Ellip
ticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+
c)^(1/2)/b/(a^2-b^2)/(b-(-a^2+b^2)^(1/2))/d/(e*sin(d*x+c))^(1/2)-1/2*a^2*e
*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*si
n(d*x+c)^(1/2)/b/(a^2-b^2)/(b+(-a^2+b^2)^(1/2))/d/(e*sin(d*x+c))^(1/2)-Ell
ipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/(a^2-b^2)/d
/sin(d*x+c)^(1/2)-b*(e*sin(d*x+c))^(3/2)/(a^2-b^2)/d/e/(a+b*cos(d*x+c))
```

## Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 15.12 (sec) , antiderivative size = 786, normalized size of antiderivative = 1.79

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^2}\,dx = \frac{b\sin(c+dx)\sqrt{e\sin(c+dx)}}{(-a^2+b^2)\,d(a+b\cos(c+dx))} + \frac{\sqrt{e\sin(c+dx)}\left(\frac{\cos^2(c+dx)\left(3\sqrt{2}a(a^2-b^2)^{3/4}\left(2\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-2\arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-\log\left(\sqrt{a^2-b^2}-\sqrt{\right.\right.\right.}{}\right.}{}$$

input
```
Integrate[Sqrt[e*Sin[c + d*x]]/(a + b*Cos[c + d*x])^2,x]
```

output
```
(b*Sin[c + d*x]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)*d*(a + b*Cos[c + d*x])
) + (Sqrt[e*Sin[c + d*x]]*((Cos[c + d*x]^2*(3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*
(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*
ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[S
qrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*
Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sq
rt[Sin[c + d*x]] + b*Sin[c + d*x]]) + 8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4
, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2))*(
a + b*Sqrt[1 - Sin[c + d*x]^2]))/(12*Sqrt[b]*(-a^2 + b^2)*(a + b*Cos[c + d
*x])*(1 - Sin[c + d*x]^2)) + (4*a*Cos[c + d*x]*(((1/8 + I/8)*(2*ArcTan[1 -
 ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + (
(1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 +
b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c +
 d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Si
n[c + d*x]] + I*b*Sin[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)) + (a*Appell
F1[3/4, 1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Si
n[c + d*x]^(3/2))/(3*(a^2 - b^2)))*(a + b*Sqrt[1 - Sin[c + d*x]^2]))/((a +
 b*Cos[c + d*x])*Sqrt[1 - Sin[c + d*x]^2])))/(2*(a - b)*(a + b)*d*Sqrt[Sin
[c + d*x]])
```

## Rubi [A] (warning: unable to verify)

Time = 1.69 (sec) , antiderivative size = 403, normalized size of antiderivative = 0.92, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.720$, Rules used = {3042, 3173, 27, 3042, 3346, 3042, 3121, 3042, 3119, 3180, 266, 827, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{ 3173}$$

$$-\frac{\int -\frac{(2a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}{2(a+b\cos(c+dx))}dx}{a^2-b^2}-\frac{b(e\sin(c+dx))^{3/2}}{de\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(2a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx}{2\left(a^2-b^2\right)}-\frac{b(e\sin(c+dx))^{3/2}}{de\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{-e\cos\left(c+dx+\frac{\pi}{2}\right)}\left(2a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2\left(a^2-b^2\right)}-\frac{b(e\sin(c+dx))^{3/2}}{de\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3346}$$

$$\frac{a\int \frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx+\int \sqrt{e\sin(c+dx)}dx}{2\left(a^2-b^2\right)}-\frac{b(e\sin(c+dx))^{3/2}}{de\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx+\int \sqrt{e\sin(c+dx)}dx}{2\left(a^2-b^2\right)}-\frac{b(e\sin(c+dx))^{3/2}}{de\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3121}$$

$$\frac{a\int \frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx+\frac{\sqrt{e\sin(c+dx)}\int \sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}}{2\left(a^2-b^2\right)}-\frac{b(e\sin(c+dx))^{3/2}}{de\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx+\frac{\sqrt{e\sin(c+dx)}\int \sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}}{2\left(a^2-b^2\right)}-\frac{b(e\sin(c+dx))^{3/2}}{de\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3119}$$

$$\frac{a\int \frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx+\frac{2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}}{2\left(a^2-b^2\right)}-\frac{b(e\sin(c+dx))^{3/2}}{de\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3180}$$

$$\frac{a\left(-\frac{be\int \frac{\sqrt{e\sin(c+dx)}}{b^2\sin^2(c+dx)e^2+\left(a^2-b^2\right)e^2}d(e\sin(c+dx))}{d}-\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}d}{2b}\right.}{2\left(a^2-b^2\right)}$$

$$\frac{b(e\sin(c+dx))^{3/2}}{de\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 266$$

$$\frac{a\left(-\frac{2be\int \frac{e^2\sin^2(c+dx)}{b^2e^4\sin^4(c+dx)+\left(a^2-b^2\right)e^2}d\sqrt{e\sin(c+dx)}}{d}-\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}}{2b}\right.}{2\left(a^2-b^2\right)}$$

$$\frac{b(e\sin(c+dx))^{3/2}}{de\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 827$$

$$\frac{a\left(-\frac{2be\left(\frac{\int \frac{1}{be^2\sin^2(c+dx)+\sqrt{b^2-a^2}e}d\sqrt{e\sin(c+dx)}}{2b}-\frac{\int \frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2b}\right)}{d}-\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\right.}{2\left(a^2-b^2\right)}$$

$$\frac{b(e\sin(c+dx))^{3/2}}{de\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 218$$

$$\frac{a\left(-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\int \frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2b}\right)}{d}-\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int \frac{1}{\sqrt{e\sin(c-}}}{}\right.}{2\left(a^2-b^2\right)}$$

$$\frac{b(e\sin(c+dx))^{3/2}}{de\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 221$$

$$\frac{a\left(-\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b}-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\text{arctanh}\left(\right.}{2b^{3/2}\sqrt{e}}\right.}{d}\right.}{2\left(a^2-b^2\right)}$$

$$\frac{b(e\sin(c+dx))^{3/2}}{de\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$\downarrow$ 3042

$$\frac{a\left(-\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b}-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\text{arctanh}\left(\right.}{2b^{3/2}\sqrt{e}}\right.}{d}\right.}{2\left(a^2-b^2\right)}$$

$$\frac{b(e\sin(c+dx))^{3/2}}{de\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$\downarrow$ 3286

$$\frac{a\left(-\frac{ae\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b\sqrt{e\sin(c+dx)}}+\frac{ae\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b\sqrt{e\sin(c+dx)}}-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c}{\sqrt[4]{b^2-}}\right.}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-}}\right.}{}\right.}{2\left(a^2-b^2\right)}$$

$$\frac{b(e\sin(c+dx))^{3/2}}{de\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$\downarrow$ 3042

$$\frac{a\left(-\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b\sqrt{e\sin(c+dx)}}+\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b\sqrt{e\sin(c+dx)}}-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}\cdots\right.}{\cdots}\right)}{2\left(a^2-b^2\right)}$$

$$\frac{b(e\sin(c+dx))^{3/2}}{de\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)}$$

$\downarrow$ 3284

$$\frac{a\left(-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{d}+\frac{ae\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\sin(c+dx)}}+\frac{ae\sqrt{\sin(c+\cdots}}{\cdots}\right)}{2\left(a^2-b^2\right)}$$

$$\frac{b(e\sin(c+dx))^{3/2}}{de\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)}$$

input
```
Int[Sqrt[e*Sin[c + d*x]]/(a + b*Cos[c + d*x])^2,x]
```

output
```
-((b*(e*Sin[c + d*x])^(3/2))/((a^2 - b^2)*d*e*(a + b*Cos[c + d*x]))) + ((2
*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(d*Sqrt[Sin[c + d*
x]]) + a*((-2*b*e*(ArcTan[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4
)]/(2*b^(3/2)*(-a^2 + b^2)^(1/4)*Sqrt[e]) - ArcTanh[(Sqrt[b]*Sqrt[e]*Sin[c
 + d*x])/(-a^2 + b^2)^(1/4)]/(2*b^(3/2)*(-a^2 + b^2)^(1/4)*Sqrt[e])))/d +
(a*e*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[
Sin[c + d*x]])/(b*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]]) + (a*e*El
lipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c +
 d*x]])/(b*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]])))/(2*(a^2 - b^2)
)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3173
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Simp[1/((a^2 - b^2)*(m + 1))
   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m +
p + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

rule 3180
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[a*(g/(2*b))   Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Simp[a*(g/(2*b))   In
t[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Simp[b*(g/f)   Su
bst[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1305 vs. $2(381) = 762$.

Time = 2.55 (sec) , antiderivative size = 1306, normalized size of antiderivative = 2.98

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1306 |

input
```
int((e*sin(d*x+c))^(1/2)/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)
```

output
```
(-4*e^3*a*b*(1/4*(e*sin(d*x+c))^(3/2)/(a^2*e^2-b^2*e^2)/(-b^2*cos(d*x+c)^2
*e^2+a^2*e^2)+1/32/(a^2*e^2-b^2*e^2)/b^2/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)
*(ln((e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+
(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(
d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arctan(2^(1/2)/(e^2*(a
^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2)/(e^2*(a^2-b^2)
/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)))+(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*e*
(1/2/b^2*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(cos
(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x
+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/2/b^2*(1-sin(d*x+c))^(1
/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1
/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)
^(1/2)/b),1/2*2^(1/2))+2*a^2*(1/2*b^2/e/a^2/(a^2-b^2)*sin(d*x+c)*(cos(d*x+
c)^2*e*sin(d*x+c))^(1/2)/(-cos(d*x+c)^2*b^2+a^2)-1/2/a^2/(a^2-b^2)*(1-sin(
d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(
d*x+c))^(1/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))+1/4/a^2/(a^2-b^2
)*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)
^2*e*sin(d*x+c))^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-3/8/(a^
2-b^2)/b^2*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(c
os(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-si...
```

## Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
Timed out
```

## Sympy [F]

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^2}\,dx$$

input
```
integrate((e*sin(d*x+c))**(1/2)/(a+b*cos(d*x+c))**2,x)
```

output
```
Integral(sqrt(e*sin(c + d*x))/(a + b*cos(c + d*x))**2, x)
```

## Maxima [F]

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\sqrt{e\sin(dx+c)}}{(b\cos(dx+c)+a)^2}\,dx$$

input
```
integrate((e*sin(d*x+c))^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(e*sin(d*x + c))/(b*cos(d*x + c) + a)^2, x)
```

## Giac [F]

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^2}\,dx=\int \frac{\sqrt{e\sin\left(dx+c\right)}}{\left(b\cos\left(dx+c\right)+a\right)^2}\,dx$$

input
```
integrate((e*sin(d*x+c))^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(sqrt(e*sin(d*x + c))/(b*cos(d*x + c) + a)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^2}\,dx=\int \frac{\sqrt{e\,\sin\left(c+d\,x\right)}}{\left(a+b\,\cos\left(c+d\,x\right)\right)^2}\,dx$$

input
```
int((e*sin(c + d*x))^(1/2)/(a + b*cos(c + d*x))^2,x)
```

output
```
int((e*sin(c + d*x))^(1/2)/(a + b*cos(c + d*x))^2, x)
```

## Reduce [F]

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^2}\,dx=\sqrt{e}\left(\int \frac{\sqrt{\sin\left(dx+c\right)}}{\cos\left(dx+c\right)^2 b^2+2\cos\left(dx+c\right)ab+a^2}dx\right)$$

input
```
int((e*sin(d*x+c))^(1/2)/(a+b*cos(d*x+c))^2,x)
```

output
```
sqrt(e)*int(sqrt(sin(c + d*x))/(cos(c + d*x)**2*b**2 + 2*cos(c + d*x)*a*b
+ a**2),x)
```

# 3.74 $\int \frac{1}{(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 445

$$\begin{aligned}
&\int \frac{1}{(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx \\
&= -\frac{3a\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2\left(-a^2+b^2\right)^{7/4}d\sqrt{e}} - \frac{3a\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2\left(-a^2+b^2\right)^{7/4}d\sqrt{e}} \\
&\quad - \frac{\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{\left(a^2-b^2\right)d\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{3a^2\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2\left(a^2-b^2\right)\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{3a^2\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2\left(a^2-b^2\right)\left(a^2-b\left(b+\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{b\sqrt{e\sin(c+dx)}}{\left(a^2-b^2\right)de(a+b\cos(c+dx))}
\end{aligned}$$

output
```
-3/2*a*b^(1/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2
))/(-a^2+b^2)^(7/4)/d/e^(1/2)-3/2*a*b^(1/2)*arctanh(b^(1/2)*(e*sin(d*x+c))
^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a^2+b^2)^(7/4)/d/e^(1/2)-InverseJacobiA
M(1/2*c-1/4*Pi+1/2*d*x,2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)/d/(e*sin(d*x+c)
)^(1/2)-3/2*a^2*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/
2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)/(a^2-b*(b-(-a^2+b^2)^(1/2)))/d/(e*
sin(d*x+c))^(1/2)-3/2*a^2*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^
2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)/(a^2-b*(b+(-a^2+b^2)^(1/
2)))/d/(e*sin(d*x+c))^(1/2)-b*(e*sin(d*x+c))^(1/2)/(a^2-b^2)/d/e/(a+b*cos(
d*x+c))
```

## Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 12.42 (sec) , antiderivative size = 1182, normalized size of antiderivative = 2.66

$$\int \frac{1}{(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx = \text{Too large to display}$$

input
```
Integrate[1/((a + b*Cos[c + d*x])^2*Sqrt[e*Sin[c + d*x]]),x]
```

output
```
-((b*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]
])) + (Sqrt[Sin[c + d*x]]*((-2*b*Cos[c + d*x]^2*(a + b*Sqrt[1 - Sin[c + d*
x]^2])*((a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)
^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1
/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c
+ d*x]] + b*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b
^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 -
 b^2)^(3/4)) + (5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2
, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Sin[c + d*x]]*Sqrt[1 - Sin[c + d
*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*
Sin[c + d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Sin[c
 + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4,
1/2, 1, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)])*Sin[c + d
*x]^2)*(a^2 + b^2*(-1 + Sin[c + d*x]^2)))))/((a + b*Cos[c + d*x])*(1 - Sin
[c + d*x]^2)) + (4*a*Cos[c + d*x]*(a + b*Sqrt[1 - Sin[c + d*x]^2])*(((-1/8
 + I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 +
 b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b
^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqr
t[Sin[c + d*x]] + I*b*Sin[c + d*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[
b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]]))/(-a^2 + ...
```

## Rubi [A] (warning: unable to verify)

Time = 1.74 (sec) , antiderivative size = 423, normalized size of antiderivative = 0.95, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.720$, Rules used = {3042, 3173, 27, 3042, 3346, 3042, 3121, 3042, 3120, 3181, 266, 756, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3173}$$

$$-\frac{\int -\frac{2a-b\cos(c+dx)}{2(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{a^2-b^2}-\frac{b\sqrt{e\sin(c+dx)}}{de\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{2a-b\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{2\left(a^2-b^2\right)}-\frac{b\sqrt{e\sin(c+dx)}}{de\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{2a+b\sin\left(c+dx-\frac{\pi}{2}\right)}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{2\left(a^2-b^2\right)}-\frac{b\sqrt{e\sin(c+dx)}}{de\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3346}$$

$$\frac{3a\int \frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx-\int \frac{1}{\sqrt{e\sin(c+dx)}}dx}{2\left(a^2-b^2\right)}-\frac{b\sqrt{e\sin(c+dx)}}{de\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3a\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx-\int \frac{1}{\sqrt{e\sin(c+dx)}}dx}{2\left(a^2-b^2\right)}-\frac{b\sqrt{e\sin(c+dx)}}{de\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3121}$$

$$\frac{3a\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx-\frac{\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}}dx}{\sqrt{e\sin(c+dx)}}}{2\left(a^2-b^2\right)}-\frac{b\sqrt{e\sin(c+dx)}}{de\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3a\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx-\frac{\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}}dx}{\sqrt{e\sin(c+dx)}}}{2\left(a^2-b^2\right)}-\frac{b\sqrt{e\sin(c+dx)}}{de\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3120}$$

$$\frac{3a\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx-\frac{2\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}}{2\left(a^2-b^2\right)}-\frac{b\sqrt{e\sin(c+dx)}}{de\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$\big\downarrow$ 3181

$$3a\left(-\frac{be\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b^2\sin^2(c+dx)e^2+\left(a^2-b^2\right)e^2\right)}d(e\sin(c+dx))}{d}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx\right.}}{2\sqrt{b^2-a^2}}\right.}{2\left(a^2-b^2\right)}$$

$$\frac{b\sqrt{e\sin(c+dx)}}{de\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)}$$

$\big\downarrow$ 266

$$3a\left(-\frac{2be\int \frac{1}{b^2e^4\sin^4(c+dx)+\left(a^2-b^2\right)e^2}d\sqrt{e\sin(c+dx)}}{d}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}}{2\sqrt{b^2-a^2}}\right.}{2\left(a^2-b^2\right)}$$

$$\frac{b\sqrt{e\sin(c+dx)}}{de\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)}$$

$\big\downarrow$ 756

$$3a\left(-\frac{2be\left(-\frac{\int \frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2e\sqrt{b^2-a^2}}-\frac{\int \frac{1}{be^2\sin^2(c+dx)+\sqrt{b^2-a^2}e}d\sqrt{e\sin(c+dx)}}{2e\sqrt{b^2-a^2}}\right)}{d}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}\right.}{2\left(a^2-b^2\right)}$$

$$\frac{b\sqrt{e\sin(c+dx)}}{de\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)}$$

$\big\downarrow$ 218

$$3a\left(-\frac{2be\left(-\frac{\int \frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2e\sqrt{b^2-a^2}}-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}\right)}{d}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int}{\sqrt{e\sin(}}\right.}{2\left(a^2-b^2\right)}$$

$$\frac{b\sqrt{e\sin(c+dx)}}{de\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)}$$

$\big\downarrow$ 221

$$\frac{3a\left(-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}}-\frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}-\frac{\text{arctanh}\left(\right.}{2\sqrt{b}e^{3/2}}\right.}{d}\right.}{2\left(a^2-b^2\right)}$$

$$\frac{b\sqrt{e\sin(c+dx)}}{de\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)}$$

$\downarrow$ 3042

$$\frac{3a\left(-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}}-\frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}-\frac{\text{arctanh}\left(\right.}{2\sqrt{b}e^{3/2}}\right.}{d}\right.}{2\left(a^2-b^2\right)}$$

$$\frac{b\sqrt{e\sin(c+dx)}}{de\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)}$$

$\downarrow$ 3286

$$\frac{3a\left(-\frac{a\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}}-\frac{a\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}}-\frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin}{\sqrt[4]{b^2-}}\right.}{2\sqrt{b}e^{3/2}\left(b^2-a\right.}\right.}{}\right.}{2\left(a^2-b^2\right)}$$

$$\frac{b\sqrt{e\sin(c+dx)}}{de\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)}$$

$\downarrow$ 3042

$$\frac{3a\left(-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}}-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}}-\frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin}{\sqrt[4]{b^2-}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a\right.}\right.}{}\right.}{2\left(a^2-b^2\right)}$$

$$\frac{b\sqrt{e\sin(c+dx)}}{de\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)}$$

$\Big\downarrow$ 3284

$$\frac{3a\left(-\frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}\right)}{d}+\frac{a\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{b^2-a^2}\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\sin(c+dx)}}-\frac{a\sqrt{\sin(c}{}\right.}{2\left(a^2-b^2\right)}$$

$$\frac{b\sqrt{e\sin(c+dx)}}{de\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)}$$

input

```
Int[1/((a + b*Cos[c + d*x])^2*Sqrt[e*Sin[c + d*x]]),x]
```

output

```
-((b*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)*d*e*(a + b*Cos[c + d*x]))) + ((-2*
EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(d*Sqrt[e*Sin[c + d*x
]]) + 3*a*((-2*b*e*(-1/2*ArcTan[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(-a^2 + b^2
)^(1/4)]/(Sqrt[b]*(-a^2 + b^2)^(3/4)*e^(3/2)) - ArcTanh[(Sqrt[b]*Sqrt[e]*S
in[c + d*x])/(-a^2 + b^2)^(1/4)]/(2*Sqrt[b]*(-a^2 + b^2)^(3/4)*e^(3/2))))/
d + (a*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqr
t[Sin[c + d*x]])/(Sqrt[-a^2 + b^2]*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c +
 d*x]]) - (a*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2,
2]*Sqrt[Sin[c + d*x]])/(Sqrt[-a^2 + b^2]*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*S
in[c + d*x]])))/(2*(a^2 - b^2))
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3173
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Simp[1/((a^2 - b^2)*(m + 1))
   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m +
p + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

rule 3181
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[-a/(2*q)   Int[1/(
Sqrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Simp[b*(g/f)   Subst[
Int[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - S
imp[a/(2*q)   Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1279 vs. $2(386) = 772$.

Time = 2.28 (sec) , antiderivative size = 1280, normalized size of antiderivative = 2.88

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1280 |

input
```
int(1/(a+cos(d*x+c)*b)^2/(e*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-4*a*b*e^3*(1/4*(e*sin(d*x+c))^(1/2)/(a^2*e^2-b^2*e^2)/(-b^2*cos(d*x+c)^2
*e^2+a^2*e^2)+3/32/(a^2*e^2-b^2*e^2)^2*(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*(
ln((e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e
^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*
x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arctan(2^(1/2)/(e^2*(a^2
-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b
^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)))+(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*(1/2
/b/(-a^2+b^2)^(1/2)*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)
^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi
((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-1/2/b/(-a^2+b^
2)^(1/2)*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(cos
(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x
+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+2*a^2*(1/2*b^2/e/a^2/(a^2
-b^2)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-cos(d*x+c)^2*b^2+a^2)+1/4/a^2/(a
^2-b^2)*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(cos(
d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-5
/8/(a^2-b^2)/b/(-a^2+b^2)^(1/2)*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2
)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b
)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/
4/a^2/(a^2-b^2)*b/(-a^2+b^2)^(1/2)*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c)...
```

## Fricas [F]

$$\int \frac{1}{(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx = \int \frac{1}{(b\cos(dx+c)+a)^2\sqrt{e\sin(dx+c)}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*sin(d*x + c))/((b^2*e*cos(d*x + c)^2 + 2*a*b*e*cos(d*x + c
) + a^2*e)*sin(d*x + c)), x)
```

## Sympy [F]

$$\int \frac{1}{(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx = \int \frac{1}{\sqrt{e\sin(c+dx)}\,(a+b\cos(c+dx))^2}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))**2/(e*sin(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(e*sin(c + d*x))*(a + b*cos(c + d*x))**2), x)
```

## Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
Timed out
```

## Giac [F]

$$\int \frac{1}{(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx = \int \frac{1}{(b\cos(dx+c)+a)^2\sqrt{e\sin(dx+c)}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^2*sqrt(e*sin(d*x + c))), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx = \int \frac{1}{\sqrt{e\,\sin(c+d\,x)}\,(a+b\,\cos(c+d\,x))^2}\,dx$$

input
```
int(1/((e*sin(c + d*x))^(1/2)*(a + b*cos(c + d*x))^2),x)
```

output
```
int(1/((e*sin(c + d*x))^(1/2)*(a + b*cos(c + d*x))^2), x)
```

## Reduce [F]

$$\int \frac{1}{(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx = \frac{\sqrt{e}\left(\int \frac{\sqrt{\sin(dx+c)}}{\cos(dx+c)^2\sin(dx+c)b^2+2\cos(dx+c)\sin(dx+c)ab+\sin(dx+c)a^2}dx\right)}{e}$$

input
```
int(1/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(1/2),x)
```

output
```
(sqrt(e)*int(sqrt(sin(c + d*x))/(cos(c + d*x)**2*sin(c + d*x)*b**2 + 2*cos
(c + d*x)*sin(c + d*x)*a*b + sin(c + d*x)*a**2),x))/e
```

# 3.75 $\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 507

$$\begin{aligned}
\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx &= \frac{5ab^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2\left(-a^2+b^2\right)^{9/4}de^{3/2}} \\
&- \frac{5ab^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2\left(-a^2+b^2\right)^{9/4}de^{3/2}} \\
&- \frac{b}{(a^2-b^2)\,de(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}} \\
&+ \frac{5ab-(2a^2+3b^2)\cos(c+dx)}{(a^2-b^2)^2\,de\sqrt{e\sin(c+dx)}} \\
&- \frac{5a^2b\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2\left(a^2-b^2\right)^2\left(b-\sqrt{-a^2+b^2}\right)de\sqrt{e\sin(c+dx)}} \\
&- \frac{5a^2b\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2\left(a^2-b^2\right)^2\left(b+\sqrt{-a^2+b^2}\right)de\sqrt{e\sin(c+dx)}} \\
&- \frac{(2a^2+3b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{e\sin(c+dx)}}{\left(a^2-b^2\right)^2de^2\sqrt{\sin(c+dx)}}
\end{aligned}$$

output
```
5/2*a*b^(3/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2)
)/(-a^2+b^2)^(9/4)/d/e^(3/2)-5/2*a*b^(3/2)*arctanh(b^(1/2)*(e*sin(d*x+c))^
(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a^2+b^2)^(9/4)/d/e^(3/2)-b/(a^2-b^2)/d/e
/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(1/2)+(5*a*b-(2*a^2+3*b^2)*cos(d*x+c))/(a
^2-b^2)^2/d/e/(e*sin(d*x+c))^(1/2)+5/2*a^2*b*EllipticPi(cos(1/2*c+1/4*Pi+1
/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^2/(b-
(-a^2+b^2)^(1/2))/d/e/(e*sin(d*x+c))^(1/2)+5/2*a^2*b*EllipticPi(cos(1/2*c+
1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^
2)^2/(b+(-a^2+b^2)^(1/2))/d/e/(e*sin(d*x+c))^(1/2)+(2*a^2+3*b^2)*EllipticE
(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^2/d/e^2
/sin(d*x+c)^(1/2)
```

## Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 7.12 (sec) , antiderivative size = 865, normalized size of antiderivative = 1.71

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx = \frac{\sin^2(c+dx)\left(-\frac{2(-2ab+a^2\cos(c+dx)+b^2\cos(c+dx))\csc(c+dx)}{(a^2-b^2)^2}+\frac{}{(a^2-b^2}\right.}{d(e\sin(c+dx))^{3/2}}$$

$$-\frac{\sin^{\frac{3}{2}}(c+dx)\left(\frac{(2a^2b+3b^3)\cos^2(c+dx)\left(3\sqrt{2}a(a^2-b^2)^{3/4}\left(2\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-2\arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-\log\left(\sqrt{a^2-}\right.\right.\right.}{}\right.}{}$$

input
```
Integrate[1/((a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(3/2)),x]
```

output
```
(Sin[c + d*x]^2*((-2*(-2*a*b + a^2*Cos[c + d*x] + b^2*Cos[c + d*x])*Csc[c
+ d*x])/(a^2 - b^2)^2 + (b^3*Sin[c + d*x])/((a^2 - b^2)^2*(a + b*Cos[c + d
*x]))))/(d*(e*Sin[c + d*x])^(3/2)) - (Sin[c + d*x]^(3/2)*(((2*a^2*b + 3*b^
3)*Cos[c + d*x]^2*(3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sq
rt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[
b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*
Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]] + Log[Sqrt[
a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[
c + d*x]]) + 8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Si
n[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2))*(a + b*Sqrt[1 - Sin[c + d*
x]^2]))/(12*b^(3/2)*(-a^2 + b^2)*(a + b*Cos[c + d*x])*(1 - Sin[c + d*x]^2)
) + (2*(2*a^3 + 8*a*b^2)*Cos[c + d*x]*(((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)
*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*S
qrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1
 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]] +
Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x
]] + I*b*Sin[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)) + (a*AppellF1[3/4, 1
/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x
]^(3/2))/(3*(a^2 - b^2)))*(a + b*Sqrt[1 - Sin[c + d*x]^2]))/((a + b*Cos[c
+ d*x])*Sqrt[1 - Sin[c + d*x]^2])))/(2*(a - b)^2*(a + b)^2*d*(e*Sin[c +...
```

## Rubi [A] (warning: unable to verify)

Time = 2.30 (sec) , antiderivative size = 490, normalized size of antiderivative = 0.97, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.840$, Rules used = {3042, 3173, 27, 3042, 3345, 27, 3042, 3346, 3042, 3121, 3042, 3119, 3180, 266, 827, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3173}$$

$$-\frac{\int -\frac{2a-3b\cos(c+dx)}{2(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}dx}{a^2-b^2}-\frac{b}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{2a-3b\cos(c+dx)}{(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}dx}{2\left(a^2-b^2\right)}-\frac{b}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{2a+3b\sin\left(c+dx-\frac{\pi}{2}\right)}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{2\left(a^2-b^2\right)}-\frac{b}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}$$

$$\Big\downarrow \ 3345$$

$$\frac{\frac{2\left(5ab-\left(2a^2+3b^2\right)\cos(c+dx)\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{2\int \frac{\left(2a\left(a^2+4b^2\right)+b\left(2a^2+3b^2\right)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{2(a+b\cos(c+dx))}dx}{e^2\left(a^2-b^2\right)}}{2\left(a^2-b^2\right)}-\frac{b}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{2\left(5ab-\left(2a^2+3b^2\right)\cos(c+dx)\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{\int \frac{\left(2a\left(a^2+4b^2\right)+b\left(2a^2+3b^2\right)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx}{e^2\left(a^2-b^2\right)}}{2\left(a^2-b^2\right)}-\frac{b}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{2\left(5ab-\left(2a^2+3b^2\right)\cos(c+dx)\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{\int \frac{\sqrt{-e\cos\left(c+dx+\frac{\pi}{2}\right)}\left(2a\left(a^2+4b^2\right)+b\left(2a^2+3b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{e^2\left(a^2-b^2\right)}}{2\left(a^2-b^2\right)}-\frac{b}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}$$

$$\Big\downarrow \ 3346$$

$$\frac{\frac{2\left(5ab-\left(2a^2+3b^2\right)\cos(c+dx)\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{\left(2a^2+3b^2\right)\int \sqrt{e\sin(c+dx)}dx+5ab^2\int \frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx}{e^2\left(a^2-b^2\right)}}{2\left(a^2-b^2\right)}-\frac{b}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{2(5ab-(2a^2+3b^2)\cos(c+dx))}{de(a^2-b^2)\sqrt{e\sin(c+dx)}}-\frac{(2a^2+3b^2)\int\sqrt{e\sin(c+dx)}dx+5ab^2\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{e^2(a^2-b^2)}}{2(a^2-b^2)}-\frac{b}{de(a^2-b^2)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}$$

$\downarrow$ 3121

$$\frac{\frac{2(5ab-(2a^2+3b^2)\cos(c+dx))}{de(a^2-b^2)\sqrt{e\sin(c+dx)}}-\frac{\frac{(2a^2+3b^2)\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}+5ab^2\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{e^2(a^2-b^2)}}{2(a^2-b^2)}-\frac{b}{de(a^2-b^2)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}$$

$\downarrow$ 3042

$$\frac{\frac{2(5ab-(2a^2+3b^2)\cos(c+dx))}{de(a^2-b^2)\sqrt{e\sin(c+dx)}}-\frac{\frac{(2a^2+3b^2)\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}+5ab^2\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{e^2(a^2-b^2)}}{2(a^2-b^2)}-\frac{b}{de(a^2-b^2)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}$$

$\downarrow$ 3119

$$\frac{\frac{2(5ab-(2a^2+3b^2)\cos(c+dx))}{de(a^2-b^2)\sqrt{e\sin(c+dx)}}-\frac{5ab^2\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx+\frac{2(2a^2+3b^2)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}}{e^2(a^2-b^2)}}{2(a^2-b^2)}-\frac{b}{de(a^2-b^2)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}$$

$\downarrow$ 3180

$$\frac{\frac{2(5ab-(2a^2+3b^2)\cos(c+dx))}{de(a^2-b^2)\sqrt{e\sin(c+dx)}}-\frac{5ab^2\left(-\frac{be\int\frac{\sqrt{e\sin(c+dx)}}{b^2\sin^2(c+dx)e^2+(a^2-b^2)e^2}d(e\sin(c+dx))}{d}-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int\frac{}{\sqrt{e\sin(}}}{}\right.}{e^2(a^2-b^2)}}{2(a^2-b^2)}$$

$$\frac{b}{de(a^2-b^2)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}$$

$\downarrow$ 266

$$\frac{\frac{2\left(5ab-\left(2a^2+3b^2\right)\cos(c+dx)\right)}{de(a^2-b^2)\sqrt{e\sin(c+dx)}}-\frac{5ab^2\left(-\frac{2be\int\frac{e^2\sin^2(c+dx)}{b^2e^4\sin^4(c+dx)+\left(a^2-b^2\right)e^2}d\sqrt{e\sin(c+dx)}}{d}-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int\frac{}{\sqrt{e\sin}\cdots}}{}\cdots\right.}{e^2(a^2-b^2)}}{2\left(a^2-b^2\right)}\cdots$$

$$\frac{b}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{827}$$

$$\frac{\frac{2\left(5ab-\left(2a^2+3b^2\right)\cos(c+dx)\right)}{de(a^2-b^2)\sqrt{e\sin(c+dx)}}-\frac{5ab^2\left(-\frac{2be\left(\frac{\int\frac{1}{be^2\sin^2(c+dx)+\sqrt{b^2-a^2}e}d\sqrt{e\sin(c+dx)}}{2b}-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2b}\right)}{d}-\frac{ae\int\frac{}{\sqrt{e\sin}\cdots}}{}\cdots\right.}{}}{2\left(a^2-b^2\right)}\cdots$$

$$\frac{b}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{2\left(5ab-\left(2a^2+3b^2\right)\cos(c+dx)\right)}{de(a^2-b^2)\sqrt{e\sin(c+dx)}}-\frac{5ab^2\left(-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2b}\right)}{d}-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}\cdots\right.}}{2b}\cdots\right.}{e^2(a^2-\cdots}}{2\left(a^2-b^2\right)}\cdots$$

$$\frac{b}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{2\left(5ab-\left(2a^2+3b^2\right)\cos(c+dx)\right)}{de(a^2-b^2)\sqrt{e\sin(c+dx)}}-\frac{5ab^2\left(-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b}-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{\cdots}}{\cdots}\right)}{2b^{3/2}\sqrt{\cdots}}\cdots\right)}{\cdots}\cdots\right)}{e^2(a^2-b^2)}\cdots}{2\left(a^2-b^2\right)}\cdots-\frac{b}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}$$

$\Big\downarrow$ 3042

$$\frac{\frac{2\left(5ab-\left(2a^2+3b^2\right)\cos(c+dx)\right)}{de(a^2-b^2)\sqrt{e\sin(c+dx)}}-\frac{5ab^2\left(-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b}-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{\cdots}}{\cdots}\right)}{2b^{3/2}\sqrt{\cdots}}\cdots\right)}{\cdots}\cdots\right)}{e^2(a^2-b^2)}\cdots}{2\left(a^2-b^2\right)}\cdots-\frac{b}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}$$

$\Big\downarrow$ 3286

$$\frac{\frac{2\left(5ab-\left(2a^2+3b^2\right)\cos(c+dx)\right)}{de(a^2-b^2)\sqrt{e\sin(c+dx)}}-\frac{5ab^2\left(-\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b\sqrt{e\sin(c+dx)}}+\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}\cdots}{2b\sqrt{e\sin(c+dx)}}\cdots\right)}{e^2(\cdots}\cdots}{2\left(a^2-b^2\right)}\cdots-\frac{b}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}$$

$\downarrow$ 3042

$$\frac{\frac{2(5ab-(2a^2+3b^2)\cos(c+dx))}{de(a^2-b^2)\sqrt{e\sin(c+dx)}}-\frac{5ab^2\left(-\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b\sqrt{e\sin(c+dx)}}+\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}}{2b\sqrt{e\sin(c+dx)}}\right.}{e^2(}}{2(a^2-b^2)}$$

$$\frac{b}{de(a^2-b^2)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}$$

$\downarrow$ 3284

$$\frac{\frac{2(5ab-(2a^2+3b^2)\cos(c+dx))}{de(a^2-b^2)\sqrt{e\sin(c+dx)}}-\frac{5ab^2\left(-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{d}+\frac{ae\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{b^2-a^2}},\frac{1}{2}\right.}{bd\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\sin(c+d}}\right.}{e^2(}}{2(a^2-b^2)}$$

$$\frac{b}{de(a^2-b^2)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}$$

input
```
Int[1/((a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(3/2)),x]
```

output
```
-(b/((a^2 - b^2)*d*e*(a + b*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]])) + ((2*(5*
a*b - (2*a^2 + 3*b^2)*Cos[c + d*x]))/((a^2 - b^2)*d*e*Sqrt[e*Sin[c + d*x]]
) - ((2*(2*a^2 + 3*b^2)*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*
x]])/(d*Sqrt[Sin[c + d*x]]) + 5*a*b^2*((-2*b*e*(ArcTan[(Sqrt[b]*Sqrt[e]*Si
n[c + d*x])/(-a^2 + b^2)^(1/4)]/(2*b^(3/2)*(-a^2 + b^2)^(1/4)*Sqrt[e]) - A
rcTanh[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/(2*b^(3/2)*(-a^2
 + b^2)^(1/4)*Sqrt[e])))/d + (a*e*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]),
 (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(b*(b - Sqrt[-a^2 + b^2])*d*Sq
rt[e*Sin[c + d*x]]) + (a*e*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - P
i/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(b*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Si
n[c + d*x]])))/((a^2 - b^2)*e^2))/(2*(a^2 - b^2))
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3173
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Simp[1/((a^2 - b^2)*(m + 1))
   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m +
p + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

rule 3180
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[a*(g/(2*b))   Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Simp[a*(g/(2*b))   In
t[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Simp[b*(g/f)   Su
bst[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3345
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Simp[1/(g^2*(a^2 - b^2)*(p +
 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
 2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[p, -1] && IntegerQ[2*m]
```

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2001 vs. $2(446) = 892$.

Time = 2.92 (sec) , antiderivative size = 2002, normalized size of antiderivative = 3.95

| method | result | size |
|---|---|---|
| default | Expression too large to display | 2002 |

input
```
int(1/(a+cos(d*x+c)*b)^2/(e*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-4*e^3*a*b*(-b^2/e^4/(a-b)^2/(a+b)^2*(1/4*(e*sin(d*x+c))^(3/2)/(-b^2*cos(
d*x+c)^2*e^2+a^2*e^2)+5/32/b^2/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*(ln((e*si
n(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-
b^2)/b^2)^(1/2))/(e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1
/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^
2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4
)*(e*sin(d*x+c))^(1/2)-1)))-1/e^4/(a^2-b^2)^2/(e*sin(d*x+c))^(1/2))-1/4/e*
a^2*(-8*a^4*cos(d*x+c)^2-4*a^4*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)
*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-2*a^2*b^2*(1
-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-si
n(d*x+c))^(1/2),1/2*2^(1/2))+6*b^4*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(
1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))+5*(-a^2+
b^2)^(1/2)*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*El
lipticPi((1-sin(d*x+c))^(1/2),-b/(-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*a^2*b-
5*(-a^2+b^2)^(1/2)*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^
(1/2)*EllipticPi((1-sin(d*x+c))^(1/2),b/(b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*
a^2*b+12*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(5/2)*Elli
pticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^4-6*(1-sin(d*x+c))^(1/2)*(2+2*si
n(d*x+c))^(1/2)*sin(d*x+c)^(5/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2
))*b^4+5*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(5/2)*E...
```

## Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

## Sympy [F]

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx = \int \frac{1}{(e\sin{(c+dx)})^{\frac{3}{2}}\,(a+b\cos{(c+dx)})^2}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))**2/(e*sin(d*x+c))**(3/2),x)
```

output
```
Integral(1/((e*sin(c + d*x))**(3/2)*(a + b*cos(c + d*x))**2), x)
```

## Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

## Giac [F]

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\cos{(dx+c)}+a)^2\,(e\sin{(dx+c)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^2*(e*sin(d*x + c))^(3/2)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx=\int \frac{1}{{\left(e\,\sin\left(c+d\,x\right)\right)}^{3/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^2} \,d x$$

input
```
int(1/((e*sin(c + d*x))^(3/2)*(a + b*cos(c + d*x))^2),x)
```

output
```
int(1/((e*sin(c + d*x))^(3/2)*(a + b*cos(c + d*x))^2), x)
```

## Reduce [F]

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx=\frac{\sqrt{e}\left(\int \frac{\sqrt{\sin(dx+c)}}{\cos(dx+c)^2\sin(dx+c)^2b^2+2\cos(dx+c)\sin(dx+c)^2ab+\sin(dx+c)^2a^2}dx\right)}{e^2}$$

input
```
int(1/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(3/2),x)
```

output
```
(sqrt(e)*int(sqrt(sin(c + d*x))/(cos(c + d*x)**2*sin(c + d*x)**2*b**2 + 2*
cos(c + d*x)*sin(c + d*x)**2*a*b + sin(c + d*x)**2*a**2),x))/e**2
```

# 3.76 $\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 530

$$\begin{aligned}
&\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx = \\
&-\frac{7ab^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2\left(-a^2+b^2\right)^{11/4}de^{5/2}}-\frac{7ab^{5/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2\left(-a^2+b^2\right)^{11/4}de^{5/2}} \\
&-\frac{b}{(a^2-b^2)\,de(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}} \\
&+\frac{7ab-(2a^2+5b^2)\cos(c+dx)}{3\left(a^2-b^2\right)^2de(e\sin(c+dx))^{3/2}} \\
&+\frac{(2a^2+5b^2)\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3\left(a^2-b^2\right)^2de^2\sqrt{e\sin(c+dx)}} \\
&-\frac{7a^2b^2\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2\left(a^2-b^2\right)^2\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)de^2\sqrt{e\sin(c+dx)}} \\
&-\frac{7a^2b^2\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2\left(a^2-b^2\right)^2\left(a^2-b\left(b+\sqrt{-a^2+b^2}\right)\right)de^2\sqrt{e\sin(c+dx)}}
\end{aligned}$$

output
```
-7/2*a*b^(5/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2
))/(-a^2+b^2)^(11/4)/d/e^(5/2)-7/2*a*b^(5/2)*arctanh(b^(1/2)*(e*sin(d*x+c)
)^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a^2+b^2)^(11/4)/d/e^(5/2)-b/(a^2-b^2)/
d/e/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(3/2)+1/3*(7*a*b-(2*a^2+5*b^2)*cos(d*x
+c))/(a^2-b^2)^2/d/e/(e*sin(d*x+c))^(3/2)+1/3*(2*a^2+5*b^2)*InverseJacobiA
M(1/2*c-1/4*Pi+1/2*d*x,2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^2/d/e^2/(e*sin(
d*x+c))^(1/2)+7/2*a^2*b^2*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^
2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^2/(a^2-b*(b-(-a^2+b^2)^(
1/2)))/d/e^2/(e*sin(d*x+c))^(1/2)+7/2*a^2*b^2*EllipticPi(cos(1/2*c+1/4*Pi+
1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^2/(a
^2-b*(b+(-a^2+b^2)^(1/2)))/d/e^2/(e*sin(d*x+c))^(1/2)
```

## Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 16.50 (sec) , antiderivative size = 1257, normalized size of antiderivative = 2.37

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[1/((a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(5/2)),x]
```

output
```
((b^3/((a^2 - b^2)^2*(a + b*Cos[c + d*x])) - (2*(-2*a*b + a^2*Cos[c + d*x]
 + b^2*Cos[c + d*x])*Csc[c + d*x]^2)/(3*(a^2 - b^2)^2))*Sin[c + d*x]^3)/(d
*(e*Sin[c + d*x])^(5/2)) + (Sin[c + d*x]^(5/2)*((2*(2*a^2*b + 5*b^3)*Cos[c
 + d*x]^2*(a + b*Sqrt[1 - Sin[c + d*x]^2])*((a*(-2*ArcTan[1 - (Sqrt[2]*Sqr
t[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b
]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*S
qrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]] + Log[Sqrt[a
^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c
 + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)) + (5*b*(a^2 - b^2)*Appell
F1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*S
qrt[Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4,
 -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] + 2*(2*b
^2*AppellF1[5/4, -1/2, 2, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2
+ b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (b^2*Sin[
c + d*x]^2)/(-a^2 + b^2)])*Sin[c + d*x]^2)*(a^2 + b^2*(-1 + Sin[c + d*x]^2
)))))/((a + b*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) + (2*(2*a^3 - 16*a*b^2)*
Cos[c + d*x]*(a + b*Sqrt[1 - Sin[c + d*x]^2])*(((-1/8 + I/8)*Sqrt[b]*(2*Ar
cTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcT
an[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt
[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + ...
```

## Rubi [A] (warning: unable to verify)

Time = 2.27 (sec) , antiderivative size = 513, normalized size of antiderivative = 0.97, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.840$, Rules used = {3042, 3173, 27, 3042, 3345, 27, 3042, 3346, 3042, 3121, 3042, 3120, 3181, 266, 756, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{ 3173}$$

$$-\frac{\int -\frac{2a-5b\cos(c+dx)}{2(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}dx}{a^2-b^2}-\frac{b}{de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{2a-5b\cos(c+dx)}{(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}dx}{2\left(a^2-b^2\right)}-\frac{b}{de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{2a+5b\sin\left(c+dx-\frac{\pi}{2}\right)}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{2\left(a^2-b^2\right)}-\frac{b}{de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 3345$$

$$\frac{\frac{2\left(7ab-\left(2a^2+5b^2\right)\cos(c+dx)\right)}{3de(a^2-b^2)(e\sin(c+dx))^{3/2}}-\frac{2\int -\frac{2a\left(a^2-8b^2\right)+b\left(2a^2+5b^2\right)\cos(c+dx)}{2(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{3e^2(a^2-b^2)}}{2\left(a^2-b^2\right)}-\frac{b}{de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{2a\left(a^2-8b^2\right)+b\left(2a^2+5b^2\right)\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{3e^2(a^2-b^2)}+\frac{2\left(7ab-\left(2a^2+5b^2\right)\cos(c+dx)\right)}{3de(a^2-b^2)(e\sin(c+dx))^{3/2}}}{2\left(a^2-b^2\right)}-\frac{b}{de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{2a\left(a^2-8b^2\right)-b\left(2a^2+5b^2\right)\sin\left(c+dx-\frac{\pi}{2}\right)}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{3e^2(a^2-b^2)}+\frac{2\left(7ab-\left(2a^2+5b^2\right)\cos(c+dx)\right)}{3de(a^2-b^2)(e\sin(c+dx))^{3/2}}}{2\left(a^2-b^2\right)}-\frac{b}{de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 3346$$

$$\frac{\frac{\left(2a^2+5b^2\right)\int \frac{1}{\sqrt{e\sin(c+dx)}}dx-21ab^2\int \frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{3e^2(a^2-b^2)}+\frac{2\left(7ab-\left(2a^2+5b^2\right)\cos(c+dx)\right)}{3de(a^2-b^2)(e\sin(c+dx))^{3/2}}}{2\left(a^2-b^2\right)}-\frac{b}{de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\left(2a^2+5b^2\right)\int \frac{1}{\sqrt{e\sin(c+dx)}}dx-21ab^2\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{3e^2(a^2-b^2)}+\frac{2(7ab-(2a^2+5b^2)\cos(c+dx))}{3de(a^2-b^2)(e\sin(c+dx))^{3/2}}}{2\left(a^2-b^2\right)}-\frac{b}{de\left(a^2-b^2\right)\left(e\sin(c+dx)\right)^{3/2}(a+b\cos(c+dx))}$$

$\Big\downarrow$ 3121

$$\frac{\frac{\frac{\left(2a^2+5b^2\right)\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}}dx}{\sqrt{e\sin(c+dx)}}-21ab^2\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{3e^2(a^2-b^2)}+\frac{2(7ab-(2a^2+5b^2)\cos(c+dx))}{3de(a^2-b^2)(e\sin(c+dx))^{3/2}}}{2\left(a^2-b^2\right)}-\frac{b}{de\left(a^2-b^2\right)\left(e\sin(c+dx)\right)^{3/2}(a+b\cos(c+dx))}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{\left(2a^2+5b^2\right)\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}}dx}{\sqrt{e\sin(c+dx)}}-21ab^2\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{3e^2(a^2-b^2)}+\frac{2(7ab-(2a^2+5b^2)\cos(c+dx))}{3de(a^2-b^2)(e\sin(c+dx))^{3/2}}}{2\left(a^2-b^2\right)}-\frac{b}{de\left(a^2-b^2\right)\left(e\sin(c+dx)\right)^{3/2}(a+b\cos(c+dx))}$$

$\Big\downarrow$ 3120

$$\frac{\frac{\frac{2\left(2a^2+5b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-21ab^2\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{3e^2(a^2-b^2)}+\frac{2(7ab-(2a^2+5b^2)\cos(c+dx))}{3de(a^2-b^2)(e\sin(c+dx))^{3/2}}}{2\left(a^2-b^2\right)}-\frac{b}{de\left(a^2-b^2\right)\left(e\sin(c+dx)\right)^{3/2}(a+b\cos(c+dx))}$$

$\Big\downarrow$ 3181

$$\frac{\frac{\frac{2\left(2a^2+5b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-21ab^2\left(-\frac{be\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b^2\sin^2(c+dx)e^2+\left(a^2-b^2\right)e^2\right)}d(e\sin(c+dx))}{d}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}\right.}}{2\sqrt{b^2-a^2}}\right.}{3e^2(a^2-b^2)}}{2\left(a^2-b^2\right)}-\frac{b}{de\left(a^2-b^2\right)\left(e\sin(c+dx)\right)^{3/2}(a+b\cos(c+dx))}$$

$\Big\downarrow$ 266

$$\frac{\frac{\frac{2\left(2a^2+5b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-21ab^2\left(-\frac{2be\int\frac{1}{b^2e^4\sin^4(c+dx)+\left(a^2-b^2\right)e^2}d\sqrt{e\sin(c+dx)}}{d}-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}}{2\sqrt{b^2-a^2}}\right.}{3e^2(a^2-b^2)}}{2\left(a^2-b^2\right)}$$

$$\frac{b}{de\left(a^2-b^2\right)\left(e\sin(c+dx)\right)^{3/2}\left(a+b\cos(c+dx)\right)}$$

$\downarrow$ 756

$$\frac{\frac{\frac{2\left(2a^2+5b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-21ab^2\left(-\frac{2be\left(-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2e\sqrt{b^2-a^2}}-\frac{\int\frac{1}{be^2\sin^2(c+dx)+\sqrt{b^2-a^2}e}d\sqrt{e\sin(c+d}}{2e\sqrt{b^2-a^2}}\right.}{d}\right.}{3e^2(a^2-b^2)}}{2\left(a^2-b^2\right)}$$

$$\frac{b}{de\left(a^2-b^2\right)\left(e\sin(c+dx)\right)^{3/2}\left(a+b\cos(c+dx)\right)}$$

$\downarrow$ 218

$$\frac{\frac{\frac{2\left(2a^2+5b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-21ab^2\left(-\frac{2be\left(-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2e\sqrt{b^2-a^2}}-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}\right)}{d}-\frac{a\int}{\sqrt{e\sin}}\right.}{3e^2(a^2-b^2)}}{2\left(a^2-b^2\right)}$$

$$\frac{b}{de\left(a^2-b^2\right)\left(e\sin(c+dx)\right)^{3/2}\left(a+b\cos(c+dx)\right)}$$

$\downarrow$ 221

$$\frac{\frac{2(2a^2+5b^2)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-21ab^2\left(-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}}-\right.}{3e^2(a^2-b^2)}$$

$$2(a^2-b^2)$$

$$\frac{b}{de(a^2-b^2)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}$$

$\downarrow$ 3042

$$\frac{\frac{2(2a^2+5b^2)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-21ab^2\left(-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}}-\right.}{3e^2(a^2-b^2)}$$

$$2(a^2-b^2)$$

$$\frac{b}{de(a^2-b^2)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}$$

$\downarrow$ 3286

$$\frac{\frac{2(2a^2+5b^2)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-21ab^2\left(-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}}-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c+\right.}}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+d}}\right.}{3e^2(a^2-b^2)}$$

$$2(a^2-b^2)$$

$$\frac{b}{de(a^2-b^2)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{2\left(2a^2+5b^2\right)\sqrt{\sin(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-21ab^2\left(-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}}-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c+\right.}}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx}}\right.}{3e^2(a^2-b^2)}}{2\left(a^2-b^2\right)}$$

$$\frac{b}{de\left(a^2-b^2\right)\left(e\sin(c+dx)\right)^{3/2}(a+b\cos(c+dx))}$$

$\big\downarrow$ 3284

$$\frac{\frac{\frac{2\left(2a^2+5b^2\right)\sqrt{\sin(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-21ab^2\left(-\frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}\right)}{d}+\frac{a\sqrt{\sin(c+dx)}\operatorname{Elliptic}}{d\sqrt{b^2-a^2}\left(b\right.}\right.}{3e^2(a^2-b^2)}}{2\left(a^2-b^2\right)}$$

$$\frac{b}{de\left(a^2-b^2\right)\left(e\sin(c+dx)\right)^{3/2}(a+b\cos(c+dx))}$$

input
```
Int[1/((a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(5/2)),x]
```

output
```
-(b/((a^2 - b^2)*d*e*(a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2))) + ((2*(
7*a*b - (2*a^2 + 5*b^2)*Cos[c + d*x]))/(3*(a^2 - b^2)*d*e*(e*Sin[c + d*x])
^(3/2)) + ((2*(2*a^2 + 5*b^2)*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c
+ d*x]])/(d*Sqrt[e*Sin[c + d*x]]) - 21*a*b^2*((-2*b*e*(-1/2*ArcTan[(Sqrt[b
]*Sqrt[e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/(Sqrt[b]*(-a^2 + b^2)^(3/4)*e^
(3/2)) - ArcTanh[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/(2*Sqr
t[b]*(-a^2 + b^2)^(3/4)*e^(3/2))))/d + (a*EllipticPi[(2*b)/(b - Sqrt[-a^2
+ b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(Sqrt[-a^2 + b^2]*(b -
 Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]]) - (a*EllipticPi[(2*b)/(b + Sqrt
[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(Sqrt[-a^2 + b^2
]*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]])))/(3*(a^2 - b^2)*e^2))/(2
*(a^2 - b^2))
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3173
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Simp[1/((a^2 - b^2)*(m + 1))
   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m +
p + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

rule 3181
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[-a/(2*q)   Int[1/(
Sqrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Simp[b*(g/f)   Subst[
Int[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - S
imp[a/(2*q)   Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3345
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Simp[1/(g^2*(a^2 - b^2)*(p +
 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
 2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[p, -1] && IntegerQ[2*m]
```

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

## Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1469 vs. $2(465) = 930$.

Time = 3.05 (sec) , antiderivative size = 1470, normalized size of antiderivative = 2.77

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1470 |

input
```
int(1/(a+cos(d*x+c)*b)^2/(e*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(-4*e^3*a*b*(-1/e^4/(a-b)^2/(a+b)^2*b^2*(1/4*(e*sin(d*x+c))^(1/2)/(-b^2*co
s(d*x+c)^2*e^2+a^2*e^2)+7/32*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2
^(1/2)*(ln((e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^
(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(
e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arctan(2^(1/2)/(
e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2)/(e^2*(a^
2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)))-1/3/e^4/(a^2-b^2)^2/(e*sin(d*x
+c))^(3/2))-(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/e^2*(-1/3*(a^2+b^2)/(a^2-b^2
)^2/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(cos(d*x+c)^2-1)*((1-sin(d*x+c))^(1/
2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(5/2)*EllipticF((1-sin(d*x+c))^(1/2),
1/2*2^(1/2))+2*cos(d*x+c)^2*sin(d*x+c))+b^2*(a^2+b^2)/(a-b)^2/(a+b)^2*(-1/
2/b/(-a^2+b^2)^(1/2)*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c
)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticP
i((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/2/b/(-a^2+b
^2)^(1/2)*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(co
s(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*
x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2)))+2*a^2*b^2/(a-b)/(a+b)*(
1/2*b^2/e/a^2/(a^2-b^2)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-cos(d*x+c)^2*b
^2+a^2)+1/4/a^2/(a^2-b^2)*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(
d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((1-sin(d*x+c))...
```

## Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))**2/(e*sin(d*x+c))**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

## Giac [F]

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\cos(dx+c)+a)^2\,(e\sin(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^2*(e*sin(d*x + c))^(5/2)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(e\,\sin\left(c+d\,x\right)\right)^{5/2}\,\left(a+b\,\cos\left(c+d\,x\right)\right)^2}\,dx$$

input
```
int(1/((e*sin(c + d*x))^(5/2)*(a + b*cos(c + d*x))^2),x)
```

output
```
int(1/((e*sin(c + d*x))^(5/2)*(a + b*cos(c + d*x))^2), x)
```

## Reduce [F]

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx = \frac{\sqrt{e}\left(\int \frac{\sqrt{\sin(dx+c)}}{\cos(dx+c)^2\sin(dx+c)^3b^2+2\cos(dx+c)\sin(dx+c)^3ab+\sin(dx+c)^3a^2}dx\right.}{e^3}$$

input
```
int(1/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(5/2),x)
```

output
```
(sqrt(e)*int(sqrt(sin(c + d*x))/(cos(c + d*x)**2*sin(c + d*x)**3*b**2 + 2*
cos(c + d*x)*sin(c + d*x)**3*a*b + sin(c + d*x)**3*a**2),x))/e**3
```

# 3.77 $\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 590

$$\begin{aligned}
\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}}\,dx = & \frac{9ab^{7/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2\left(-a^2+b^2\right)^{13/4}de^{7/2}} \\
& - \frac{9ab^{7/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2\left(-a^2+b^2\right)^{13/4}de^{7/2}} \\
& - \frac{b}{(a^2-b^2)\,de(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}} \\
& + \frac{9ab-(2a^2+7b^2)\cos(c+dx)}{5\left(a^2-b^2\right)^2de(e\sin(c+dx))^{5/2}} \\
& - \frac{3(15ab^3+(2a^4-10a^2b^2-7b^4)\cos(c+dx))}{5\left(a^2-b^2\right)^3de^3\sqrt{e\sin(c+dx)}} \\
& + \frac{9a^2b^3\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2\left(a^2-b^2\right)^3\left(b-\sqrt{-a^2+b^2}\right)de^3\sqrt{e\sin(c+dx)}} \\
& + \frac{9a^2b^3\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2\left(a^2-b^2\right)^3\left(b+\sqrt{-a^2+b^2}\right)de^3\sqrt{e\sin(c+dx)}} \\
& - \frac{3(2a^4-10a^2b^2-7b^4)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5\left(a^2-b^2\right)^3de^4\sqrt{\sin(c+dx)}}
\end{aligned}$$

output
```
9/2*a*b^(7/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2)
)/(-a^2+b^2)^(13/4)/d/e^(7/2)-9/2*a*b^(7/2)*arctanh(b^(1/2)*(e*sin(d*x+c))
^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a^2+b^2)^(13/4)/d/e^(7/2)-b/(a^2-b^2)/d
/e/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(5/2)+1/5*(9*a*b-(2*a^2+7*b^2)*cos(d*x+
c))/(a^2-b^2)^2/d/e/(e*sin(d*x+c))^(5/2)-3/5*(15*a*b^3+(2*a^4-10*a^2*b^2-7
*b^4)*cos(d*x+c))/(a^2-b^2)^3/d/e^3/(e*sin(d*x+c))^(1/2)-9/2*a^2*b^3*Ellip
ticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+
c)^(1/2)/(a^2-b^2)^3/(b-(-a^2+b^2)^(1/2))/d/e^3/(e*sin(d*x+c))^(1/2)-9/2*a
^2*b^3*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/
2))*sin(d*x+c)^(1/2)/(a^2-b^2)^3/(b+(-a^2+b^2)^(1/2))/d/e^3/(e*sin(d*x+c))
^(1/2)+3/5*(2*a^4-10*a^2*b^2-7*b^4)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^
(1/2))*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^3/d/e^4/sin(d*x+c)^(1/2)
```

## Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 7.38 (sec) , antiderivative size = 950, normalized size of antiderivative = 1.61

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[1/((a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(7/2)),x]
```

output
```
(Sin[c + d*x]^4*((-2*(20*a*b^3 + 3*a^4*Cos[c + d*x] - 15*a^2*b^2*Cos[c + d
*x] - 8*b^4*Cos[c + d*x])*Csc[c + d*x])/(5*(a^2 - b^2)^3) - (2*(-2*a*b + a
^2*Cos[c + d*x] + b^2*Cos[c + d*x])*Csc[c + d*x]^3)/(5*(a^2 - b^2)^2) - (b
^5*Sin[c + d*x])/((a^2 - b^2)^3*(a + b*Cos[c + d*x]))))/(d*(e*Sin[c + d*x]
)^(7/2)) - (3*Sin[c + d*x]^(7/2)*(((2*a^4*b - 10*a^2*b^3 - 7*b^5)*Cos[c +
d*x]^2*(3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[
Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin
[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^
2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2]
+ Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]])
+ 8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^
2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2))*(a + b*Sqrt[1 - Sin[c + d*x]^2]))/(12
*b^(3/2)*(-a^2 + b^2)*(a + b*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) + (2*(2*a
^5 - 10*a^3*b^2 - 22*a*b^4)*Cos[c + d*x]*(((1/8 + I/8)*(2*ArcTan[1 - ((1 +
 I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I
)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] -
 (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]]
 + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c +
d*x]] + I*b*Sin[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)) + (a*AppellF1[3/4
, 1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c...
```

## Rubi [A] (warning: unable to verify)

Time = 2.81 (sec) , antiderivative size = 583, normalized size of antiderivative = 0.99, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.960, Rules used = {3042, 3173, 27, 3042, 3345, 27, 3042, 3345, 27, 3042, 3346, 3042, 3121, 3042, 3119, 3180, 266, 827, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(e\sin(c+dx))^{7/2}(a+b\cos(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{7/2}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3173}$$

$$-\frac{\int -\frac{2a-7b\cos(c+dx)}{2(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}dx}{a^2-b^2}-\frac{b}{de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\int \frac{2a-7b\cos(c+dx)}{(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}dx}{2\left(a^2-b^2\right)}-\frac{b}{de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\int \frac{2a+7b\sin\left(c+dx-\frac{\pi}{2}\right)}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{7/2}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{2\left(a^2-b^2\right)}-\frac{b}{de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}$$

$$\Big\downarrow \textbf{3345}$$

$$\frac{\frac{2(9ab-(2a^2+7b^2)\cos(c+dx))}{5de(a^2-b^2)(e\sin(c+dx))^{5/2}}-\frac{2\int -\frac{3\left(2a\left(a^2-4b^2\right)+b\left(2a^2+7b^2\right)\cos(c+dx)\right)}{2(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}dx}{5e^2(a^2-b^2)}}{2\left(a^2-b^2\right)}-\frac{b}{de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\frac{3\int \frac{2a\left(a^2-4b^2\right)+b\left(2a^2+7b^2\right)\cos(c+dx)}{(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}dx}{5e^2(a^2-b^2)}+\frac{2(9ab-(2a^2+7b^2)\cos(c+dx))}{5de(a^2-b^2)(e\sin(c+dx))^{5/2}}}{2\left(a^2-b^2\right)}-\frac{b}{de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\frac{3\int \frac{2a\left(a^2-4b^2\right)-b\left(2a^2+7b^2\right)\sin\left(c+dx-\frac{\pi}{2}\right)}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{5e^2(a^2-b^2)}+\frac{2(9ab-(2a^2+7b^2)\cos(c+dx))}{5de(a^2-b^2)(e\sin(c+dx))^{5/2}}}{2\left(a^2-b^2\right)}-\frac{b}{de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}$$

$$\Big\downarrow \textbf{3345}$$

$$\frac{\frac{3\left(-\frac{2\int \frac{\left(2a\left(a^4-5b^2a^2-11b^4\right)+b\left(2a^4-10b^2a^2-7b^4\right)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{2(a+b\cos(c+dx))}dx}{e^2\left(a^2-b^2\right)}-\frac{2\left(\left(2a^4-10a^2b^2-7b^4\right)\cos(c+dx)+15ab^3\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}\right)}{5e^2(a^2-b^2)}+\frac{2(9ab-(2a^2+7b^2)\cos}{5de(a^2-b^2)(e\sin(c+}}{2\left(a^2-b^2\right)}
\frac{b}{de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}$$

$\downarrow$ 27

$$\frac{3\left(-\frac{\int \frac{\left(2a\left(a^4-5b^2a^2-11b^4\right)+b\left(2a^4-10b^2a^2-7b^4\right)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx}{e^2\left(a^2-b^2\right)}-\frac{2\left(\left(2a^4-10a^2b^2-7b^4\right)\cos(c+dx)+15ab^3\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}\right)}{5e^2(a^2-b^2)}+\frac{2(9ab-(2a^2+7b^2)\cos(c+dx))}{5de(a^2-b^2)(e\sin(c+dx))^{5/2}}}{2\left(a^2-b^2\right)}$$

$$\frac{b}{de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}$$

$\downarrow$ 3042

$$\frac{3\left(-\frac{\int \frac{\sqrt{-e\cos\left(c+dx+\frac{\pi}{2}\right)}\left(2a\left(a^4-5b^2a^2-11b^4\right)+b\left(2a^4-10b^2a^2-7b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{e^2\left(a^2-b^2\right)}-\frac{2\left(\left(2a^4-10a^2b^2-7b^4\right)\cos(c+dx)+15ab^3\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}\right)}{5e^2(a^2-b^2)}+\frac{2(9ab-(2a^2+7b^2)\cos(c+dx))}{5de(a^2-b^2)(e\sin(c+dx))^{5/2}}}{2\left(a^2-b^2\right)}$$

$$\frac{b}{de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}$$

$\downarrow$ 3346

$$\frac{3\left(-\frac{\left(2a^4-10a^2b^2-7b^4\right)\int\sqrt{e\sin(c+dx)}dx-15ab^4\int\frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx}{e^2\left(a^2-b^2\right)}-\frac{2\left(\left(2a^4-10a^2b^2-7b^4\right)\cos(c+dx)+15ab^3\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}\right)}{5e^2(a^2-b^2)}+\frac{2(9ab-(2a^2+7b^2)\cos(c+dx))}{5de(a^2-b^2)(e\sin(c+dx))^{5/2}}}{2\left(a^2-b^2\right)}$$

$$\frac{b}{de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}$$

$\downarrow$ 3042

$$\frac{3\left(-\frac{\left(2a^4-10a^2b^2-7b^4\right)\int\sqrt{e\sin(c+dx)}dx-15ab^4\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{e^2\left(a^2-b^2\right)}-\frac{2\left(\left(2a^4-10a^2b^2-7b^4\right)\cos(c+dx)+15ab^3\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}\right)}{5e^2(a^2-b^2)}+\frac{2(9ab-(2a^2+7b^2)\cos(c+dx))}{5de(a^2-b^2)(e\sin(c+dx))^{5/2}}}{2\left(a^2-b^2\right)}$$

$$\frac{b}{de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}$$

$\downarrow$ 3121

$$\frac{\frac{3\left(-\frac{\frac{\left(2a^4-10a^2b^2-7b^4\right)\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}-15ab^4\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{e^2\left(a^2-b^2\right)}-\frac{2\left(\left(2a^4-10a^2b^2-7b^4\right)\cos(c+dx)+15ab^3\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}\right)}{5e^2(a^2-b^2)}+\frac{2(9ab-(2a^2+7b^2}{5de(a^2-b^2)(e\sin}}{2\left(a^2-b^2\right)}$$

$$\frac{b}{de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}$$

$\Big\downarrow$ 3042

$$\frac{\frac{3\left(-\frac{\frac{\left(2a^4-10a^2b^2-7b^4\right)\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}-15ab^4\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{e^2\left(a^2-b^2\right)}-\frac{2\left(\left(2a^4-10a^2b^2-7b^4\right)\cos(c+dx)+15ab^3\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}\right)}{5e^2(a^2-b^2)}+\frac{2(9ab-(2a^2+7b^2}{5de(a^2-b^2)(e\sin}}{2\left(a^2-b^2\right)}$$

$$\frac{b}{de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}$$

$\Big\downarrow$ 3119

$$\frac{\frac{3\left(-\frac{\frac{2\left(2a^4-10a^2b^2-7b^4\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-15ab^4\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{e^2\left(a^2-b^2\right)}-\frac{2\left(\left(2a^4-10a^2b^2-7b^4\right)\cos(c+dx)+15ab^3\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}\right)}{5e^2(a^2-b^2)}+\frac{2(9ab-(2a^2}{5de(a^2-b^2)}}{2\left(a^2-b^2\right)}$$

$$\frac{b}{de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}$$

$\Big\downarrow$ 3180

$$\frac{\frac{3\left(-\frac{\frac{2\left(2a^4-10a^2b^2-7b^4\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-15ab^4\left(-\frac{be\int\frac{\sqrt{e\sin(c+dx)}}{b^2\sin^2(c+dx)e^2+\left(a^2-b^2\right)e^2}d(e\sin(c+dx))}{d}-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(\right.}}{2b}\right.}{e^2\left(a^2-b^2\right)}\right.}{5e^2(a^2-b^2)}}{2\left(a^2-b^2\right)}$$

$$\frac{b}{de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}$$

$\Big\downarrow$ 266

$$3\left(-\frac{\frac{2\left(2a^4-10a^2b^2-7b^4\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-15ab^4\left(-\frac{2be\int\frac{e^2\sin^2(c+dx)}{b^2e^4\sin^4(c+dx)+\left(a^2-b^2\right)e^2}d\sqrt{e\sin(c+dx)}}{d}-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin\right.}}{2b}\right.}{e^2\left(a^2-b^2\right)}\right.$$

$$5e^2(a^2-b^2)$$

$$2\left(a^2-b^2\right.$$

$$\frac{b}{de\left(a^2-b^2\right)\left(e\sin(c+dx)\right)^{5/2}\left(a+b\cos(c+dx)\right)}$$

$\Big\downarrow$ 827

$$3\left(-\frac{\frac{2\left(2a^4-10a^2b^2-7b^4\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-15ab^4\left(-\frac{2be\left(\frac{\int\frac{1}{be^2\sin^2(c+dx)+\sqrt{b^2-a^2}e}d\sqrt{e\sin(c+dx)}}{2b}-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin}}{2b}\right.}{d}\right.}{e^2\left(a^2-b^2\right)}\right.$$

$$5e^2(a^2-$$

$$\frac{b}{de\left(a^2-b^2\right)\left(e\sin(c+dx)\right)^{5/2}\left(a+b\cos(c+dx)\right)}$$

$\Big\downarrow$ 218

$$3\left(-\frac{\frac{2\left(2a^4-10a^2b^2-7b^4\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-15ab^4\left(-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2b}\right)}{d}-\frac{ae\int}{}\right.}{e^2\left(a^2-b^2\right)}\right.}{5e^2(a^2-b^2)}$$

$$\frac{b}{de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 221$$

$$3\left(-\frac{\frac{2\left(2a^4-10a^2b^2-7b^4\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-15ab^4\left(-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-\right.}}}{2b}\right.}{e^2\left(a^2-b^2\right)}\right.}{5e^2(a^2-b^2)}$$

$$2\,(a$$

$$\frac{b}{de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}$$

$\Big\downarrow$ 3042

$$3\left(-\frac{\frac{2\left(2a^4-10a^2b^2-7b^4\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-15ab^4\left(-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-\cdots}\right)}\cdots}{2b}\cdots\right.}{e^2\left(a^2-b^2\right)}\cdots\right)$$

$$\frac{\cdots}{5e^2(a^2-b^2)}$$

$$\frac{\cdots}{2\,(a\cdots}$$

$$\frac{b}{de\left(a^2-b^2\right)\left(e\sin(c+dx)\right)^{5/2}\left(a+b\cos(c+dx)\right)}$$

$\Big\downarrow$ 3286

$$3\left(-\frac{\frac{2\left(2a^4-10a^2b^2-7b^4\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-15ab^4\left(-\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b\sqrt{e\sin(c+dx)}}+\frac{ae\sqrt{\sin(c+dx)}\int\frac{}{\sqrt{\sin(c+dx)}}}{2b\sqrt{e\sin}}\right.}{e^2\left(a^2-b^2\right)}\right.$$

$$\overline{\qquad 5e^2(a^2-b^2)}$$

$$\frac{b}{de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 3042$$

$$3\left(-\frac{\frac{2\left(2a^4-10a^2b^2-7b^4\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-15ab^4\left(-\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b\sqrt{e\sin(c+dx)}}+\frac{ae\sqrt{\sin(c+dx)}\int\frac{}{\sqrt{\sin(c+dx)}}}{2b\sqrt{e\sin}}\right.}{e^2\left(a^2-b^2\right)}\right.$$

$$\overline{\qquad 5e^2(a^2-b^2)}$$

$$\frac{b}{de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3284}$$

$$\frac{2\left(9ab-\left(2a^2+7b^2\right)\cos(c+dx)\right)}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}}+\frac{3\left(-\frac{\frac{2\left(2a^4-10a^2b^2-7b^4\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-15ab^4\left(-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\text{arctanh}\left(\cdots\right.}{2b^{3/2}\sqrt{e}\cdots}\right)}{d}\cdots\right.}{\cdots}\right.}{\cdots}$$

$$\frac{b}{de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}$$

input

```
Int[1/((a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(7/2)),x]
```

output

```
-(b/((a^2 - b^2)*d*e*(a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(5/2))) + ((2*(
9*a*b - (2*a^2 + 7*b^2)*Cos[c + d*x]))/(5*(a^2 - b^2)*d*e*(e*Sin[c + d*x])
^(5/2)) + (3*((-2*(15*a*b^3 + (2*a^4 - 10*a^2*b^2 - 7*b^4)*Cos[c + d*x]))/
((a^2 - b^2)*d*e*Sqrt[e*Sin[c + d*x]]) - ((2*(2*a^4 - 10*a^2*b^2 - 7*b^4)*
EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(d*Sqrt[Sin[c + d*x
]]) - 15*a*b^4*((-2*b*e*(ArcTan[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(-a^2 + b^2
)^(1/4)]/(2*b^(3/2)*(-a^2 + b^2)^(1/4)*Sqrt[e]) - ArcTanh[(Sqrt[b]*Sqrt[e]
*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/(2*b^(3/2)*(-a^2 + b^2)^(1/4)*Sqrt[e]))
)/d + (a*e*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]
*Sqrt[Sin[c + d*x]])/(b*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]]) + (
a*e*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[S
in[c + d*x]])/(b*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]])))/((a^2 -
b^2)*e^2)))/(5*(a^2 - b^2)*e^2))/(2*(a^2 - b^2))
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3173
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Simp[1/((a^2 - b^2)*(m + 1))
   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m +
p + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

rule 3180
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[a*(g/(2*b))   Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Simp[a*(g/(2*b))   In
t[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Simp[b*(g/f)   Su
bst[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3345
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Simp[1/(g^2*(a^2 - b^2)*(p +
 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
 2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[p, -1] && IntegerQ[2*m]
```

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

## Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1744 vs. $2(522) = 1044$.

Time = 3.40 (sec) , antiderivative size = 1745, normalized size of antiderivative = 2.96

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1745 |

input
```
int(1/(a+cos(d*x+c)*b)^2/(e*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
(-2*e^3*a*b*(2*b^4/e^6/(a-b)^3/(a+b)^3*(1/4*(e*sin(d*x+c))^(3/2)/(-b^2*cos
(d*x+c)^2*e^2+a^2*e^2)+9/32/b^2/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*(ln((e*s
in(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2
-b^2)/b^2)^(1/2))/(e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(
1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b
^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/
4)*(e*sin(d*x+c))^(1/2)-1)))+4/e^6/(a-b)^3/(a+b)^3*b^2/(e*sin(d*x+c))^(1/2
)-2/5/e^4/(a+b)^2/(a-b)^2/(e*sin(d*x+c))^(5/2))-(cos(d*x+c)^2*e*sin(d*x+c)
)^(1/2)/e^3*(1/5*(a^2+b^2)/(a^2-b^2)^2/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/s
in(d*x+c)/(cos(d*x+c)^2-1)*(6*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*
sin(d*x+c)^(7/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-3*(1-sin(d*x+
c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(7/2)*EllipticF((1-sin(d*x+c))
^(1/2),1/2*2^(1/2))+6*cos(d*x+c)^4*sin(d*x+c)-8*cos(d*x+c)^2*sin(d*x+c))+b
^2*(3*a^2+b^2)/(a^2-b^2)^3*(2*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*
sin(d*x+c)^(1/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-(1-sin(d*x+c)
)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(
1/2),1/2*2^(1/2))-2*cos(d*x+c)^2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)-2*a^2*
b^4/(a-b)^2/(a+b)^2*(1/2*b^2/e/a^2/(a^2-b^2)*sin(d*x+c)*(cos(d*x+c)^2*e*si
n(d*x+c))^(1/2)/(-cos(d*x+c)^2*b^2+a^2)-1/2/a^2/(a^2-b^2)*(1-sin(d*x+c))^(
1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c)...
```

## Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
Timed out
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))**2/(e*sin(d*x+c))**(7/2),x)
```

output
```
Timed out
```

## Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
Timed out
```

## Giac [F]

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}}\,dx = \int \frac{1}{(b\cos(dx+c)+a)^2(e\sin(dx+c))^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^2*(e*sin(d*x + c))^(7/2)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}}\,dx = \int \frac{1}{(e\sin(c+dx))^{7/2}\,(a+b\cos(c+dx))^2}\,dx$$

input
```
int(1/((e*sin(c + d*x))^(7/2)*(a + b*cos(c + d*x))^2),x)
```

output
```
int(1/((e*sin(c + d*x))^(7/2)*(a + b*cos(c + d*x))^2), x)
```

## Reduce [F]

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}}\,dx = \frac{\sqrt{e}\left(\int \frac{\sqrt{\sin(dx+c)}}{\cos(dx+c)^2\sin(dx+c)^4b^2+2\cos(dx+c)\sin(dx+c)^4ab+\sin(dx+c)^4a^2}dx\right.}{e^4}$$

input
```
int(1/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(7/2),x)
```

output
```
(sqrt(e)*int(sqrt(sin(c + d*x))/(cos(c + d*x)**2*sin(c + d*x)**4*b**2 + 2*
cos(c + d*x)*sin(c + d*x)**4*a*b + sin(c + d*x)**4*a**2),x))/e**4
```

# 3.78 $\int \frac{(e\sin(c+dx))^{13/2}}{(a+b\cos(c+dx))^3}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 590

$$\begin{aligned}\int \frac{(e\sin(c+dx))^{13/2}}{(a+b\cos(c+dx))^3}\,dx &= \frac{11(9a^4-11a^2b^2+2b^4)\,e^{13/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{13/2}\sqrt[4]{-a^2+b^2}d}\\
&-\frac{11(9a^4-11a^2b^2+2b^4)\,e^{13/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{13/2}\sqrt[4]{-a^2+b^2}d}\\
&-\frac{11a(9a^4-11a^2b^2+2b^4)\,e^7\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b^7\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}\\
&-\frac{11a(9a^4-11a^2b^2+2b^4)\,e^7\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b^7\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}\\
&+\frac{11a(45a^2-37b^2)\,e^6E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{20b^6d\sqrt{\sin(c+dx)}}\\
&-\frac{11e^5(5(9a^2-2b^2)-27ab\cos(c+dx))\,(e\sin(c+dx))^{3/2}}{60b^5d}\\
&+\frac{11e^3(9a+2b\cos(c+dx))(e\sin(c+dx))^{7/2}}{28b^3d(a+b\cos(c+dx))}+\frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2}\end{aligned}$$

output
```
11/8*(9*a^4-11*a^2*b^2+2*b^4)*e^(13/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)
/(-a^2+b^2)^(1/4)/e^(1/2))/b^(13/2)/(-a^2+b^2)^(1/4)/d-11/8*(9*a^4-11*a^2*
b^2+2*b^4)*e^(13/2)*arctanh(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/
e^(1/2))/b^(13/2)/(-a^2+b^2)^(1/4)/d+11/8*a*(9*a^4-11*a^2*b^2+2*b^4)*e^7*E
llipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(
d*x+c)^(1/2)/b^7/(b-(-a^2+b^2)^(1/2))/d/(e*sin(d*x+c))^(1/2)+11/8*a*(9*a^4
-11*a^2*b^2+2*b^4)*e^7*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b
^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^7/(b+(-a^2+b^2)^(1/2))/d/(e*sin(d*x
+c))^(1/2)-11/20*a*(45*a^2-37*b^2)*e^6*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x)
,2^(1/2))*(e*sin(d*x+c))^(1/2)/b^6/d/sin(d*x+c)^(1/2)-11/60*e^5*(45*a^2-10
*b^2-27*a*b*cos(d*x+c))*(e*sin(d*x+c))^(3/2)/b^5/d+11/28*e^3*(9*a+2*b*cos(
d*x+c))*(e*sin(d*x+c))^(7/2)/b^3/d/(a+b*cos(d*x+c))+1/2*e*(e*sin(d*x+c))^(
11/2)/b/d/(a+b*cos(d*x+c))^2
```

## Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 16.72 (sec) , antiderivative size = 930, normalized size of antiderivative = 1.58

$$\int \frac{(e\sin(c+dx))^{13/2}}{(a+b\cos(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
Integrate[(e*Sin[c + d*x])^(13/2)/(a + b*Cos[c + d*x])^3,x]
```

output
```
(11*(e*Sin[c + d*x])^(13/2)*(((45*a^3 - 37*a*b^2)*Cos[c + d*x]^2*(3*Sqrt[2
]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(
a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2
 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*S
qrt[Sin[c + d*x]] + b*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b
]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]]) + 8*b^(5/2)*Appe
llF1[3/4, -1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]
*Sin[c + d*x]^(3/2))*(a + b*Sqrt[1 - Sin[c + d*x]^2]))/(12*b^(3/2)*(-a^2 +
 b^2)*(a + b*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) + (2*(18*a^2*b - 10*b^3)*
Cos[c + d*x]*(((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x
]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]]
)/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2
)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1
 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]]))/
(Sqrt[b]*(-a^2 + b^2)^(1/4)) + (a*AppellF1[3/4, 1/2, 1, 7/4, Sin[c + d*x]^
2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2))/(3*(a^2 - b^2)))
*(a + b*Sqrt[1 - Sin[c + d*x]^2]))/((a + b*Cos[c + d*x])*Sqrt[1 - Sin[c +
d*x]^2])))/(40*b^5*d*Sin[c + d*x]^(13/2)) + (Csc[c + d*x]^6*(e*Sin[c + d*x
])^(13/2)*(((-168*a^2 + 65*b^2)*Sin[c + d*x])/(42*b^5) - (19*(a^3*Sin[c +
d*x] - a*b^2*Sin[c + d*x]))/(4*b^5*(a + b*Cos[c + d*x])) + (a^4*Sin[c +...
```

## Rubi [A] (warning: unable to verify)

Time = 2.53 (sec) , antiderivative size = 543, normalized size of antiderivative = 0.92, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.960$, Rules used = {3042, 3172, 25, 3042, 3342, 27, 3042, 3344, 27, 3042, 3346, 3042, 3121, 3042, 3119, 3180, 266, 827, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sin(c+dx))^{13/2}}{(a+b\cos(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{13/2}}{\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3172}$$

$$\frac{11e^2\int -\frac{\cos(c+dx)(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^2}dx}{4b}+\frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2}-\frac{11e^2\int \frac{\cos(c+dx)(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^2}dx}{4b}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2}-\frac{11e^2\int \frac{\left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{9/2}\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4b}$$

$$\Big\downarrow \textbf{3342}$$

$$\frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2}-\frac{11e^2\left(-\frac{e^2\int -\frac{(2b+9a\cos(c+dx))(e\sin(c+dx))^{5/2}}{2(a+b\cos(c+dx))}dx}{b^2}-\frac{e(e\sin(c+dx))^{7/2}(9a+2b\cos(c+dx))}{7b^2d(a+b\cos(c+dx))}\right)}{4b}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2}-\frac{11e^2\left(\frac{e^2\int \frac{(2b+9a\cos(c+dx))(e\sin(c+dx))^{5/2}}{a+b\cos(c+dx)}dx}{2b^2}-\frac{e(e\sin(c+dx))^{7/2}(9a+2b\cos(c+dx))}{7b^2d(a+b\cos(c+dx))}\right)}{4b}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2}-\frac{11e^2\left(\frac{e^2\int \frac{\left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(2b+9a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2b^2}-\frac{e(e\sin(c+dx))^{7/2}(9a+2b\cos(c+dx))}{7b^2d(a+b\cos(c+dx))}\right)}{4b}$$

$$\Big\downarrow \textbf{3344}$$

$$\frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2}-\frac{11e^2\left(\frac{e^2\left(\frac{2e^2\int-\frac{\left(2b\left(9a^2-5b^2\right)+a\left(45a^2-37b^2\right)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{2(a+b\cos(c+dx))}dx}{5b^2}+\frac{2e(e\sin(c+dx))^{3/2}\left(5\left(9a^2-2b^2\right)-27ab\cos(c+dx)\right)}{15b^2d}\right)}{2b^2}-\frac{e(e\sin(c+dx))^{7}}{7b^2d(a-}\right.}{4b}$$

$\downarrow$ 27

$$\frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2}-\frac{11e^2\left(\frac{e^2\left(\frac{2e(e\sin(c+dx))^{3/2}\left(5\left(9a^2-2b^2\right)-27ab\cos(c+dx)\right)}{15b^2d}-\frac{e^2\int\frac{\left(2b\left(9a^2-5b^2\right)+a\left(45a^2-37b^2\right)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx}{5b^2}\right)}{2b^2}-\frac{e(e\sin(c+dx))^{7/2}}{7b^2d(a+b}\right.}{4b}$$

$\downarrow$ 3042

$$\frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2}-\frac{11e^2\left(\frac{e^2\left(\frac{2e(e\sin(c+dx))^{3/2}\left(5\left(9a^2-2b^2\right)-27ab\cos(c+dx)\right)}{15b^2d}-\frac{e^2\int\frac{\sqrt{-e\cos\left(c+dx+\frac{\pi}{2}\right)}\left(2b\left(9a^2-5b^2\right)+a\left(45a^2-37b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5b^2}\right)}{2b^2}-\frac{e(e\sin(c}{}\right.}{4b}$$

$\downarrow$ 3346

$$\frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2}-\frac{11e^2\left(\frac{e^2\left(\frac{2e(e\sin(c+dx))^{3/2}\left(5\left(9a^2-2b^2\right)-27ab\cos(c+dx)\right)}{15b^2d}-\frac{e^2\left(\frac{a\left(45a^2-37b^2\right)\int\sqrt{e\sin(c+dx)}dx}{b}-\frac{5\left(9a^4-11a^2b^2+2b^4\right)\int\frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx}{b}\right)}{5b^2}\right)}{2b^2}-\ldots\right)}{4b}$$

$\downarrow$ 3042

$$\frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2}-\frac{11e^2\left(\frac{e^2\left(\frac{2e(e\sin(c+dx))^{3/2}\left(5\left(9a^2-2b^2\right)-27ab\cos(c+dx)\right)}{15b^2d}-\frac{e^2\left(\frac{a\left(45a^2-37b^2\right)\int\sqrt{e\sin(c+dx)}dx}{b}-\frac{5\left(9a^4-11a^2b^2+2b^4\right)\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{b}\right)}{5b^2}\right)}{2b^2}\ldots\right)}{4b}$$

$\downarrow$ 3121

$$\frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2}-\frac{11e^2\left(\frac{e^2\left(\frac{2e(e\sin(c+dx))^{3/2}\left(5\left(9a^2-2b^2\right)-27ab\cos(c+dx)\right)}{15b^2d}-\frac{e^2\left(\frac{a\left(45a^2-37b^2\right)\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{b\sqrt{\sin(c+dx)}}-\frac{5\left(9a^4-11a^2b^2+2b^4\right)\int\frac{\sqrt{e\cos\left(c+dx-\ldots\right)}}{a-b\sin\left(c+dx-\ldots\right)}}{b}\ldots\right)}{5b^2}\ldots\right)}{2b^2}\ldots\right)}{4b}$$

$\downarrow$ 3042

$$\frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2}-\frac{11e^2\left(\frac{e^2\left(\frac{2e(e\sin(c+dx))^{3/2}\left(5\left(9a^2-2b^2\right)-27ab\cos(c+dx)\right)}{15b^2d}-\frac{e^2\left(\frac{a\left(45a^2-37b^2\right)\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{b\sqrt{\sin(c+dx)}}-\frac{5\left(9a^4-11a^2b^2+2b^4\right)\int\frac{\sqrt{e\cos\left(c+dx-\ldots\right)}}{a-b\sin\left(c+dx-\ldots\right)}}{b}\ldots\right)}{5b^2}\ldots\right)}{2b^2}\ldots\right)}{4b}$$

$\downarrow$ 3119

$$\frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2}-$$

$$11e^2\left(\frac{e^2\left(\frac{2e(e\sin(c+dx))^{3/2}\left(5\left(9a^2-2b^2\right)-27ab\cos(c+dx)\right)}{15b^2d}-\frac{e^2\left(\frac{2a\left(45a^2-37b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{5\left(9a^4-11a^2b^2+2b^4\right)\int\frac{\sqrt{e\cos\left(c+\right.}}{a-b\sin\left(c+\right.}}{b}\right.}{5b^2}\right.}{2b^2}\right.$$

$$\overline{\qquad 4b \qquad}$$

$\downarrow$ 3180

$$\frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2}-$$

$$11e^2\left(\frac{e^2\left(\frac{2e(e\sin(c+dx))^{3/2}\left(5\left(9a^2-2b^2\right)-27ab\cos(c+dx)\right)}{15b^2d}-\frac{e^2\left(\frac{2a\left(45a^2-37b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{5\left(9a^4-11a^2b^2+2b^4\right)\left(-\frac{be\int\frac{}{b^2\sin}}{}\right.}{}\right.}{}\right.}{2b^2}\right.$$

$\downarrow$ 266

$$\frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2}-$$

$$11e^2\left(\frac{e^2\left(\frac{2e(e\sin(c+dx))^{3/2}\left(5\left(9a^2-2b^2\right)-27ab\cos(c+dx)\right)}{15b^2d}-\frac{e^2\left(\frac{2a\left(45a^2-37b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{5\left(9a^4-11a^2b^2+2b^4\right)\left(-\frac{2be\int\frac{}{b^2e^{4}}}{}\right.}{}\right.}{}\right)}{2b^2}\right.$$

$\downarrow$ 827

$$\frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2}-$$

$$11e^2\left(e^2\left(\frac{2e(e\sin(c+dx))^{3/2}\left(5\left(9a^2-2b^2\right)-27ab\cos(c+dx)\right)}{15b^2d}-\frac{e^2\left(\frac{2a\left(45a^2-37b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{5\left(9a^4-11a^2b^2+2b^4\right)\left(-\frac{2be\left(\frac{\int}{be}\right.}{}\right.}{}\right.}{}\right.\right.$$

$\downarrow$ 218

$$\frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2}-$$

$$11e^2\left(\frac{e^2\left(\frac{2e(e\sin(c+dx))^{3/2}\left(5\left(9a^2-2b^2\right)-27ab\cos(c+dx)\right)}{15b^2d}-\frac{e^2\left(\frac{2a\left(45a^2-37b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{5\left(9a^4-11a^2b^2+2b^4\right)\left(-\frac{2be\left(\frac{\text{arcta}}{2b}\right.}{}\right.}{}\right.}{}\right.}{}\right.$$

↓ 221

$$\frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2} -$$

$$11e^2 \left( \frac{e^2 \left( \frac{2e(e\sin(c+dx))^{3/2}\left(5\left(9a^2-2b^2\right)-27ab\cos(c+dx)\right)}{15b^2d} - \frac{e^2\left( \frac{2a\left(45a^2-37b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}} - \frac{5\left(9a^4-11a^2b^2+2b^4\right)\left(-\frac{ae\int\frac{\cdots}{\sqrt{e\,\mathrm{si}\cdots}}}{\cdots}\right.}{\cdots}\right.}{\cdots}\right)}{\cdots}\right.$$

↓ 3042

$$\frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2} -$$

$$11e^2 \Bigg( e^2 \Bigg( \frac{2e(e\sin(c+dx))^{3/2}\left(5\left(9a^2-2b^2\right)-27ab\cos(c+dx)\right)}{15b^2 d} - \frac{e^2 \left( \frac{2a\left(45a^2-37b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}} - \frac{5\left(9a^4-11a^2b^2+2b^4\right)\left(-\frac{ae\int \frac{}{\sqrt{e\sin\cdots}}}{\cdots}\right.}{\cdots} \right.}{\cdots}$$

↓ 3286

$$\frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2}-$$

$$11e^2\left(\frac{e^2\left(\frac{2e(e\sin(c+dx))^{3/2}\left(5\left(9a^2-2b^2\right)-27ab\cos(c+dx)\right)}{15b^2d}-\frac{e^2\left(\frac{2a\left(45a^2-37b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{5\left(9a^4-11a^2b^2+2b^4\right)\left(-\frac{ae\sqrt{\sin(c-}}{}\right.}{}\right)}{}\right)}{}\right.$$

↓ 3042

$$\frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2}-$$

$$11e^2\left(\frac{e^2\left(\frac{2e(e\sin(c+dx))^{3/2}\left(5\left(9a^2-2b^2\right)-27ab\cos(c+dx)\right)}{15b^2d}-\frac{e^2\left(\frac{2a\left(45a^2-37b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{5\left(9a^4-11a^2b^2+2b^4\right)\left(-\frac{ae\sqrt{\sin(c-}}{}\right.}{}\right)}{}\right)}{}\right.$$

↓ 3284

$$\frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2}-$$

$$11e^2\left(e^2\left(\frac{2e(e\sin(c+dx))^{3/2}\left(5\left(9a^2-2b^2\right)-27ab\cos(c+dx)\right)}{15b^2d}-\frac{e^2\left(\frac{2a\left(45a^2-37b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{5\left(9a^4-11a^2b^2+2b^4\right)\left(-\frac{2be\left(\frac{\text{arcta}}{2b}\right.}{}\right.}{}\right.}{}\right.\right.$$

input
```
Int[(e*Sin[c + d*x])^(13/2)/(a + b*Cos[c + d*x])^3,x]
```

output
```
(e*(e*Sin[c + d*x])^(11/2))/(2*b*d*(a + b*Cos[c + d*x])^2) - (11*e^2*(-1/7
*(e*(9*a + 2*b*Cos[c + d*x])*(e*Sin[c + d*x])^(7/2))/(b^2*d*(a + b*Cos[c +
 d*x])) + (e^2*((2*e*(5*(9*a^2 - 2*b^2) - 27*a*b*Cos[c + d*x])*(e*Sin[c +
d*x])^(3/2))/(15*b^2*d) - (e^2*((2*a*(45*a^2 - 37*b^2)*EllipticE[(c - Pi/2
 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(b*d*Sqrt[Sin[c + d*x]]) - (5*(9*a^4 -
 11*a^2*b^2 + 2*b^4)*((-2*b*e*(ArcTan[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(-a^2
 + b^2)^(1/4)]/(2*b^(3/2)*(-a^2 + b^2)^(1/4)*Sqrt[e]) - ArcTanh[(Sqrt[b]*S
qrt[e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/(2*b^(3/2)*(-a^2 + b^2)^(1/4)*Sqr
t[e])))/d + (a*e*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)
/2, 2]*Sqrt[Sin[c + d*x]])/(b*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]
]) + (a*e*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*
Sqrt[Sin[c + d*x]])/(b*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]])))/b)
)/(5*b^2)))/(2*b^2)))/(4*b)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3172
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Simp[g^2*((p - 1)/(b*(m + 1)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Fre
eQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && I
ntegersQ[2*m, 2*p]
```

rule 3180
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[a*(g/(2*b))   Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Simp[a*(g/(2*b))   In
t[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Simp[b*(g/f)   Su
bst[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3342
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*C
os[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x])/(b^2*f*(m + 1)*(m + p + 1))), x] + Simp[g^2*((
p - 1)/(b^2*(m + 1)*(m + p + 1)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

rule 3344
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Simp[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Si
n[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^
2*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1
, 0] && IntegerQ[2*m]
```

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

## Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2994 vs. $2(517) = 1034$.

Time = 107.10 (sec) , antiderivative size = 2995, normalized size of antiderivative = 5.08

| method | result | size |
|---|---|---|
| default | Expression too large to display | 2995 |

input
```
int((e*sin(d*x+c))^(13/2)/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)
```

output
```
(2*e^3*b*(-1/21/b^6*(e*sin(d*x+c))^(3/2)*e^2*(3*cos(d*x+c)^2*b^2+42*a^2-17
*b^2)+e^4/b^6*(-1/8*(e*sin(d*x+c))^(3/2)*e^2*(-21*cos(d*x+c)^2*a^4*b^2+23*
a^2*b^4*cos(d*x+c)^2-2*b^6*cos(d*x+c)^2+17*a^6-15*a^4*b^2-2*b^4*a^2)/(-b^2
*cos(d*x+c)^2*e^2+a^2*e^2)^2+1/8*(99/8*a^4-121/8*a^2*b^2+11/4*b^4)/b^2/(e^
2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*(ln((e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)
*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)+(e^
2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1
/2)))+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2
*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1))))-(cos(
d*x+c)^2*e*sin(d*x+c))^(1/2)*e^7*a*(1/5/b^6/(cos(d*x+c)^2*e*sin(d*x+c))^(1
/2)*(100*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*Elli
pticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2-78*(1-sin(d*x+c))^(1/2)*(2+2*s
in(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/
2))*b^2-50*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*El
lipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2+39*(1-sin(d*x+c))^(1/2)*(2+2
*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(
1/2))*b^2+6*b^2*cos(d*x+c)^4-6*cos(d*x+c)^2*b^2)+3*(7*a^4-10*a^2*b^2+3*b^4
)/b^6*(-1/2/b^2*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/
2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-
sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-1/2/b^2*(1-sin(...
```

## Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{13/2}}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))^(13/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
Timed out
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{13/2}}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))**(13/2)/(a+b*cos(d*x+c))**3,x)
```

output
```
Timed out
```

## Maxima [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{13/2}}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))^(13/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
Timed out
```

## Giac [F]

$$\int \frac{(e\sin(c+dx))^{13/2}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{(e\sin(dx+c))^{\frac{13}{2}}}{(b\cos(dx+c)+a)^3}\,dx$$

input
```
integrate((e*sin(d*x+c))^(13/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((e*sin(d*x + c))^(13/2)/(b*cos(d*x + c) + a)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{13/2}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{(e\,\sin(c+d\,x))^{13/2}}{(a+b\,\cos(c+d\,x))^3}\,dx$$

input
```
int((e*sin(c + d*x))^(13/2)/(a + b*cos(c + d*x))^3,x)
```

output
```
int((e*sin(c + d*x))^(13/2)/(a + b*cos(c + d*x))^3, x)
```

## Reduce [F]

$$\int \frac{(e\sin(c+dx))^{13/2}}{(a+b\cos(c+dx))^3}\,dx = \sqrt{e}\left(\int \frac{\sqrt{\sin(dx+c)}\,\sin(dx+c)^6}{\cos(dx+c)^3 b^3+3\cos(dx+c)^2 a b^2+3\cos(dx+c) a^2 b+a^3}dx\right)e^6$$

input
```
int((e*sin(d*x+c))^(13/2)/(a+b*cos(d*x+c))^3,x)
```

output
```
sqrt(e)*int((sqrt(sin(c + d*x))*sin(c + d*x)**6)/(cos(c + d*x)**3*b**3 + 3
*cos(c + d*x)**2*a*b**2 + 3*cos(c + d*x)*a**2*b + a**3),x)*e**6
```

# 3.79 $\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^3}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 604

$$\begin{aligned}\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^3}\,dx = & -\frac{9(7a^4-9a^2b^2+2b^4)\,e^{11/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{11/2}\,(-a^2+b^2)^{3/4}\,d} \\ & -\frac{9(7a^4-9a^2b^2+2b^4)\,e^{11/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{11/2}\,(-a^2+b^2)^{3/4}\,d} \\ & +\frac{3a(21a^2-13b^2)\,e^6\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{4b^6d\sqrt{e\sin(c+dx)}} \\ & -\frac{9a(7a^4-9a^2b^2+2b^4)\,e^6\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b^6\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}} \\ & -\frac{9a(7a^4-9a^2b^2+2b^4)\,e^6\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b^6\left(a^2-b\left(b+\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}} \\ & -\frac{3e^5(3(7a^2-2b^2)-7ab\cos(c+dx))\sqrt{e\sin(c+dx)}}{4b^5d} \\ & +\frac{9e^3(7a+2b\cos(c+dx))(e\sin(c+dx))^{5/2}}{20b^3d(a+b\cos(c+dx))}+\frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2}\end{aligned}$$

output
```
-9/8*(7*a^4-9*a^2*b^2+2*b^4)*e^(11/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/
(-a^2+b^2)^(1/4)/e^(1/2))/b^(11/2)/(-a^2+b^2)^(3/4)/d-9/8*(7*a^4-9*a^2*b^2
+2*b^4)*e^(11/2)*arctanh(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(
1/2))/b^(11/2)/(-a^2+b^2)^(3/4)/d+3/4*a*(21*a^2-13*b^2)*e^6*InverseJacobiA
M(1/2*c-1/4*Pi+1/2*d*x,2^(1/2))*sin(d*x+c)^(1/2)/b^6/d/(e*sin(d*x+c))^(1/2
)+9/8*a*(7*a^4-9*a^2*b^2+2*b^4)*e^6*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2
*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^6/(a^2-b*(b-(-a^2+b^2)
^(1/2)))/d/(e*sin(d*x+c))^(1/2)+9/8*a*(7*a^4-9*a^2*b^2+2*b^4)*e^6*Elliptic
Pi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^
(1/2)/b^6/(a^2-b*(b+(-a^2+b^2)^(1/2)))/d/(e*sin(d*x+c))^(1/2)-3/4*e^5*(21*
a^2-6*b^2-7*a*b*cos(d*x+c))*(e*sin(d*x+c))^(1/2)/b^5/d+9/20*e^3*(7*a+2*b*c
os(d*x+c))*(e*sin(d*x+c))^(5/2)/b^3/d/(a+b*cos(d*x+c))+1/2*e*(e*sin(d*x+c)
)^(9/2)/b/d/(a+b*cos(d*x+c))^2
```

## Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 17.64 (sec) , antiderivative size = 2024, normalized size of antiderivative = 3.35

$$\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[(e*Sin[c + d*x])^(11/2)/(a + b*Cos[c + d*x])^3,x]
```

output
```
(((2*a*Cos[c + d*x])/b^4 + (-a^2 + b^2)^2/(2*b^5*(a + b*Cos[c + d*x])^2) -
 (17*a*(a^2 - b^2))/(4*b^5*(a + b*Cos[c + d*x])) - Cos[2*(c + d*x)]/(5*b^3
))*Csc[c + d*x]^5*(e*Sin[c + d*x])^(11/2))/d + (3*(e*Sin[c + d*x])^(11/2)*
((2*(25*a^3 - 37*a*b^2)*Cos[c + d*x]^2*(a + b*Sqrt[1 - Sin[c + d*x]^2])*((
a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] +
 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Lo
g[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] +
 b*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)
*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/
4)) + (5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Si
n[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]^2])/(
(-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d
*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Sin[c + d*x]^2
, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9
/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)])*Sin[c + d*x]^2)*(a
^2 + b^2*(-1 + Sin[c + d*x]^2)))))/((a + b*Cos[c + d*x])*(1 - Sin[c + d*x]
^2)) + (2*(30*a^2*b - 16*b^3)*Cos[c + d*x]*(a + b*Sqrt[1 - Sin[c + d*x]^2]
)*(((-1/8 + I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]]
)/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/
(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^...
```

## Rubi [A] (warning: unable to verify)

Time = 2.60 (sec) , antiderivative size = 562, normalized size of antiderivative = 0.93, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.960$, Rules used = {3042, 3172, 25, 3042, 3342, 27, 3042, 3344, 27, 3042, 3346, 3042, 3121, 3042, 3120, 3181, 266, 756, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{11/2}}{\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \; 3172$$

$$\frac{9e^2 \int -\frac{\cos(c+dx)(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^2}dx}{4b}+\frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2}$$

$\Big\downarrow$ 25

$$\frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2}-\frac{9e^2 \int \frac{\cos(c+dx)(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^2}dx}{4b}$$

$\Big\downarrow$ 3042

$$\frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2}-\frac{9e^2 \int \frac{\left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4b}$$

$\Big\downarrow$ 3342

$$\frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2}-\frac{9e^2\left(-\frac{e^2\int -\frac{(2b+7a\cos(c+dx))(e\sin(c+dx))^{3/2}}{2(a+b\cos(c+dx))}dx}{b^2}-\frac{e(e\sin(c+dx))^{5/2}(7a+2b\cos(c+dx))}{5b^2d(a+b\cos(c+dx))}\right)}{4b}$$

$\Big\downarrow$ 27

$$\frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2}-\frac{9e^2\left(\frac{e^2\int \frac{(2b+7a\cos(c+dx))(e\sin(c+dx))^{3/2}}{a+b\cos(c+dx)}dx}{2b^2}-\frac{e(e\sin(c+dx))^{5/2}(7a+2b\cos(c+dx))}{5b^2d(a+b\cos(c+dx))}\right)}{4b}$$

$\Big\downarrow$ 3042

$$\frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2}-\frac{9e^2\left(\frac{e^2\int \frac{\left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(2b+7a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2b^2}-\frac{e(e\sin(c+dx))^{5/2}(7a+2b\cos(c+dx))}{5b^2d(a+b\cos(c+dx))}\right)}{4b}$$

$\Big\downarrow$ 3344

$$\frac{\frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2}-9e^2\left(\frac{e^2\left(\frac{2e^2\int -\frac{2b(7a^2-3b^2)+a(21a^2-13b^2)\cos(c+dx)}{2(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{3b^2}+\frac{2e\sqrt{e\sin(c+dx)}\left(3(7a^2-2b^2)-7ab\cos(c+dx)\right)}{3b^2d}\right)}{2b^2}-\frac{e(e\sin(c+dx))^{5/2}(7a+2b\cos(c+dx}{5b^2d(a+b\cos(c+dx))}\right.}{4b}$$

$\downarrow$ 27

$$\frac{\frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2}-9e^2\left(\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(3(7a^2-2b^2)-7ab\cos(c+dx)\right)}{3b^2d}-\frac{e^2\int\frac{2b(7a^2-3b^2)+a(21a^2-13b^2)\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{3b^2}\right)}{2b^2}-\frac{e(e\sin(c+dx))^{5/2}(7a+2b\cos(c+dx))}{5b^2d(a+b\cos(c+dx))}\right)}{4b}$$

$\downarrow$ 3042

$$\frac{\frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2}-9e^2\left(\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(3(7a^2-2b^2)-7ab\cos(c+dx)\right)}{3b^2d}-\frac{e^2\int\frac{2b(7a^2-3b^2)-a(21a^2-13b^2)\sin\left(c+dx-\frac{\pi}{2}\right)}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{3b^2}\right)}{2b^2}-\frac{e(e\sin(c+dx))^{5/2}(7a+2b\cos(c+d}{5b^2d(a+b\cos(c+dx))}\right.}{4b}$$

$\downarrow$ 3346

$$\frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2}-\frac{9e^2\left(\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(7a^2-2b^2\right)-7ab\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{a\left(21a^2-13b^2\right)\int\frac{1}{\sqrt{e\sin(c+dx)}}dx}{b}-\frac{3\left(7a^4-9a^2b^2+2b^4\right)\int\frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{b}\right)}{3b^2}\right.}{2b^2}\right.}{4b}$$

$\downarrow$ 3042

$$\frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2}-\frac{9e^2\left(\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(7a^2-2b^2\right)-7ab\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{a\left(21a^2-13b^2\right)\int\frac{1}{\sqrt{e\sin(c+dx)}}dx}{b}-\frac{3\left(7a^4-9a^2b^2+2b^4\right)\int\frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\right.\right.}}{b}\right.}{3b^2}\right.}{2b^2}\right.}{4b}$$

$\downarrow$ 3121

$$\frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2}-\frac{9e^2\left(\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(7a^2-2b^2\right)-7ab\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{a\left(21a^2-13b^2\right)\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{b\sqrt{e\sin(c+dx)}}-\frac{3\left(7a^4-9a^2b^2+2b^4\right)\int\frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\right.}}{b}\right.}{3b^2}\right.}{2b^2}\right.}{4b}$$

$\downarrow$ 3042

$$\frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2}-$$

$$\frac{9e^2\left(\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(7a^2-2b^2\right)-7ab\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{a\left(21a^2-13b^2\right)\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{b\sqrt{e\sin(c+dx)}}-\frac{3\left(7a^4-9a^2b^2+2b^4\right)\int\frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\right.}}{b}\right.}{3b^2}\right.}{2b^2}\right.}{4b}$$

$\downarrow$ 3120

$$\frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2}-$$

$$\frac{9e^2\left(\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(7a^2-2b^2\right)-7ab\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{2a\left(21a^2-13b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{3\left(7a^4-9a^2b^2+2b^4\right)\int\frac{}{\sqrt{e\cos\left(c+dx\right.}}}{b}\right.}{3b^2}\right.}{2b^2}\right.}{4b}$$

$\downarrow$ 3181

$$\frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2}-$$

$$9e^2\left(\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(7a^2-2b^2\right)-7ab\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{2a\left(21a^2-13b^2\right)\sqrt{\sin(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{3\left(7a^4-9a^2b^2+2b^4\right)\left(-\frac{be\int\frac{}{\sqrt{e\sin(}}}{}\right.}{}\right)}{}\right)}{2}\right.$$

$$\Big\downarrow \text{266}$$

$$\frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2}-$$

$$9e^2\left(\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(7a^2-2b^2\right)-7ab\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{2a\left(21a^2-13b^2\right)\sqrt{\sin(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{3\left(7a^4-9a^2b^2+2b^4\right)\left(-\frac{2be\int\frac{}{b^2e^4\,\mathrm{s}}}{}\right.}{}\right)}{}\right)}{2b^2}\right.$$

$$\downarrow 756$$

$$\frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2} - \frac{9e^2\left(\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(7a^2-2b^2\right)-7ab\cos(c+dx)\right)}{3b^2d} - \frac{e^2\left(\frac{2a\left(21a^2-13b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}} - \frac{3\left(7a^4-9a^2b^2+2b^4\right)\left(-\frac{2be\left(-\frac{\int \frac{}{\sqrt{}}}{}\right.}{}\right.}{}\right)}{}\right)}{}\right)}{}$$

$$\downarrow 218$$

$$\frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2}-$$

$$9e^2\left(\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(7a^2-2b^2\right)-7ab\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{2a\left(21a^2-13b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{3\left(7a^4-9a^2b^2+2b^4\right)\left(-\frac{2be\left(-\frac{\int\ldots}{\sqrt{\ldots}}\right.}{\ldots}\right.}{\ldots}\right.}{\ldots}\right.}{\ldots}\right.$$

$\downarrow$ 221

$$\frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2}-$$

$$3\left(7a^4-9a^2b^2+2b^4\right)\left(-\frac{a\int\frac{}{\sqrt{e\sin(c}}}{}\right.$$

$$e^2\left(\frac{2a\left(21a^2-13b^2\right)\sqrt{\sin(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\right.$$

$$e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(7a^2-2b^2\right)-7ab\cos(c+dx)\right)}{3b^2d}-\right.$$

$$9e^2$$

↓ 3042

$$\frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2}-$$

$$9e^2\left(e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(7a^2-2b^2\right)-7ab\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{2a\left(21a^2-13b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{3\left(7a^4-9a^2b^2+2b^4\right)\left(-\frac{a\int\frac{}{\sqrt{e\sin(c}}}{}\right.}{}\right.}{}\right.\right.$$

↓ 3286

$$\frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2} -$$

$$9e^2 \left( \frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(7a^2-2b^2\right)-7ab\cos(c+dx)\right)}{3b^2d} - \frac{e^2\left(\frac{2a\left(21a^2-13b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}} - \frac{3\left(7a^4-9a^2b^2+2b^4\right)\left(-\frac{a\sqrt{\sin(c+d}}{}\right.}{}\right.}{}\right.}{}\right.$$

$\downarrow$ 3042

$$\frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2} -$$

$$9e^2 \left( \frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(7a^2-2b^2\right)-7ab\cos(c+dx)\right)}{3b^2d} - \frac{e^2\left(\frac{2a\left(21a^2-13b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}} - \frac{3\left(7a^4-9a^2b^2+2b^4\right)\left(-\frac{a\sqrt{\sin(c+dx}\cdots}{\cdots}\right.}{\cdots}\right)}{\cdots}\right)}{\cdots}\right.$$

↓ 3284

$$\frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2} -$$

$$9e^2 \left( \frac{e^2 \left( \frac{2e\sqrt{e\sin(c+dx)}\left(3\left(7a^2-2b^2\right)-7ab\cos(c+dx)\right)}{3b^2d} - \frac{e^2 \left( \frac{2a\left(21a^2-13b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}} - \frac{3\left(7a^4-9a^2b^2+2b^4\right)\left(-\frac{2be\left(-\frac{\text{arcta}}{2}\right.}{}\right.}{}\right.}{}\right.}{}\right.$$

input
```
Int[(e*Sin[c + d*x])^(11/2)/(a + b*Cos[c + d*x])^3,x]
```

output
```
(e*(e*Sin[c + d*x])^(9/2))/(2*b*d*(a + b*Cos[c + d*x])^2) - (9*e^2*(-1/5*(
e*(7*a + 2*b*Cos[c + d*x])*(e*Sin[c + d*x])^(5/2))/(b^2*d*(a + b*Cos[c + d
*x])) + (e^2*((2*e*(3*(7*a^2 - 2*b^2) - 7*a*b*Cos[c + d*x])*Sqrt[e*Sin[c +
 d*x]])/(3*b^2*d) - (e^2*((2*a*(21*a^2 - 13*b^2)*EllipticF[(c - Pi/2 + d*x
)/2, 2]*Sqrt[Sin[c + d*x]])/(b*d*Sqrt[e*Sin[c + d*x]]) - (3*(7*a^4 - 9*a^2
*b^2 + 2*b^4)*((-2*b*e*(-1/2*ArcTan[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(-a^2 +
 b^2)^(1/4)]/(Sqrt[b]*(-a^2 + b^2)^(3/4)*e^(3/2)) - ArcTanh[(Sqrt[b]*Sqrt[
e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/(2*Sqrt[b]*(-a^2 + b^2)^(3/4)*e^(3/2)
)))/d + (a*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]
*Sqrt[Sin[c + d*x]])/(Sqrt[-a^2 + b^2]*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin
[c + d*x]]) - (a*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)
/2, 2]*Sqrt[Sin[c + d*x]])/(Sqrt[-a^2 + b^2]*(b + Sqrt[-a^2 + b^2])*d*Sqrt
[e*Sin[c + d*x]])))/b))/(3*b^2)))/(2*b^2)))/(4*b)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3172
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Simp[g^2*((p - 1)/(b*(m + 1)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Fre
eQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && I
ntegersQ[2*m, 2*p]
```

rule 3181
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[-a/(2*q)   Int[1/(
Sqrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Simp[b*(g/f)   Subst[
Int[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - S
imp[a/(2*q)   Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3342
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*C
os[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x])/(b^2*f*(m + 1)*(m + p + 1))), x] + Simp[g^2*((
p - 1)/(b^2*(m + 1)*(m + p + 1)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

rule 3344
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Simp[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Si
n[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^
2*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1
, 0] && IntegerQ[2*m]
```

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

## Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2753 vs. $2(530) = 1060$.

Time = 101.66 (sec) , antiderivative size = 2754, normalized size of antiderivative = 4.56

| method | result | size |
|---|---|---|
| default | Expression too large to display | 2754 |

input
```
int((e*sin(d*x+c))^(11/2)/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)
```

output
```
(2*e^3*b*(-1/5/b^6*(e*sin(d*x+c))^(1/2)*e^2*(cos(d*x+c)^2*b^2+30*a^2-11*b^
2)+e^4/b^6*(-1/8*(e*sin(d*x+c))^(1/2)*e^2*(-19*cos(d*x+c)^2*a^4*b^2+21*a^2
*b^4*cos(d*x+c)^2-2*b^6*cos(d*x+c)^2+15*a^6-13*a^4*b^2-2*b^4*a^2)/(-b^2*co
s(d*x+c)^2*e^2+a^2*e^2)^2+9/64*(7*a^4-9*a^2*b^2+2*b^4)*(e^2*(a^2-b^2)/b^2)
^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*(ln((e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/
4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)-(
e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^
(1/2)))+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)
+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1))))-(co
s(d*x+c)^2*e*sin(d*x+c))^(1/2)*e^6*a*(1/b^6/(cos(d*x+c)^2*e*sin(d*x+c))^(1
/2)*(10*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*Ellip
ticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2-7*(1-sin(d*x+c))^(1/2)*(2+2*sin
(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2)
)*b^2-2*b^2*cos(d*x+c)^2*sin(d*x+c))+3/b^6*(7*a^4-10*a^2*b^2+3*b^4)*(-1/2/
b/(-a^2+b^2)^(1/2)*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^
(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi(
(1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/2/b/(-a^2+b^2
)^(1/2)*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(cos(
d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+
c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2)))-3/b^6*(5*a^6-11*a^4*b^...
```

## Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))^(11/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
Timed out
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))**(11/2)/(a+b*cos(d*x+c))**3,x)
```

output
```
Timed out
```

## Maxima [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))^(11/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
Timed out
```

## Giac [F]

$$\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{(e\sin(dx+c))^{\frac{11}{2}}}{(b\cos(dx+c)+a)^3}\,dx$$

input
```
integrate((e*sin(d*x+c))^(11/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((e*sin(d*x + c))^(11/2)/(b*cos(d*x + c) + a)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^3}\,dx$$

input
```
int((e*sin(c + d*x))^(11/2)/(a + b*cos(c + d*x))^3,x)
```

output
```
int((e*sin(c + d*x))^(11/2)/(a + b*cos(c + d*x))^3, x)
```

## Reduce [F]

$$\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^3}\,dx = \sqrt{e}\left(\int \frac{\sqrt{\sin(dx+c)}\,\sin(dx+c)^5}{\cos(dx+c)^3 b^3+3\cos(dx+c)^2 a b^2+3\cos(dx+c)\,a^2 b+a^3}dx\right)e^5$$

input
```
int((e*sin(d*x+c))^(11/2)/(a+b*cos(d*x+c))^3,x)
```

output
```
sqrt(e)*int((sqrt(sin(c + d*x))*sin(c + d*x)**5)/(cos(c + d*x)**3*b**3 + 3
*cos(c + d*x)**2*a*b**2 + 3*cos(c + d*x)*a**2*b + a**3),x)*e**5
```

# 3.80 $\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^3}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 498

$$\begin{aligned}\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^3}\,dx &= -\frac{7(5a^2-2b^2)\,e^{9/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{9/2}\sqrt[4]{-a^2+b^2}d}\\
&+\frac{7(5a^2-2b^2)\,e^{9/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{9/2}\sqrt[4]{-a^2+b^2}d}\\
&+\frac{7a(5a^2-2b^2)\,e^5\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b^5\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}\\
&+\frac{7a(5a^2-2b^2)\,e^5\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b^5\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}\\
&-\frac{35ae^4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{4b^4d\sqrt{\sin(c+dx)}}\\
&+\frac{7e^3(5a+2b\cos(c+dx))(e\sin(c+dx))^{3/2}}{12b^3d(a+b\cos(c+dx))}+\frac{e(e\sin(c+dx))^{7/2}}{2bd(a+b\cos(c+dx))^2}\end{aligned}$$

output
```
-7/8*(5*a^2-2*b^2)*e^(9/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^
(1/4)/e^(1/2))/b^(9/2)/(-a^2+b^2)^(1/4)/d+7/8*(5*a^2-2*b^2)*e^(9/2)*arctan
h(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(9/2)/(-a^2+b^2
)^(1/4)/d-7/8*a*(5*a^2-2*b^2)*e^5*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b
/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^5/(b-(-a^2+b^2)^(1/2))/d
/(e*sin(d*x+c))^(1/2)-7/8*a*(5*a^2-2*b^2)*e^5*EllipticPi(cos(1/2*c+1/4*Pi+
1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^5/(b+(-a^2+b
^2)^(1/2))/d/(e*sin(d*x+c))^(1/2)+35/4*a*e^4*EllipticE(cos(1/2*c+1/4*Pi+1/
2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/b^4/d/sin(d*x+c)^(1/2)+7/12*e^3*(5*a+
2*b*cos(d*x+c))*(e*sin(d*x+c))^(3/2)/b^3/d/(a+b*cos(d*x+c))+1/2*e*(e*sin(d
*x+c))^(7/2)/b/d/(a+b*cos(d*x+c))^2
```

## Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 15.86 (sec) , antiderivative size = 837, normalized size of antiderivative = 1.68

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^3}\,dx = \frac{\csc^4(c+dx)(e\sin(c+dx))^{9/2}\left(\frac{2\sin(c+dx)}{3b^3}+\frac{11a\sin(c+dx)}{4b^3(a+b\cos(c+dx))}+\frac{-a^2\sin(c+dx)+b^2\sin\cdots}{2b^3(a+b\cos(c+dx\cdots}\right.}{d}$$

$$-\frac{7(e\sin(c+dx))^{9/2}\left(\frac{5a\cos^2(c+dx)\left(3\sqrt{2}a(a^2-b^2)^{3/4}\left(2\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-2\arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-\log\left(\sqrt{a^2-b\cdots}\right.\right.\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(e*Sin[c + d*x])^(9/2)/(a + b*Cos[c + d*x])^3,x]
```

output
```
(Csc[c + d*x]^4*(e*Sin[c + d*x])^(9/2)*((2*Sin[c + d*x])/(3*b^3) + (11*a*S
in[c + d*x])/(4*b^3*(a + b*Cos[c + d*x])) + (-(a^2*Sin[c + d*x]) + b^2*Sin
[c + d*x])/(2*b^3*(a + b*Cos[c + d*x])^2)))/d - (7*(e*Sin[c + d*x])^(9/2)*
((5*a*Cos[c + d*x]^2*(3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]
*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sq
rt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[
2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]] + Log[Sq
rt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*S
in[c + d*x]]) + 8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin[c + d*x]^2, (b^2
*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2))*(a + b*Sqrt[1 - Sin[c +
 d*x]^2]))/(12*b^(3/2)*(-a^2 + b^2)*(a + b*Cos[c + d*x])*(1 - Sin[c + d*x]
^2)) + (4*b*Cos[c + d*x]*(((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt
[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[S
in[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]
*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]] + Log[Sqrt[-a^2
 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[
c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)) + (a*AppellF1[3/4, 1/2, 1, 7/4, S
in[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2))/(3*(
a^2 - b^2)))*(a + b*Sqrt[1 - Sin[c + d*x]^2]))/((a + b*Cos[c + d*x])*Sqrt[
1 - Sin[c + d*x]^2])))/(8*b^3*d*Sin[c + d*x]^(9/2))
```

## Rubi [A] (warning: unable to verify)

Time = 2.01 (sec) , antiderivative size = 466, normalized size of antiderivative = 0.94, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.840$, Rules used = {3042, 3172, 25, 3042, 3342, 27, 3042, 3346, 3042, 3121, 3042, 3119, 3180, 266, 827, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{9/2}}{\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3172}$$

$$\frac{7e^2\int -\frac{\cos(c+dx)(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^2}dx}{4b}+\frac{e(e\sin(c+dx))^{7/2}}{2bd(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{e(e\sin(c+dx))^{7/2}}{2bd(a+b\cos(c+dx))^2}-\frac{7e^2\int \frac{\cos(c+dx)(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^2}dx}{4b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e(e\sin(c+dx))^{7/2}}{2bd(a+b\cos(c+dx))^2}-\frac{7e^2\int \frac{\left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4b}$$

$$\Big\downarrow \text{3342}$$

$$\frac{e(e\sin(c+dx))^{7/2}}{2bd(a+b\cos(c+dx))^2}-\frac{7e^2\left(-\frac{e^2\int -\frac{(2b+5a\cos(c+dx))\sqrt{e\sin(c+dx)}}{2(a+b\cos(c+dx))}dx}{b^2}-\frac{e(e\sin(c+dx))^{3/2}(5a+2b\cos(c+dx))}{3b^2d(a+b\cos(c+dx))}\right)}{4b}$$

$$\Big\downarrow \text{27}$$

$$\frac{e(e\sin(c+dx))^{7/2}}{2bd(a+b\cos(c+dx))^2}-\frac{7e^2\left(\frac{e^2\int \frac{(2b+5a\cos(c+dx))\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx}{2b^2}-\frac{e(e\sin(c+dx))^{3/2}(5a+2b\cos(c+dx))}{3b^2d(a+b\cos(c+dx))}\right)}{4b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e(e\sin(c+dx))^{7/2}}{2bd(a+b\cos(c+dx))^2}-\frac{7e^2\left(\frac{e^2\int \frac{\sqrt{-e\cos\left(c+dx+\frac{\pi}{2}\right)}\left(2b+5a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2b^2}-\frac{e(e\sin(c+dx))^{3/2}(5a+2b\cos(c+dx))}{3b^2d(a+b\cos(c+dx))}\right)}{4b}$$

$$\Big\downarrow \text{3346}$$

$$\frac{e(e\sin(c+dx))^{7/2}}{2bd(a+b\cos(c+dx))^2}-\frac{7e^2\left(\frac{e^2\left(\frac{5a\int \sqrt{e\sin(c+dx)}dx}{b}-\frac{\left(5a^2-2b^2\right)\int \frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx}{b}\right)}{2b^2}-\frac{e(e\sin(c+dx))^{3/2}(5a+2b\cos(c+dx))}{3b^2d(a+b\cos(c+dx))}\right)}{4b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e(e\sin(c+dx))^{7/2}}{2bd(a+b\cos(c+dx))^2}-\frac{7e^2\left(\frac{e^2\left(\frac{5a\int\sqrt{e\sin(c+dx)}dx}{b}-\frac{(5a^2-2b^2)\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{b}\right)}{2b^2}-\frac{e(e\sin(c+dx))^{3/2}(5a+2b\cos(c+dx))}{3b^2d(a+b\cos(c+dx))}\right)}{4b}$$

$$\Big\downarrow \text{3121}$$

$$\frac{e(e\sin(c+dx))^{7/2}}{2bd(a+b\cos(c+dx))^2}-\frac{7e^2\left(\frac{e^2\left(\frac{5a\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{b\sqrt{\sin(c+dx)}}-\frac{(5a^2-2b^2)\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{b}\right)}{2b^2}-\frac{e(e\sin(c+dx))^{3/2}(5a+2b\cos(c+dx))}{3b^2d(a+b\cos(c+dx))}\right)}{4b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e(e\sin(c+dx))^{7/2}}{2bd(a+b\cos(c+dx))^2}-\frac{7e^2\left(\frac{e^2\left(\frac{5a\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{b\sqrt{\sin(c+dx)}}-\frac{(5a^2-2b^2)\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{b}\right)}{2b^2}-\frac{e(e\sin(c+dx))^{3/2}(5a+2b\cos(c+dx))}{3b^2d(a+b\cos(c+dx))}\right)}{4b}$$

$$\Big\downarrow \text{3119}$$

$$\frac{e(e\sin(c+dx))^{7/2}}{2bd(a+b\cos(c+dx))^2}-\frac{7e^2\left(\frac{e^2\left(\frac{10aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{\left(5a^2-2b^2\right)\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{b}\right)}{2b^2}-\frac{e(e\sin(c+dx))^{3/2}(5a+2b\cos(c+dx))}{3b^2d(a+b\cos(c+dx))}\right)}{4b}$$

$\downarrow$ 3180

$$\frac{e(e\sin(c+dx))^{7/2}}{2bd(a+b\cos(c+dx))^2}-\frac{7e^2\left(\frac{e^2\left(\frac{10aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{\left(5a^2-2b^2\right)\left(-\frac{be\int\frac{\sqrt{e\sin(c+dx)}}{b^2\sin^2(c+dx)e^2+\left(a^2-b^2\right)e^2}d(e\sin(c+dx))}{d}-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx\right.}}{2b}\right.}{b}\right.}{2b^2}\right.}{4b}$$

$\downarrow$ 266

$$\frac{e(e\sin(c+dx))^{7/2}}{2bd(a+b\cos(c+dx))^2}-\frac{7e^2\left(\frac{e^2\left(\frac{10aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{\left(5a^2-2b^2\right)\left(-\frac{2be\int\frac{e^2\sin^2(c+dx)}{b^2e^4\sin^4(c+dx)+\left(a^2-b^2\right)e^2}d\sqrt{e\sin(c+dx)}}{d}-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+d\right.}}{2b}\right.}{b}\right.}{2b^2}\right.}{4b}$$

$$\Big\downarrow \text{827}$$

$$\frac{e(e\sin(c+dx))^{7/2}}{2bd(a+b\cos(c+dx))^2}-$$

$$\frac{7e^2\left(\frac{e^2\left(\frac{10aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{\left(5a^2-2b^2\right)\left(-\frac{2be\left(\frac{\int\frac{1}{be^2\sin^2(c+dx)+\sqrt{b^2-a^2}e}d\sqrt{e\sin(c+dx)}}{2b}-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+}}{2b}\right.}{d}\right.}{b}\right.}{2b^2}\right.}{4b}$$

$$\Big\downarrow \text{218}$$

$$\frac{e(e\sin(c+dx))^{7/2}}{2bd(a+b\cos(c+dx))^2} -$$

$$\left(5a^2-2b^2\right)\left(-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2b}\right)}{d}-\frac{ae\int\frac{}{\sqrt{e\,\mathrm{s}}}}{}\right.$$

$$7e^2\left(\frac{e^2\left(\frac{10aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{\ldots}{b}\right.}{2b^2}\right.$$

$$\overline{\qquad\qquad 4b \qquad\qquad}$$

$$\downarrow 221$$

$$\frac{e(e\sin(c+dx))^{7/2}}{2bd(a+b\cos(c+dx))^2}-$$

$$7e^2\left(\frac{e^2\left(\frac{10aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{(5a^2-2b^2)\left(-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}d}{2b}\right.}{b}\right.}{2b^2}\right.$$

$$\overline{\qquad\qquad\qquad 4b \qquad\qquad}$$

$\downarrow$ 3042

$$\frac{e(e\sin(c+dx))^{7/2}}{2bd(a+b\cos(c+dx))^2} -$$

$$\frac{7e^2\left(\frac{e^2\left(\frac{10aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{\left(5a^2-2b^2\right)\left(-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}d}{2b}\right.}{b}}{2b^2}\right.}{4b}$$

↓ 3286

$$\frac{e(e\sin(c+dx))^{7/2}}{2bd(a+b\cos(c+dx))^2} -$$

$$7e^2 \left( e^2 \left( \frac{10aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}} - \frac{(5a^2-2b^2)\left(-\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b\sqrt{e\sin(c+dx)}} + \frac{ae\sqrt{\sin(c+dx)}\int\frac{}{\sqrt{\sin(c+dx)}\left(b\sin\right.}}{2b\sqrt{e\sin(c+d}}\right.}{b} \right.}{2b^2} \right.}{4b}$$

↓ 3042

$$\frac{e(e\sin(c+dx))^{7/2}}{2bd(a+b\cos(c+dx))^2} - \frac{7e^2\left(\frac{e^2\left(\frac{10aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}} - \frac{(5a^2-2b^2)\left(-\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b\sqrt{e\sin(c+dx)}} + \frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(b\sin\right.}}{2b\sqrt{e\sin(c+d}}\right.}{b}\right)}{2b^2}\right.}{4b}$$

$\downarrow$ 3284

$$\frac{e(e\sin(c+dx))^{7/2}}{2bd(a+b\cos(c+dx))^2} - \frac{7e^2\left(\frac{e^2\left(\frac{10aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}} - \frac{(5a^2-2b^2)\left(-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{d} + \frac{ae\sqrt{\sin(c+dx)}\,\text{Ellipti}\cdots}{bd\left(b-\sqrt{\cdots}\right.}\cdots\right)}{b}\cdots\right)}{2b^2}\cdots\right)}{4b}$$

input
```
Int[(e*Sin[c + d*x])^(9/2)/(a + b*Cos[c + d*x])^3,x]
```

output
```
(e*(e*Sin[c + d*x])^(7/2))/(2*b*d*(a + b*Cos[c + d*x])^2) - (7*e^2*(-1/3*(
e*(5*a + 2*b*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2))/(b^2*d*(a + b*Cos[c + d
*x])) + (e^2*((10*a*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])
/(b*d*Sqrt[Sin[c + d*x]]) - ((5*a^2 - 2*b^2)*((-2*b*e*(ArcTan[(Sqrt[b]*Sqr
t[e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/(2*b^(3/2)*(-a^2 + b^2)^(1/4)*Sqrt[
e]) - ArcTanh[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/(2*b^(3/2
)*(-a^2 + b^2)^(1/4)*Sqrt[e])))/d + (a*e*EllipticPi[(2*b)/(b - Sqrt[-a^2 +
 b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(b*(b - Sqrt[-a^2 + b^2
])*d*Sqrt[e*Sin[c + d*x]]) + (a*e*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]),
 (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(b*(b + Sqrt[-a^2 + b^2])*d*Sq
rt[e*Sin[c + d*x]])))/b))/(2*b^2)))/(4*b)
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3172
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Simp[g^2*((p - 1)/(b*(m + 1)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Fre
eQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && I
ntegersQ[2*m, 2*p]
```

rule 3180
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[a*(g/(2*b))   Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Simp[a*(g/(2*b))   In
t[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Simp[b*(g/f)   Su
bst[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3342
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*C
os[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x])/(b^2*f*(m + 1)*(m + p + 1))), x] + Simp[g^2*((
p - 1)/(b^2*(m + 1)*(m + p + 1)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

## Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2735 vs. 2(432) = 864.

Time = 100.15 (sec) , antiderivative size = 2736, normalized size of antiderivative = 5.49

| method | result | size |
|---|---|---|
| default | Expression too large to display | 2736 |

input
```
int((e*sin(d*x+c))^(9/2)/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)
```

output
```
(2*e^3*b*(1/3*(e*sin(d*x+c))^(3/2)/b^4-e^2/b^4*(-1/8*(e*sin(d*x+c))^(3/2)*
e^2*(-13*cos(d*x+c)^2*a^2*b^2+2*b^4*cos(d*x+c)^2+9*a^4+2*a^2*b^2)/(-b^2*co
s(d*x+c)^2*e^2+a^2*e^2)^2+1/8*(35/8*a^2-7/4*b^2)/b^2/(e^2*(a^2-b^2)/b^2)^(
1/4)*2^(1/2)*(ln((e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1
/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(
1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arctan(2^(
1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2)/(e
^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1))))-(cos(d*x+c)^2*e*sin(d*x
+c))^(1/2)*e^5*a*((11*a^4-14*a^2*b^2+3*b^4)/b^4*(1/2*b^2/e/a^2/(a^2-b^2)*s
in(d*x+c)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-cos(d*x+c)^2*b^2+a^2)-1/2/a^
2/(a^2-b^2)*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(
cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2
))+1/4/a^2/(a^2-b^2)*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c
)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1
/2*2^(1/2))-3/8/(a^2-b^2)/b^2*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*
sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*
EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/4/
a^2/(a^2-b^2)*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)
/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-si
n(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-3/8/(a^2-b^2)/b^2...
```

## Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))^(9/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
Timed out
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))**(9/2)/(a+b*cos(d*x+c))**3,x)
```

output
```
Timed out
```

## Maxima [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))^(9/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
Timed out
```

## Giac [F]

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{(e\sin(dx+c))^{\frac{9}{2}}}{(b\cos(dx+c)+a)^3}\,dx$$

input
```
integrate((e*sin(d*x+c))^(9/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((e*sin(d*x + c))^(9/2)/(b*cos(d*x + c) + a)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{(e\,\sin(c+d\,x))^{9/2}}{(a+b\,\cos(c+d\,x))^3}\,dx$$

input
```
int((e*sin(c + d*x))^(9/2)/(a + b*cos(c + d*x))^3,x)
```

output
```
int((e*sin(c + d*x))^(9/2)/(a + b*cos(c + d*x))^3, x)
```

## Reduce [F]

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^3}\,dx = \sqrt{e}\left(\int \frac{\sqrt{\sin(dx+c)}\,\sin(dx+c)^4}{\cos(dx+c)^3 b^3+3\cos(dx+c)^2 a\,b^2+3\cos(dx+c)\,a^2b+a^3}dx\right)e^4$$

input
```
int((e*sin(d*x+c))^(9/2)/(a+b*cos(d*x+c))^3,x)
```

output
```
sqrt(e)*int((sqrt(sin(c + d*x))*sin(c + d*x)**4)/(cos(c + d*x)**3*b**3 + 3
*cos(c + d*x)**2*a*b**2 + 3*cos(c + d*x)*a**2*b + a**3),x)*e**4
```

# 3.81 $\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^3}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 512

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^3}\,dx = \frac{5(3a^2-2b^2)\,e^{7/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{7/2}\left(-a^2+b^2\right)^{3/4}d} + \frac{5(3a^2-2b^2)\,e^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{7/2}\left(-a^2+b^2\right)^{3/4}d} - \frac{15ae^4\operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{4b^4d\sqrt{e\sin(c+dx)}} + \frac{5a(3a^2-2b^2)\,e^4\operatorname{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b^4\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}} + \frac{5a(3a^2-2b^2)\,e^4\operatorname{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b^4\left(a^2-b\left(b+\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}} + \frac{5e^3(3a+2b\cos(c+dx))\sqrt{e\sin(c+dx)}}{4b^3d(a+b\cos(c+dx))} + \frac{e(e\sin(c+dx))^{5/2}}{2bd(a+b\cos(c+dx))^2}$$

output
```
5/8*(3*a^2-2*b^2)*e^(7/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(
1/4)/e^(1/2))/b^(7/2)/(-a^2+b^2)^(3/4)/d+5/8*(3*a^2-2*b^2)*e^(7/2)*arctanh
(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(7/2)/(-a^2+b^2)
^(3/4)/d-15/4*a*e^4*InverseJacobiAM(1/2*c-1/4*Pi+1/2*d*x,2^(1/2))*sin(d*x+
c)^(1/2)/b^4/d/(e*sin(d*x+c))^(1/2)-5/8*a*(3*a^2-2*b^2)*e^4*EllipticPi(cos
(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/
b^4/(a^2-b*(b-(-a^2+b^2)^(1/2)))/d/(e*sin(d*x+c))^(1/2)-5/8*a*(3*a^2-2*b^2
)*e^4*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2
))*sin(d*x+c)^(1/2)/b^4/(a^2-b*(b+(-a^2+b^2)^(1/2)))/d/(e*sin(d*x+c))^(1/2
)+5/4*e^3*(3*a+2*b*cos(d*x+c))*(e*sin(d*x+c))^(1/2)/b^3/d/(a+b*cos(d*x+c))
+1/2*e*(e*sin(d*x+c))^(5/2)/b/d/(a+b*cos(d*x+c))^2
```

## Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 18.28 (sec) , antiderivative size = 1954, normalized size of antiderivative = 3.82

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
Integrate[(e*Sin[c + d*x])^(7/2)/(a + b*Cos[c + d*x])^3,x]
```

output
```
(((-a^2 + b^2)/(2*b^3*(a + b*Cos[c + d*x])^2) + (9*a)/(4*b^3*(a + b*Cos[c
+ d*x])))*Csc[c + d*x]^3*(e*Sin[c + d*x])^(7/2))/d - ((e*Sin[c + d*x])^(7/
2)*((14*a*Cos[c + d*x]^2*(a + b*Sqrt[1 - Sin[c + d*x]^2])*((a*(-2*ArcTan[1
 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 +
(Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b
^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x
]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c +
d*x]] + b*Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)) + (5*b*(a^
2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/
(-a^2 + b^2)]*Sqrt[Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((-5*(a^2 - b^2
)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 +
 b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Sin[c + d*x]^2, (b^2*Sin[c +
 d*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*
x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)])*Sin[c + d*x]^2)*(a^2 + b^2*(-1 +
 Sin[c + d*x]^2)))))/((a + b*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) + (12*b*C
os[c + d*x]*(a + b*Sqrt[1 - Sin[c + d*x]^2])*(((-1/8 + I/8)*Sqrt[b]*(2*Arc
Tan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTa
n[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[
-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*
Sin[c + d*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/...
```

## Rubi [A] (warning: unable to verify)

Time = 2.16 (sec) , antiderivative size = 483, normalized size of antiderivative = 0.94, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.840, Rules used = {3042, 3172, 25, 3042, 3342, 27, 3042, 3346, 3042, 3121, 3042, 3120, 3181, 266, 756, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{7/2}}{\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3172}$$

$$\frac{5e^2\int -\frac{\cos(c+dx)(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^2}dx}{4b}+\frac{e(e\sin(c+dx))^{5/2}}{2bd(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 25$$

$$\frac{e(e\sin(c+dx))^{5/2}}{2bd(a+b\cos(c+dx))^2}-\frac{5e^2\int \frac{\cos(c+dx)(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^2}dx}{4b}$$

$$\Big\downarrow \; 3042$$

$$\frac{e(e\sin(c+dx))^{5/2}}{2bd(a+b\cos(c+dx))^2}-\frac{5e^2\int \frac{\left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4b}$$

$$\Big\downarrow \; 3342$$

$$\frac{e(e\sin(c+dx))^{5/2}}{2bd(a+b\cos(c+dx))^2}-\frac{5e^2\left(-\frac{e^2\int -\frac{2b+3a\cos(c+dx)}{2(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{b^2}-\frac{e\sqrt{e\sin(c+dx)}(3a+2b\cos(c+dx))}{b^2d(a+b\cos(c+dx))}\right)}{4b}$$

$$\Big\downarrow \; 27$$

$$\frac{e(e\sin(c+dx))^{5/2}}{2bd(a+b\cos(c+dx))^2}-\frac{5e^2\left(\frac{e^2\int \frac{2b+3a\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{2b^2}-\frac{e\sqrt{e\sin(c+dx)}(3a+2b\cos(c+dx))}{b^2d(a+b\cos(c+dx))}\right)}{4b}$$

$$\Big\downarrow \; 3042$$

$$\frac{e(e\sin(c+dx))^{5/2}}{2bd(a+b\cos(c+dx))^2}-\frac{5e^2\left(\frac{e^2\int \frac{2b-3a\sin\left(c+dx-\frac{\pi}{2}\right)}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{2b^2}-\frac{e\sqrt{e\sin(c+dx)}(3a+2b\cos(c+dx))}{b^2d(a+b\cos(c+dx))}\right)}{4b}$$

$$\Big\downarrow \; 3346$$

$$\frac{e(e\sin(c+dx))^{5/2}}{2bd(a+b\cos(c+dx))^2}-\frac{5e^2\left(\frac{e^2\left(\frac{3a\int \frac{1}{\sqrt{e\sin(c+dx)}}dx}{b}-\frac{\left(3a^2-2b^2\right)\int \frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{b}\right)}{2b^2}-\frac{e\sqrt{e\sin(c+dx)}(3a+2b\cos(c+dx))}{b^2d(a+b\cos(c+dx))}\right)}{4b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{e(e\sin(c+dx))^{5/2}}{2bd(a+b\cos(c+dx))^2}-\frac{5e^2\left(\frac{e^2\left(\frac{3a\int\frac{1}{\sqrt{e\sin(c+dx)}}dx}{b}-\frac{\left(3a^2-2b^2\right)\int\frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{b}\right)}{2b^2}-\frac{e\sqrt{e\sin(c+dx)}(3a+2b\cos(c+dx))}{b^2d(a+b\cos(c+dx))}\right)}{4b}}{}$$

$\big\downarrow$ 3121

$$\frac{e(e\sin(c+dx))^{5/2}}{2bd(a+b\cos(c+dx))^2}-\frac{5e^2\left(\frac{e^2\left(\frac{3a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{b\sqrt{e\sin(c+dx)}}-\frac{\left(3a^2-2b^2\right)\int\frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{b}\right)}{2b^2}-\frac{e\sqrt{e\sin(c+dx)}(3a+2b\cos(c+dx))}{b^2d(a+b\cos(c+dx))}\right)}{4b}$$

$\big\downarrow$ 3042

$$\frac{e(e\sin(c+dx))^{5/2}}{2bd(a+b\cos(c+dx))^2}-\frac{5e^2\left(\frac{e^2\left(\frac{3a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{b\sqrt{e\sin(c+dx)}}-\frac{\left(3a^2-2b^2\right)\int\frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{b}\right)}{2b^2}-\frac{e\sqrt{e\sin(c+dx)}(3a+2b\cos(c+dx))}{b^2d(a+b\cos(c+dx))}\right)}{4b}$$

$\big\downarrow$ 3120

$$\frac{e(e\sin(c+dx))^{5/2}}{2bd(a+b\cos(c+dx))^2}-\frac{5e^2\left(\frac{e^2\left(\frac{6a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{\left(3a^2-2b^2\right)\int\frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{b}\right)}{2b^2}-\frac{e\sqrt{e\sin(c+dx)}(3a+2b\cos(c+dx))}{b^2d(a+b\cos(c+dx))}\right)}{4b}$$

$\big\downarrow$ 3181

$$\frac{e(e\sin(c+dx))^{5/2}}{2bd(a+b\cos(c+dx))^2} - \frac{5e^2\left(\frac{e^2\left(\frac{6a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}} - \frac{\left(3a^2-2b^2\right)\left(-\frac{be\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b^2\sin^2(c+dx)e^2+\left(a^2-b^2\right)e^2\right)}d(e\sin(c+dx))}{d} - \frac{a\int\frac{}{\sqrt{e\sin(c+dx)}\left(\sqrt{\right.}}{2\sqrt{}}\right.}{b}\right.}{2b^2}\right.}{4b}$$

$\downarrow$ 266

$$\frac{e(e\sin(c+dx))^{5/2}}{2bd(a+b\cos(c+dx))^2} - \frac{5e^2\left(\frac{e^2\left(\frac{6a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}} - \frac{\left(3a^2-2b^2\right)\left(-\frac{2be\int\frac{1}{b^2e^4\sin^4(c+dx)+\left(a^2-b^2\right)e^2}d\sqrt{e\sin(c+dx)}}{d} - \frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c\right.}}{2\sqrt{b^2-a^2}}\right.}{b}\right.}{2b^2}\right.}{4b}$$

$\downarrow$ 756

$$\frac{e(e\sin(c+dx))^{5/2}}{2bd(a+b\cos(c+dx))^2}-$$

$$5e^2\left(\frac{e^2\left(\frac{6a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{\left(3a^2-2b^2\right)\left(-\frac{2be\left(-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2e\sqrt{b^2-a^2}}-\frac{\int\frac{1}{be^2\sin^2(c+dx)+\sqrt{b^2-a^2}e}d\sqrt{e\,s}}{2e\sqrt{b^2-a^2}}\right.}{d}\right.}{}\right.}{2b^2}\right.$$

$$\overline{\qquad 4b}$$

$\downarrow$ 218

$$\frac{e(e\sin(c+dx))^{5/2}}{2bd(a+b\cos(c+dx))^2} - \frac{5e^2\left(\frac{e^2\left(\frac{6a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}} - \frac{\left(3a^2-2b^2\right)\left(-\frac{2be\left(-\frac{\int \frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}\,d\sqrt{e\sin(c+dx)}}{2e\sqrt{b^2-a^2}} - \frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}\right)}{d} - \frac{a\int \cdots}{\cdots}\right)}{b}\cdots\right)}{2b^2}\cdots\right)}{4b}$$

$\downarrow$ 221

$$\frac{e(e\sin(c+dx))^{5/2}}{2bd(a+b\cos(c+dx))^2} - \frac{5e^2\left(\frac{e^2\left(\frac{6a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}} - \frac{(3a^2-2b^2)\left(-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}} - \frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}}{2\sqrt{b^2-a^2}}\right.}{b}\right.}{2b^2}\right.}{4b}$$

$\downarrow$ 3042

$$\frac{e(e\sin(c+dx))^{5/2}}{2bd(a+b\cos(c+dx))^2} - \frac{5e^2\left(\frac{e^2\left(\frac{6a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}} - \frac{(3a^2-2b^2)\left(-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}} - \frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right.}}{2\sqrt{b^2-a^2}}\right.}{b}\right.}{2b^2}\right.}{4b}$$

$\downarrow$ 3286

$$\frac{e(e\sin(c+dx))^{5/2}}{2bd(a+b\cos(c+dx))^2} -$$

$$\left(3a^2-2b^2\right)\left(-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}} - \frac{a\sqrt{\sin(c+dx)}\int\frac{}{\sqrt{\sin(c+dx)}\left(b\right.}}{2\sqrt{b^2-a^2}\sqrt{e\,\mathrm{si}}}\right.$$

$$5e^2\left(e^2\left(\frac{6a\sqrt{\sin(c+dx)}\,\mathrm{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}} - \frac{\cdots}{b}\right)\right.$$

$$\frac{\cdots}{2b^2}$$

$$\frac{\cdots}{4b}$$

$\downarrow$ 3042

$$\frac{e(e\sin(c+dx))^{5/2}}{2bd(a+b\cos(c+dx))^2}-$$

$$5e^2\left(\frac{e^2\left(\frac{6a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{(3a^2-2b^2)\left(-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}}-\frac{a\sqrt{\sin(c+dx)}\int\frac{}{\sqrt{\sin(c+dx)}(b}}{2\sqrt{b^2-a^2}\sqrt{e\,\text{si}}}\right.}{b}\right)}{2b^2}\right.$$

$$4b$$

$\downarrow$ 3284

$$\frac{e(e\sin(c+dx))^{5/2}}{2bd(a+b\cos(c+dx))^2}-\frac{5e^2\left(\frac{e^2\left(\frac{6a\sqrt{\sin(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{\left(3a^2-2b^2\right)\left(-\frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}\right)}{d}+\frac{a\sqrt{\sin(c+dx)}\,\mathrm{E}\ldots}{d\sqrt{b^2\ldots}}\ldots\right)}{b}\ldots\right)}{2b^2}\ldots\right)}{4b}\ldots$$

input
```
Int[(e*Sin[c + d*x])^(7/2)/(a + b*Cos[c + d*x])^3,x]
```

output
```
(e*(e*Sin[c + d*x])^(5/2))/(2*b*d*(a + b*Cos[c + d*x])^2) - (5*e^2*(-((e*(
3*a + 2*b*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]])/(b^2*d*(a + b*Cos[c + d*x]))
) + (e^2*((6*a*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(b*d*S
qrt[e*Sin[c + d*x]]) - ((3*a^2 - 2*b^2)*((-2*b*e*(-1/2*ArcTan[(Sqrt[b]*Sqr
t[e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/(Sqrt[b]*(-a^2 + b^2)^(3/4)*e^(3/2)
) - ArcTanh[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/(2*Sqrt[b]*
(-a^2 + b^2)^(3/4)*e^(3/2))))/d + (a*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2
]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(Sqrt[-a^2 + b^2]*(b - Sqrt
[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]]) - (a*EllipticPi[(2*b)/(b + Sqrt[-a^2
 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(Sqrt[-a^2 + b^2]*(b
+ Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]])))/b))/(2*b^2)))/(4*b)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3172
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Simp[g^2*((p - 1)/(b*(m + 1)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Fre
eQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && I
ntegersQ[2*m, 2*p]
```

rule 3181
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[-a/(2*q)   Int[1/(
Sqrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Simp[b*(g/f)   Subst[
Int[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - S
imp[a/(2*q)   Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3342
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*C
os[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x])/(b^2*f*(m + 1)*(m + p + 1))), x] + Simp[g^2*((
p - 1)/(b^2*(m + 1)*(m + p + 1)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

## Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2589 vs. $2(445) = 890$.

Time = 98.71 (sec) , antiderivative size = 2590, normalized size of antiderivative = 5.06

| method | result | size |
|---|---|---|
| default | Expression too large to display | 2590 |

input
```
int((e*sin(d*x+c))^(7/2)/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)
```

output
```
(2*e^3*b*(1/b^4*(e*sin(d*x+c))^(1/2)-e^2/b^4*(-1/8*(e*sin(d*x+c))^(1/2)*e^
2*(-11*cos(d*x+c)^2*a^2*b^2+2*b^4*cos(d*x+c)^2+7*a^4+2*a^2*b^2)/(-b^2*cos(
d*x+c)^2*e^2+a^2*e^2)^2+5/64*(3*a^2-2*b^2)*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*
e^2-b^2*e^2)*2^(1/2)*(ln((e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*
x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)-(e^2*(a^2-b^2
)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*ar
ctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^
(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1))))-(cos(d*x+c)^2*e
*sin(d*x+c))^(1/2)*e^4*a*(-2/b^4*(5*a^2-3*b^2)*(-1/2/b/(-a^2+b^2)^(1/2)*(1
-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e
*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),
1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/2/b/(-a^2+b^2)^(1/2)*(1-sin(d*x+c)
)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c)
)^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+
b^2)^(1/2)/b),1/2*2^(1/2)))+1/b^4*(11*a^4-14*a^2*b^2+3*b^4)*(1/2*b^2/e/a^2
/(a^2-b^2)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-cos(d*x+c)^2*b^2+a^2)+1/4/a
^2/(a^2-b^2)*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/
(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/
2))-5/8/(a^2-b^2)/b/(-a^2+b^2)^(1/2)*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))
^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)...
```

## Fricas [F(-1)]

Timed out.

$$\int \frac{(e \sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))^(7/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
Timed out
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))**(7/2)/(a+b*cos(d*x+c))**3,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{(e\sin(dx+c))^{\frac{7}{2}}}{(b\cos(dx+c)+a)^3}\,dx$$

input
```
integrate((e*sin(d*x+c))^(7/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate((e*sin(d*x + c))^(7/2)/(b*cos(d*x + c) + a)^3, x)
```

## Giac [F]

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{(e\sin(dx+c))^{\frac{7}{2}}}{(b\cos(dx+c)+a)^3}\,dx$$

input
```
integrate((e*sin(d*x+c))^(7/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((e*sin(d*x + c))^(7/2)/(b*cos(d*x + c) + a)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{\left(e\,\sin\left(c+d\,x\right)\right)^{7/2}}{\left(a+b\,\cos\left(c+d\,x\right)\right)^3}\,dx$$

input
```
int((e*sin(c + d*x))^(7/2)/(a + b*cos(c + d*x))^3,x)
```

output
```
int((e*sin(c + d*x))^(7/2)/(a + b*cos(c + d*x))^3, x)
```

## Reduce [F]

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^3}\,dx = \sqrt{e}\left(\int \frac{\sqrt{\sin(dx+c)}\,\sin(dx+c)^3}{\cos(dx+c)^3 b^3+3\cos(dx+c)^2 a\,b^2+3\cos(dx+c)\,a^2b+a^3}dx\right)e^3$$

input
```
int((e*sin(d*x+c))^(7/2)/(a+b*cos(d*x+c))^3,x)
```

output
```
sqrt(e)*int((sqrt(sin(c + d*x))*sin(c + d*x)**3)/(cos(c + d*x)**3*b**3 + 3
*cos(c + d*x)**2*a*b**2 + 3*cos(c + d*x)*a**2*b + a**3),x)*e**3
```

# 3.82 $\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^3}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 520

$$\begin{aligned}\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^3}\,dx &= -\frac{3(a^2-2b^2)\,e^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{5/2}\left(-a^2+b^2\right)^{5/4}d} \\ &+ \frac{3(a^2-2b^2)\,e^{5/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{5/2}\left(-a^2+b^2\right)^{5/4}d} \\ &- \frac{3a(a^2-2b^2)\,e^3\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b^3\left(a^2-b^2\right)\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} \\ &- \frac{3a(a^2-2b^2)\,e^3\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b^3\left(a^2-b^2\right)\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} \\ &+ \frac{3ae^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{4b^2\left(a^2-b^2\right)d\sqrt{\sin(c+dx)}} \\ &+ \frac{e(e\sin(c+dx))^{3/2}}{2bd(a+b\cos(c+dx))^2} - \frac{3ae(e\sin(c+dx))^{3/2}}{4b\left(a^2-b^2\right)d(a+b\cos(c+dx))}\end{aligned}$$

output
```
-3/8*(a^2-2*b^2)*e^(5/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1
/4)/e^(1/2))/b^(5/2)/(-a^2+b^2)^(5/4)/d+3/8*(a^2-2*b^2)*e^(5/2)*arctanh(b^
(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(5/2)/(-a^2+b^2)^(5
/4)/d+3/8*a*(a^2-2*b^2)*e^3*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-
a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^3/(a^2-b^2)/(b-(-a^2+b^2)^(1/2
))/d/(e*sin(d*x+c))^(1/2)+3/8*a*(a^2-2*b^2)*e^3*EllipticPi(cos(1/2*c+1/4*P
i+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^3/(a^2-b^2
)/(b+(-a^2+b^2)^(1/2))/d/(e*sin(d*x+c))^(1/2)-3/4*a*e^2*EllipticE(cos(1/2*
c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/b^2/(a^2-b^2)/d/sin(d*x+c)
^(1/2)+1/2*e*(e*sin(d*x+c))^(3/2)/b/d/(a+b*cos(d*x+c))^2-3/4*a*e*(e*sin(d*
x+c))^(3/2)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))
```

## Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 15.92 (sec) , antiderivative size = 831, normalized size of antiderivative = 1.60

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^3}\,dx = \frac{\csc^2(c+dx)(e\sin(c+dx))^{5/2}\left(\frac{\sin(c+dx)}{2b(a+b\cos(c+dx))^2}+\frac{3a\sin(c+dx)}{4b(-a^2+b^2)(a+b\cos(c+dx))}\right)}{d}$$

$$+\frac{3(e\sin(c+dx))^{5/2}\left(\frac{a\cos^2(c+dx)\left(3\sqrt{2}a(a^2-b^2)^{3/4}\left(2\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-2\arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-\log\left(\sqrt{a^2-b^2}\cdots\right.\right.\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(e*Sin[c + d*x])^(5/2)/(a + b*Cos[c + d*x])^3,x]
```

output
```
(Csc[c + d*x]^2*(e*Sin[c + d*x])^(5/2)*(Sin[c + d*x]/(2*b*(a + b*Cos[c + d
*x])^2) + (3*a*Sin[c + d*x])/(4*b*(-a^2 + b^2)*(a + b*Cos[c + d*x]))))/d +
 (3*(e*Sin[c + d*x])^(5/2)*((a*Cos[c + d*x]^2*(3*Sqrt[2]*a*(a^2 - b^2)^(3/
4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] -
 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Lo
g[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] +
 b*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)
*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]]) + 8*b^(5/2)*AppellF1[3/4, -1/2, 1,
7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2)
)*(a + b*Sqrt[1 - Sin[c + d*x]^2]))/(12*b^(3/2)*(-a^2 + b^2)*(a + b*Cos[c
+ d*x])*(1 - Sin[c + d*x]^2)) + (4*b*Cos[c + d*x]*(((1/8 + I/8)*(2*ArcTan[
1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1
+ ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2
 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[
c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt
[Sin[c + d*x]] + I*b*Sin[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)) + (a*App
ellF1[3/4, 1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]
*Sin[c + d*x]^(3/2))/(3*(a^2 - b^2)))*(a + b*Sqrt[1 - Sin[c + d*x]^2]))/((
a + b*Cos[c + d*x])*Sqrt[1 - Sin[c + d*x]^2])))/(8*(a - b)*b*(a + b)*d*Sin
[c + d*x]^(5/2))
```

## Rubi [A] (warning: unable to verify)

Time = 2.05 (sec) , antiderivative size = 464, normalized size of antiderivative = 0.89, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.840$, Rules used = {3042, 3172, 25, 3042, 3343, 27, 3042, 3346, 3042, 3121, 3042, 3119, 3180, 266, 827, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2}}{\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3172}$$

$$\frac{3e^2\int -\frac{\cos(c+dx)\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^2}dx}{4b}+\frac{e(e\sin(c+dx))^{3/2}}{2bd(a+b\cos(c+dx))^2}$$

$\Big\downarrow$ 25

$$\frac{e(e\sin(c+dx))^{3/2}}{2bd(a+b\cos(c+dx))^2}-\frac{3e^2\int \frac{\cos(c+dx)\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^2}dx}{4b}$$

$\Big\downarrow$ 3042

$$\frac{e(e\sin(c+dx))^{3/2}}{2bd(a+b\cos(c+dx))^2}-\frac{3e^2\int \frac{\sqrt{-e\cos\left(c+dx+\frac{\pi}{2}\right)}\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4b}$$

$\Big\downarrow$ 3343

$$\frac{e(e\sin(c+dx))^{3/2}}{2bd(a+b\cos(c+dx))^2}-\frac{3e^2\left(\frac{a(e\sin(c+dx))^{3/2}}{de(a^2-b^2)(a+b\cos(c+dx))}-\frac{\int \frac{(2b+a\cos(c+dx))\sqrt{e\sin(c+dx)}}{2(a+b\cos(c+dx))}dx}{a^2-b^2}\right)}{4b}$$

$\Big\downarrow$ 27

$$\frac{e(e\sin(c+dx))^{3/2}}{2bd(a+b\cos(c+dx))^2}-\frac{3e^2\left(\frac{a(e\sin(c+dx))^{3/2}}{de(a^2-b^2)(a+b\cos(c+dx))}-\frac{\int \frac{(2b+a\cos(c+dx))\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx}{2(a^2-b^2)}\right)}{4b}$$

$\Big\downarrow$ 3042

$$\frac{e(e\sin(c+dx))^{3/2}}{2bd(a+b\cos(c+dx))^2}-\frac{3e^2\left(\frac{a(e\sin(c+dx))^{3/2}}{de(a^2-b^2)(a+b\cos(c+dx))}-\frac{\int \frac{\sqrt{-e\cos\left(c+dx+\frac{\pi}{2}\right)}\left(2b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2(a^2-b^2)}\right)}{4b}$$

$\Big\downarrow$ 3346

$$\frac{e(e\sin(c+dx))^{3/2}}{2bd(a+b\cos(c+dx))^2}-\frac{3e^2\left(\frac{a(e\sin(c+dx))^{3/2}}{de(a^2-b^2)(a+b\cos(c+dx))}-\frac{\frac{a\int\sqrt{e\sin(c+dx)}dx}{b}-\frac{(a^2-2b^2)\int \frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx}{b}}{2(a^2-b^2)}\right)}{4b}$$

$\Big\downarrow$ 3042

$$\frac{e(e\sin(c+dx))^{3/2}}{2bd(a+b\cos(c+dx))^2}-\frac{3e^2\left(\frac{a(e\sin(c+dx))^{3/2}}{de(a^2-b^2)(a+b\cos(c+dx))}-\frac{\frac{a\int\sqrt{e\sin(c+dx)}dx}{b}-\frac{(a^2-2b^2)\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{b}}{2(a^2-b^2)}\right)}{4b}$$

$\Big\downarrow$ 3121

$$\frac{e(e\sin(c+dx))^{3/2}}{2bd(a+b\cos(c+dx))^2}-\frac{3e^2\left(\frac{a(e\sin(c+dx))^{3/2}}{de(a^2-b^2)(a+b\cos(c+dx))}-\frac{\frac{a\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{b\sqrt{\sin(c+dx)}}-\frac{(a^2-2b^2)\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{b}}{2(a^2-b^2)}\right)}{4b}$$

$\Big\downarrow$ 3042

$$\frac{e(e\sin(c+dx))^{3/2}}{2bd(a+b\cos(c+dx))^2}-\frac{3e^2\left(\frac{a(e\sin(c+dx))^{3/2}}{de(a^2-b^2)(a+b\cos(c+dx))}-\frac{\frac{a\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{b\sqrt{\sin(c+dx)}}-\frac{(a^2-2b^2)\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{b}}{2(a^2-b^2)}\right)}{4b}$$

$\Big\downarrow$ 3119

$$\frac{e(e\sin(c+dx))^{3/2}}{2bd(a+b\cos(c+dx))^2}-\frac{3e^2\left(\frac{a(e\sin(c+dx))^{3/2}}{de(a^2-b^2)(a+b\cos(c+dx))}-\frac{\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{(a^2-2b^2)\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{b}}{2(a^2-b^2)}\right)}{4b}$$

$\Big\downarrow$ 3180

$$\frac{e(e\sin(c+dx))^{3/2}}{2bd(a+b\cos(c+dx))^2}-\frac{3e^2\left(\frac{a(e\sin(c+dx))^{3/2}}{de(a^2-b^2)(a+b\cos(c+dx))}-\frac{\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{\left(a^2-2b^2\right)\left(-\frac{be\int\frac{\sqrt{e\sin(c+dx)}}{b^2\sin^2(c+dx)e^2+\left(a^2-b^2\right)e^2}d(e\sin(c+dx))}{d}-\frac{ae\int\frac{}{\sqrt{e\sin(}}}{}\right.}{}}{2(a^2-b^2)}\right.}{4b}$$

$\Big\downarrow$ 266

$$\frac{e(e\sin(c+dx))^{3/2}}{2bd(a+b\cos(c+dx))^2}-\frac{3e^2\left(\frac{a(e\sin(c+dx))^{3/2}}{de(a^2-b^2)(a+b\cos(c+dx))}-\frac{\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{\left(a^2-2b^2\right)\left(-\frac{2be\int\frac{e^2\sin^2(c+dx)}{b^2e^4\sin^4(c+dx)+\left(a^2-b^2\right)e^2}d\sqrt{e\sin(c+dx)}}{d}-\frac{ae\int\frac{}{\sqrt{e\sin}}}{}\right.}{}}{2(a^2-b^2)}\right.}{4b}$$

$\Big\downarrow$ 827

$$\frac{e(e\sin(c+dx))^{3/2}}{2bd(a+b\cos(c+dx))^2}-\frac{3e^2\left(\frac{a(e\sin(c+dx))^{3/2}}{de(a^2-b^2)(a+b\cos(c+dx))}-\frac{\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{\left(a^2-2b^2\right)\left(-\frac{2be\left(\frac{\int\frac{1}{be^2\sin^2(c+dx)+\sqrt{b^2-a^2}e}d\sqrt{e\sin(c+dx)}}{2b}-\frac{\int\frac{}{\sqrt{b^2-a^2}e}}{}\right.}{d}\right.}{}}{}\right.}{4b}$$

$\Big\downarrow$ 218

$$\frac{e(e\sin(c+dx))^{3/2}}{2bd(a+b\cos(c+dx))^2}-$$

$$\frac{3e^2\left(\frac{a(e\sin(c+dx))^{3/2}}{de(a^2-b^2)(a+b\cos(c+dx))}-\frac{\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{\left(a^2-2b^2\right)\left(-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}}{2b}\right.}{d}\right.}{}}{2(a^2-}\right.}{4b}$$

$$\downarrow 221$$

$$\frac{e(e\sin(c+dx))^{3/2}}{2bd(a+b\cos(c+dx))^2}-$$

$$\frac{3e^2\left(\frac{a(e\sin(c+dx))^{3/2}}{de(a^2-b^2)(a+b\cos(c+dx))}-\frac{\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{\left(a^2-2b^2\right)\left(-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int\frac{}{\sqrt{e\sin(c+d}}}{}\right.}{}}{2(a^2-b^2)}\right.}{4b}$$

$\Big\downarrow$ 3042

$$\frac{e(e\sin(c+dx))^{3/2}}{2bd(a+b\cos(c+dx))^2}-$$

$$\frac{3e^2\left(\frac{a(e\sin(c+dx))^{3/2}}{de(a^2-b^2)(a+b\cos(c+dx))}-\frac{\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{\left(a^2-2b^2\right)\left(-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int\frac{}{\sqrt{e\sin(c+d}}}{}\right.}{}}{2(a^2-b^2)}\right.}{4b}$$

$\Big\downarrow$ 3286

$$\frac{e(e\sin(c+dx))^{3/2}}{2bd(a+b\cos(c+dx))^2}-\frac{3e^2\left(\frac{a(e\sin(c+dx))^{3/2}}{de(a^2-b^2)(a+b\cos(c+dx))}-\frac{\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{(a^2-2b^2)\left(-\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b\sqrt{e\sin(c+dx)}}+\frac{ae\sqrt{\mathrm{si}}}{}\right.}{}}{2(}\right.}{4b}$$

$\downarrow$ 3042

$$\frac{e(e\sin(c+dx))^{3/2}}{2bd(a+b\cos(c+dx))^2}-\frac{3e^2\left(\frac{a(e\sin(c+dx))^{3/2}}{de(a^2-b^2)(a+b\cos(c+dx))}-\frac{\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{(a^2-2b^2)\left(-\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b\sqrt{e\sin(c+dx)}}+\frac{ae\sqrt{\mathrm{si}}}{}\right.}{}}{2(}\right.}{4b}$$

$\downarrow$ 3284

$$\frac{e(e\sin(c+dx))^{3/2}}{2bd(a+b\cos(c+dx))^2}-\frac{3e^2\left(\frac{a(e\sin(c+dx))^{3/2}}{de(a^2-b^2)(a+b\cos(c+dx))}-\frac{\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{(a^2-2b^2)\left(-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{d}\right.}{2(}\right)}{4b}$$

input
```
Int[(e*Sin[c + d*x])^(5/2)/(a + b*Cos[c + d*x])^3,x]
```

output
```
(e*(e*Sin[c + d*x])^(3/2))/(2*b*d*(a + b*Cos[c + d*x])^2) - (3*e^2*((a*(e*
Sin[c + d*x])^(3/2))/((a^2 - b^2)*d*e*(a + b*Cos[c + d*x])) - ((2*a*Ellipt
icE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(b*d*Sqrt[Sin[c + d*x]])
- ((a^2 - 2*b^2)*((-2*b*e*(ArcTan[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(-a^2 + b
^2)^(1/4)]/(2*b^(3/2)*(-a^2 + b^2)^(1/4)*Sqrt[e]) - ArcTanh[(Sqrt[b]*Sqrt[
e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/(2*b^(3/2)*(-a^2 + b^2)^(1/4)*Sqrt[e]
)))/d + (a*e*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2,
2]*Sqrt[Sin[c + d*x]])/(b*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]]) +
 (a*e*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt
[Sin[c + d*x]])/(b*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]])))/b)/(2*
(a^2 - b^2))))/(4*b)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3172
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Simp[g^2*((p - 1)/(b*(m + 1)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Fre
eQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && I
ntegersQ[2*m, 2*p]
```

rule 3180
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[a*(g/(2*b))   Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Simp[a*(g/(2*b))   In
t[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Simp[b*(g/f)   Su
bst[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3343
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Simp[1/((a^2 - b^2)*(m + 1))   Int[(g*Cos[e + f*x])^p
*(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ
[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2611 vs. $2(454) = 908$.

Time = 96.44 (sec) , antiderivative size = 2612, normalized size of antiderivative = 5.02

| method | result | size |
|---|---|---|
| default | Expression too large to display | 2612 |

input
```
int((e*sin(d*x+c))^(5/2)/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)
```

output
```
(e^3*b*(-1/4*(e*sin(d*x+c))^(3/2)*e^2*(-5*cos(d*x+c)^2*a^2*b^2+2*b^4*cos(d
*x+c)^2+a^4+2*a^2*b^2)/b^2/(a^2-b^2)/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)^2+3/3
2*(a^2-2*b^2)/b^4/(a^2-b^2)/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*(ln((e*sin(d
*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2
)/b^2)^(1/2))/(e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)
*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^
(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(
e*sin(d*x+c))^(1/2)-1)))-(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*e^3*a*(3/b^2*(-
1/2/b^2*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(cos(
d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+
c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-1/2/b^2*(1-sin(d*x+c))^(1/
2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/
2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^
(1/2)/b),1/2*2^(1/2)))-(7*a^2-3*b^2)/b^2*(1/2*b^2/e/a^2/(a^2-b^2)*sin(d*x+
c)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-cos(d*x+c)^2*b^2+a^2)-1/2/a^2/(a^2-
b^2)*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x
+c)^2*e*sin(d*x+c))^(1/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))+1/4/
a^2/(a^2-b^2)*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)
/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1
/2))-3/8/(a^2-b^2)/b^2*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(...
```

## Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))^(5/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
Timed out
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))**(5/2)/(a+b*cos(d*x+c))**3,x)
```

output
```
Timed out
```

## Maxima [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))^(5/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
Timed out
```

## Giac [F]

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{(e\sin(dx+c))^{\frac{5}{2}}}{(b\cos(dx+c)+a)^3}\,dx$$

input
```
integrate((e*sin(d*x+c))^(5/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((e*sin(d*x + c))^(5/2)/(b*cos(d*x + c) + a)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{(e\,\sin(c+d\,x))^{5/2}}{(a+b\,\cos(c+d\,x))^3}\,dx$$

input
```
int((e*sin(c + d*x))^(5/2)/(a + b*cos(c + d*x))^3,x)
```

output
```
int((e*sin(c + d*x))^(5/2)/(a + b*cos(c + d*x))^3, x)
```

## Reduce [F]

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^3}\,dx = \sqrt{e}\left(\int \frac{\sqrt{\sin(dx+c)}\,\sin(dx+c)^2}{\cos(dx+c)^3 b^3+3\cos(dx+c)^2 a\,b^2+3\cos(dx+c)\,a^2b+a^3}dx\right) e^2$$

input
```
int((e*sin(d*x+c))^(5/2)/(a+b*cos(d*x+c))^3,x)
```

output
```
sqrt(e)*int((sqrt(sin(c + d*x))*sin(c + d*x)**2)/(cos(c + d*x)**3*b**3 + 3
*cos(c + d*x)**2*a*b**2 + 3*cos(c + d*x)*a**2*b + a**3),x)*e**2
```

# 3.83 $\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^3}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 534

$$\begin{aligned}
\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^3}\,dx &= -\frac{(a^2+2b^2)\,e^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{3/2}\left(-a^2+b^2\right)^{7/4}d} \\
&- \frac{(a^2+2b^2)\,e^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{3/2}\left(-a^2+b^2\right)^{7/4}d} \\
&- \frac{ae^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{4b^2\left(a^2-b^2\right)d\sqrt{e\sin(c+dx)}} \\
&+ \frac{a(a^2+2b^2)\,e^2\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b^2\left(a^2-b^2\right)\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}} \\
&+ \frac{a(a^2+2b^2)\,e^2\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b^2\left(a^2-b^2\right)\left(a^2-b\left(b+\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}} \\
&+ \frac{e\sqrt{e\sin(c+dx)}}{2bd(a+b\cos(c+dx))^2} - \frac{ae\sqrt{e\sin(c+dx)}}{4b\left(a^2-b^2\right)d(a+b\cos(c+dx))}
\end{aligned}$$

output
```
-1/8*(a^2+2*b^2)*e^(3/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1
/4)/e^(1/2))/b^(3/2)/(-a^2+b^2)^(7/4)/d-1/8*(a^2+2*b^2)*e^(3/2)*arctanh(b^
(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(3/2)/(-a^2+b^2)^(7
/4)/d-1/4*a*e^2*InverseJacobiAM(1/2*c-1/4*Pi+1/2*d*x,2^(1/2))*sin(d*x+c)^(
1/2)/b^2/(a^2-b^2)/d/(e*sin(d*x+c))^(1/2)-1/8*a*(a^2+2*b^2)*e^2*EllipticPi
(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1
/2)/b^2/(a^2-b^2)/(a^2-b*(b-(-a^2+b^2)^(1/2)))/d/(e*sin(d*x+c))^(1/2)-1/8*
a*(a^2+2*b^2)*e^2*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(
1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^2/(a^2-b^2)/(a^2-b*(b+(-a^2+b^2)^(1/2)))
/d/(e*sin(d*x+c))^(1/2)+1/2*e*(e*sin(d*x+c))^(1/2)/b/d/(a+b*cos(d*x+c))^2-
1/4*a*e*(e*sin(d*x+c))^(1/2)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))
```

## Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 12.93 (sec) , antiderivative size = 1211, normalized size of antiderivative = 2.27

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
Integrate[(e*Sin[c + d*x])^(3/2)/(a + b*Cos[c + d*x])^3,x]
```

output
```
((1/(2*b*(a + b*Cos[c + d*x])^2) + a/(4*b*(-a^2 + b^2)*(a + b*Cos[c + d*x]
)))*Csc[c + d*x]*(e*Sin[c + d*x])^(3/2))/d - ((e*Sin[c + d*x])^(3/2)*((2*a
*Cos[c + d*x]^2*(a + b*Sqrt[1 - Sin[c + d*x]^2])*((a*(-2*ArcTan[1 - (Sqrt[
2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*
Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqr
t[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]] + Log[
Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b
*Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)) + (5*b*(a^2 - b^2)*
AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b
^2)]*Sqrt[Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((-5*(a^2 - b^2)*AppellF
1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] +
2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/
(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (b^
2*Sin[c + d*x]^2)/(-a^2 + b^2)])*Sin[c + d*x]^2)*(a^2 + b^2*(-1 + Sin[c +
d*x]^2))))/((a + b*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) - (4*b*Cos[c + d*x
]*(a + b*Sqrt[1 - Sin[c + d*x]^2])*(((-1/8 + I/8)*Sqrt[b]*(2*ArcTan[1 - ((
1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1
+ I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2
] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*
x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Si...
```

## Rubi [A] (warning: unable to verify)

Time = 2.09 (sec) , antiderivative size = 483, normalized size of antiderivative = 0.90, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.880$, Rules used = {3042, 3172, 25, 3042, 25, 3343, 27, 3042, 3346, 3042, 3121, 3042, 3120, 3181, 266, 756, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}}{\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3172}$$

$$\frac{e^2\int -\frac{\cos(c+dx)}{(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}}dx}{4b}+\frac{e\sqrt{e\sin(c+dx)}}{2bd(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{e\sqrt{e\sin(c+dx)}}{2bd(a+b\cos(c+dx))^2}-\frac{e^2\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}}dx}{4b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e\sqrt{e\sin(c+dx)}}{2bd(a+b\cos(c+dx))^2}-\frac{e^2\int -\frac{\sin\left(c+dx-\frac{\pi}{2}\right)}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx}{4b}$$

$$\Big\downarrow \text{25}$$

$$\frac{e^2\int \frac{\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}{\sqrt{e\cos\left(\frac{1}{2}(2c-\pi)+dx\right)}\left(a-b\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)^2}dx}{4b}+\frac{e\sqrt{e\sin(c+dx)}}{2bd(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3343}$$

$$\frac{e^2\left(-\frac{\int -\frac{2b-a\cos(c+dx)}{2(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{a^2-b^2}-\frac{a\sqrt{e\sin(c+dx)}}{de(a^2-b^2)(a+b\cos(c+dx))}\right)}{4b}+\frac{e\sqrt{e\sin(c+dx)}}{2bd(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^2\left(\frac{\int \frac{2b-a\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{2(a^2-b^2)}-\frac{a\sqrt{e\sin(c+dx)}}{de(a^2-b^2)(a+b\cos(c+dx))}\right)}{4b}+\frac{e\sqrt{e\sin(c+dx)}}{2bd(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e^2\left(\frac{\int \frac{2b+a\sin\left(c+dx-\frac{\pi}{2}\right)}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{2(a^2-b^2)}-\frac{a\sqrt{e\sin(c+dx)}}{de(a^2-b^2)(a+b\cos(c+dx))}\right)}{4b}+\frac{e\sqrt{e\sin(c+dx)}}{2bd(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3346}$$

$$\frac{e^2\left(\frac{\frac{(a^2+2b^2)\int \frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{b}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}}dx}{b}}{2(a^2-b^2)}-\frac{a\sqrt{e\sin(c+dx)}}{de(a^2-b^2)(a+b\cos(c+dx))}\right)}{4b}+\frac{e\sqrt{e\sin(c+dx)}}{2bd(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e^2\left(\frac{\frac{(a^2+2b^2)\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{b}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}}dx}{b}}{2(a^2-b^2)}-\frac{a\sqrt{e\sin(c+dx)}}{de(a^2-b^2)(a+b\cos(c+dx))}\right)}{4b}+\frac{e\sqrt{e\sin(c+dx)}}{2bd(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3121}$$

$$\frac{e^2\left(\frac{\frac{(a^2+2b^2)\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{b}-\frac{a\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}}dx}{b\sqrt{e\sin(c+dx)}}}{2(a^2-b^2)}-\frac{a\sqrt{e\sin(c+dx)}}{de(a^2-b^2)(a+b\cos(c+dx))}\right)}{4b}+\frac{e\sqrt{e\sin(c+dx)}}{2bd(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e^2\left(\frac{\frac{(a^2+2b^2)\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{b}-\frac{a\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}}dx}{b\sqrt{e\sin(c+dx)}}}{2(a^2-b^2)}-\frac{a\sqrt{e\sin(c+dx)}}{de(a^2-b^2)(a+b\cos(c+dx))}\right)}{4b}+\frac{e\sqrt{e\sin(c+dx)}}{2bd(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3120}$$

$$\frac{e^2\left(\frac{\frac{(a^2+2b^2)\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{b}-\frac{2a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}}{2(a^2-b^2)}-\frac{a\sqrt{e\sin(c+dx)}}{de(a^2-b^2)(a+b\cos(c+dx))}\right)}{4b}+\frac{e\sqrt{e\sin(c+dx)}}{2bd(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3181}$$

$$e^2\left(\frac{\frac{\left(a^2+2b^2\right)\left(-\frac{be\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b^2\sin^2(c+dx)e^2+\left(a^2-b^2\right)e^2\right)}d(e\sin(c+dx))}{d}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c-\right.}}{2\sqrt{b^2-a}}\right.}{b}}{2(a^2-b^2)}\right.$$

$$\overline{\qquad 4b \qquad}$$

$$\frac{e\sqrt{e\sin(c+dx)}}{2bd(a+b\cos(c+dx))^2}$$

$\downarrow$ 266

$$e^2\left(\frac{\frac{\left(a^2+2b^2\right)\left(-\frac{2be\int \frac{1}{b^2e^4\sin^4(c+dx)+\left(a^2-b^2\right)e^2}d\sqrt{e\sin(c+dx)}}{d}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right.}}{2\sqrt{b^2-a^2}}\right.}{b}}{2(a^2-b^2)}\right.$$

$$\overline{\qquad 4b \qquad}$$

$$\frac{e\sqrt{e\sin(c+dx)}}{2bd(a+b\cos(c+dx))^2}$$

$\downarrow$ 756

$$e^2\left(\frac{\frac{\left(a^2+2b^2\right)\left(-\frac{2be\left(-\frac{\int \frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2e\sqrt{b^2-a^2}}-\frac{\int \frac{1}{be^2\sin^2(c+dx)+\sqrt{b^2-a^2}e}d\sqrt{e\sin(c+dx)}}{2e\sqrt{b^2-a^2}}\right)}{d}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+d\right.}}{2\sqrt{b^2-a^2}}\right.}{b}}{2(a^2-b^2)}\right.$$

$$\overline{\qquad 4b \qquad}$$

$$\frac{e\sqrt{e\sin(c+dx)}}{2bd(a+b\cos(c+dx))^2}$$

$\downarrow$ 218

$$e^2\left(\frac{\frac{\left(a^2+2b^2\right)\left(-\frac{2be\left(-\frac{\int \frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2e\sqrt{b^2-a^2}}-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}\right)}{d}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int \frac{\cdots}{\sqrt{e\cdots}}\cdots}{\cdots}\cdots\right)}{b}\cdots}{2(a^2-b^2)}\cdots\right.}{4b}\cdots$$

$$\frac{e\sqrt{e\sin(c+dx)}}{2bd(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 221$$

$$e^2\left(\frac{\frac{(a^2+2b^2)\left(-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}}-\frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}-\frac{\arctan}{2\sqrt{b}}\right.}{d}\right.}{b}}{2(a^2-b^2)}\right.}{4b}$$

$$\frac{e\sqrt{e\sin(c+dx)}}{2bd(a+b\cos(c+dx))^2}$$

$\downarrow$ 3042

$$e^2\left(\frac{\frac{(a^2+2b^2)\left(-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}}-\frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}-\frac{\arctan}{2\sqrt{b}}\right.}{d}\right.}{b}}{2(a^2-b^2)}\right.$$

$$\overline{\qquad 4b \qquad}$$

$$\frac{e\sqrt{e\sin(c+dx)}}{2bd(a+b\cos(c+dx))^2}$$

$\downarrow$ 3286

$$e^2\left(\frac{\frac{\left(a^2+2b^2\right)\left(-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}}-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}}-\frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin}{\sqrt[4]{b^2-}}\right.}{2\sqrt{b}e^{3/2}\left(b^2-a\right.}\right.}{}\right.}{b}}{2(a^2-b^2)}\right.$$

$$\overline{\qquad\qquad 4b \qquad\qquad}$$

$$\frac{e\sqrt{e\sin(c+dx)}}{2bd(a+b\cos(c+dx))^2}$$

$\downarrow$ 3042

$$e^2\left(\frac{\frac{\left(a^2+2b^2\right)\left(-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}}-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}}-\frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin\cdots}{\sqrt[4]{b^2-\cdots}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a\cdots\right.}\cdots\right.}{\cdots}\cdots\right.}{b}\cdots}{2(a^2-b^2)}\cdots\right.}{4b}\cdots$$

$$\frac{e\sqrt{e\sin(c+dx)}}{2bd(a+b\cos(c+dx))^2}$$

$\downarrow$ 3284

$$e^2\left(\frac{\frac{(a^2+2b^2)\left(-\frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}\right)}{d}+\frac{a\sqrt{\sin(c+dx)}\operatorname{EllipticPi}\left(\frac{2b}{b-\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{b^2-a^2}\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\sin(c+dx)}}-\frac{a\sqrt{\sin(c+\cdots}}{d\cdots}\cdots\right)}{b}}{2(a^2-b^2)}\cdots\right)$$
$$\overline{\qquad\qquad 4b \qquad\qquad}$$

$$\frac{e\sqrt{e\sin(c+dx)}}{2bd(a+b\cos(c+dx))^2}$$

input
```
Int[(e*Sin[c + d*x])^(3/2)/(a + b*Cos[c + d*x])^3,x]
```

output
```
(e*Sqrt[e*Sin[c + d*x]])/(2*b*d*(a + b*Cos[c + d*x])^2) + (e^2*(-((a*Sqrt[
e*Sin[c + d*x]])/((a^2 - b^2)*d*e*(a + b*Cos[c + d*x]))) + ((-2*a*Elliptic
F[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(b*d*Sqrt[e*Sin[c + d*x]]) +
((a^2 + 2*b^2)*((-2*b*e*(-1/2*ArcTan[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(-a^2
+ b^2)^(1/4)]/(Sqrt[b]*(-a^2 + b^2)^(3/4)*e^(3/2)) - ArcTanh[(Sqrt[b]*Sqrt
[e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/(2*Sqrt[b]*(-a^2 + b^2)^(3/4)*e^(3/2
))))/d + (a*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2
]*Sqrt[Sin[c + d*x]])/(Sqrt[-a^2 + b^2]*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Si
n[c + d*x]]) - (a*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x
)/2, 2]*Sqrt[Sin[c + d*x]])/(Sqrt[-a^2 + b^2]*(b + Sqrt[-a^2 + b^2])*d*Sqr
t[e*Sin[c + d*x]]))))/b)/(2*(a^2 - b^2))))/(4*b)
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3172
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Simp[g^2*((p - 1)/(b*(m + 1)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Fre
eQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && I
ntegersQ[2*m, 2*p]
```

rule 3181
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[-a/(2*q)   Int[1/(
Sqrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Simp[b*(g/f)   Subst[
Int[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - S
imp[a/(2*q)   Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3343
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Simp[1/((a^2 - b^2)*(m + 1))   Int[(g*Cos[e + f*x])^p
*(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ
[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2493 vs. $2(467) = 934$.

Time = 3.36 (sec) , antiderivative size = 2494, normalized size of antiderivative = 4.67

| method | result | size |
|---|---|---|
| default | Expression too large to display | 2494 |

input
```
int((e*sin(d*x+c))^(3/2)/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)
```

output
```
(2*e^3*b*(1/8*(e*sin(d*x+c))^(1/2)*e^2*(3*cos(d*x+c)^2*a^2*b^2-2*b^4*cos(d
*x+c)^2+a^4-2*a^2*b^2)/b^2/(a^2-b^2)/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)^2-1/6
4*(a^2+2*b^2)/b^2/(a^2-b^2)*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^
(1/2)*(ln((e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(
1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e
*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arctan(2^(1/2)/(e
^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2)/(e^2*(a^2
-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)))-(cos(d*x+c)^2*e*sin(d*x+c))^(1/
2)*e^2*a*(-(7*a^2-3*b^2)/b^2*(1/2*b^2/e/a^2/(a^2-b^2)*(cos(d*x+c)^2*e*sin(
d*x+c))^(1/2)/(-cos(d*x+c)^2*b^2+a^2)+1/4/a^2/(a^2-b^2)*(1-sin(d*x+c))^(1/
2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/
2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-5/8/(a^2-b^2)/b/(-a^2+b^2)^
(1/2)*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(cos(d*
x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c)
)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/4/a^2/(a^2-b^2)*b/(-a^2+b^
2)^(1/2)*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(cos
(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x
+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+5/8/(a^2-b^2)/b/(-a^2+b^2
)^(1/2)*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(cos(
d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d...
```

## Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
Timed out
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))**(3/2)/(a+b*cos(d*x+c))**3,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{(e\sin(dx+c))^{\frac{3}{2}}}{(b\cos(dx+c)+a)^3}\,dx$$

input
```
integrate((e*sin(d*x+c))^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate((e*sin(d*x + c))^(3/2)/(b*cos(d*x + c) + a)^3, x)
```

## Giac [F]

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{(e\sin(dx+c))^{\frac{3}{2}}}{(b\cos(dx+c)+a)^3}\,dx$$

input
```
integrate((e*sin(d*x+c))^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((e*sin(d*x + c))^(3/2)/(b*cos(d*x + c) + a)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{(e\,\sin(c+d\,x))^{3/2}}{(a+b\,\cos(c+d\,x))^3}\,dx$$

input
```
int((e*sin(c + d*x))^(3/2)/(a + b*cos(c + d*x))^3,x)
```

output
```
int((e*sin(c + d*x))^(3/2)/(a + b*cos(c + d*x))^3, x)
```

## Reduce [F]

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^3}\,dx = \sqrt{e}\left(\int \frac{\sqrt{\sin(dx+c)}\,\sin(dx+c)}{\cos(dx+c)^3 b^3+3\cos(dx+c)^2 a\,b^2+3\cos(dx+c)\,a^2b+a^3}dx\right)e$$

input
```
int((e*sin(d*x+c))^(3/2)/(a+b*cos(d*x+c))^3,x)
```

output
```
sqrt(e)*int((sqrt(sin(c + d*x))*sin(c + d*x))/(cos(c + d*x)**3*b**3 + 3*co
s(c + d*x)**2*a*b**2 + 3*cos(c + d*x)*a**2*b + a**3),x)*e
```

# 3.84 $\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^3}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 529

$$\begin{aligned}
&\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^3}\,dx \\
&= -\frac{(3a^2+2b^2)\sqrt{e}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8\sqrt{b}\,(-a^2+b^2)^{9/4}\,d}+\frac{(3a^2+2b^2)\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8\sqrt{b}\,(-a^2+b^2)^{9/4}\,d} \\
&\quad+\frac{a(3a^2+2b^2)\,e\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b\,(a^2-b^2)^2\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&\quad+\frac{a(3a^2+2b^2)\,e\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b\,(a^2-b^2)^2\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&\quad+\frac{5aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{e\sin(c+dx)}}{4\,(a^2-b^2)^2\,d\sqrt{\sin(c+dx)}} \\
&\quad-\frac{b(e\sin(c+dx))^{3/2}}{2\,(a^2-b^2)\,de(a+b\cos(c+dx))^2}-\frac{5ab(e\sin(c+dx))^{3/2}}{4\,(a^2-b^2)^2\,de(a+b\cos(c+dx))}
\end{aligned}$$

output
```
-1/8*(3*a^2+2*b^2)*e^(1/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^
(1/4)/e^(1/2))/b^(1/2)/(-a^2+b^2)^(9/4)/d+1/8*(3*a^2+2*b^2)*e^(1/2)*arctan
h(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(1/2)/(-a^2+b^2
)^(9/4)/d-1/8*a*(3*a^2+2*b^2)*e*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(
b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b/(a^2-b^2)^2/(b-(-a^2+b^2)^
(1/2))/d/(e*sin(d*x+c))^(1/2)-1/8*a*(3*a^2+2*b^2)*e*EllipticPi(cos(1/2*c+1
/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b/(a^2-b
^2)^2/(b+(-a^2+b^2)^(1/2))/d/(e*sin(d*x+c))^(1/2)-5/4*a*EllipticE(cos(1/2*
c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^2/d/sin(d*x+c)^(
1/2)-1/2*b*(e*sin(d*x+c))^(3/2)/(a^2-b^2)/d/e/(a+b*cos(d*x+c))^2-5/4*a*b*(
e*sin(d*x+c))^(3/2)/(a^2-b^2)^2/d/e/(a+b*cos(d*x+c))
```

## Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 15.79 (sec) , antiderivative size = 837, normalized size of antiderivative = 1.58

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^3}\,dx$$
$$= \frac{\sqrt{e\sin(c+dx)}\left(-\frac{b\sin(c+dx)}{2(a^2-b^2)(a+b\cos(c+dx))^2}-\frac{5ab\sin(c+dx)}{4(a^2-b^2)^2(a+b\cos(c+dx))}\right)}{d}$$
$$+ \frac{\sqrt{e\sin(c+dx)}\left(\frac{5a\cos^2(c+dx)\left(3\sqrt{2}a(a^2-b^2)^{3/4}\left(2\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-2\arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-\log\left(\sqrt{a^2-b^2}\cdots\right.\right.\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[Sqrt[e*Sin[c + d*x]]/(a + b*Cos[c + d*x])^3,x]
```

output
```
(Sqrt[e*Sin[c + d*x]]*(-1/2*(b*Sin[c + d*x])/((a^2 - b^2)*(a + b*Cos[c + d
*x])^2) - (5*a*b*Sin[c + d*x])/(4*(a^2 - b^2)^2*(a + b*Cos[c + d*x]))))/d
+ (Sqrt[e*Sin[c + d*x]]*((5*a*Cos[c + d*x]^2*(3*Sqrt[2]*a*(a^2 - b^2)^(3/4
)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] -
2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log
[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] +
b*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*
Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]]) + 8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7
/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2))
*(a + b*Sqrt[1 - Sin[c + d*x]^2]))/(12*Sqrt[b]*(-a^2 + b^2)*(a + b*Cos[c +
 d*x])*(1 - Sin[c + d*x]^2)) + (2*(8*a^2 + 2*b^2)*Cos[c + d*x]*(((1/8 + I/
8)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)]
- 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] -
Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x
]] + I*b*Sin[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^
2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1
/4)) + (a*AppellF1[3/4, 1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/
(-a^2 + b^2)]*Sin[c + d*x]^(3/2))/(3*(a^2 - b^2)))*(a + b*Sqrt[1 - Sin[c +
 d*x]^2]))/((a + b*Cos[c + d*x])*Sqrt[1 - Sin[c + d*x]^2])))/(8*(a - b)^2*
(a + b)^2*d*Sqrt[Sin[c + d*x]])
```

## Rubi [A] (warning: unable to verify)

Time = 2.23 (sec) , antiderivative size = 477, normalized size of antiderivative = 0.90, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.840$, Rules used = {3042, 3173, 27, 3042, 3343, 27, 3042, 3346, 3042, 3121, 3042, 3119, 3180, 266, 827, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3173}$$

$$-\frac{\int -\frac{(4a-b\cos(c+dx))\sqrt{e\sin(c+dx)}}{2(a+b\cos(c+dx))^2}dx}{2\left(a^2-b^2\right)}-\frac{b(e\sin(c+dx))^{3/2}}{2de\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{(4a-b\cos(c+dx))\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^2}dx}{4\left(a^2-b^2\right)}-\frac{b(e\sin(c+dx))^{3/2}}{2de\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{\sqrt{-e\cos\left(c+dx+\frac{\pi}{2}\right)}\left(4a-b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4\left(a^2-b^2\right)}-\frac{b(e\sin(c+dx))^{3/2}}{2de\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{ 3343}$$

$$\frac{-\frac{\int -\frac{\left(8a^2+5b\cos(c+dx)a+2b^2\right)\sqrt{e\sin(c+dx)}}{2(a+b\cos(c+dx))}dx}{a^2-b^2}-\frac{5ab(e\sin(c+dx))^{3/2}}{de(a^2-b^2)(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b(e\sin(c+dx))^{3/2}}{2de\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\int \frac{\left(2\left(4a^2+b^2\right)+5ab\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx}{2(a^2-b^2)}-\frac{5ab(e\sin(c+dx))^{3/2}}{de(a^2-b^2)(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b(e\sin(c+dx))^{3/2}}{2de\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{\int \frac{\sqrt{-e\cos\left(c+dx+\frac{\pi}{2}\right)}\left(2\left(4a^2+b^2\right)+5ab\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2(a^2-b^2)}-\frac{5ab(e\sin(c+dx))^{3/2}}{de(a^2-b^2)(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b(e\sin(c+dx))^{3/2}}{2de\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{ 3346}$$

$$\frac{\frac{(3a^2+2b^2)\int \frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx+5a\int \sqrt{e\sin(c+dx)}dx}{2(a^2-b^2)}-\frac{5ab(e\sin(c+dx))^{3/2}}{de(a^2-b^2)(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b(e\sin(c+dx))^{3/2}}{2de\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{(3a^2+2b^2)\int \frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx+5a\int \sqrt{e\sin(c+dx)}dx}{2(a^2-b^2)}-\frac{5ab(e\sin(c+dx))^{3/2}}{de(a^2-b^2)(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b(e\sin(c+dx))^{3/2}}{2de\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3121$$

$$\frac{\frac{(3a^2+2b^2)\int \frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx+\frac{5a\sqrt{e\sin(c+dx)}\int \sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}}{2(a^2-b^2)}-\frac{5ab(e\sin(c+dx))^{3/2}}{de(a^2-b^2)(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b(e\sin(c+dx))^{3/2}}{2de\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{(3a^2+2b^2)\int \frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx+\frac{5a\sqrt{e\sin(c+dx)}\int \sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}}{2(a^2-b^2)}-\frac{5ab(e\sin(c+dx))^{3/2}}{de(a^2-b^2)(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b(e\sin(c+dx))^{3/2}}{2de\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3119$$

$$\frac{\frac{(3a^2+2b^2)\int \frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx+\frac{10aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}}{2(a^2-b^2)}-\frac{5ab(e\sin(c+dx))^{3/2}}{de(a^2-b^2)(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b(e\sin(c+dx))^{3/2}}{2de\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3180$$

$$\frac{\frac{(3a^2+2b^2)\left(-\frac{be\int \frac{\sqrt{e\sin(c+dx)}}{b^2\sin^2(c+dx)e^2+\left(a^2-b^2\right)e^2}d(e\sin(c+dx))}{d}-\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}\cdots}{2b}\cdots\right.}{2(a^2-b^2)}\cdots}{4\left(a^2-b^2\right)}$$

$$\frac{b(e\sin(c+dx))^{3/2}}{2de\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 266$$

$$\frac{\frac{(3a^2+2b^2)\left(-\frac{2be\int \frac{e^2\sin^2(c+dx)}{b^2e^4\sin^4(c+dx)+\left(a^2-b^2\right)e^2}d\sqrt{e\sin(c+dx)}}{d}-\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}\cdots}{2b}\cdots\right.}{2(a^2-b^2)}\cdots}{4\left(a^2-b^2\right)}$$

$$\frac{b(e\sin(c+dx))^{3/2}}{2de\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 827$$

$$\frac{\frac{(3a^2+2b^2)\left(-\frac{2be\left(\frac{\int \frac{1}{be^2\sin^2(c+dx)+\sqrt{b^2-a^2}e}d\sqrt{e\sin(c+dx)}}{2b}-\frac{\int \frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2b}\right)}{d}-\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}\cdots}{2b}\cdots\right.}{2(a^2-b^2)}\cdots}{4\left(a^2-b^2\right)}$$

$$\frac{b(e\sin(c+dx))^{3/2}}{2de\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{(3a^2+2b^2)\left(-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\int \frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2b}\right)}{d}-\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int \cdots}{\sqrt{e\sin(\cdots}}\cdots\right.}{2(a^2-b^2)}\cdots}{4\left(a^2-b^2\right)}$$

$$\frac{b(e\sin(c+dx))^{3/2}}{2de\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 221$$

$$\frac{(3a^2+2b^2)\left(-\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b}-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\text{arctanh}\left(\cdots\right.}{2b^{3/2}\sqrt{\cdots}}\right.}{d}\cdots\right.}{2(a^2-b^2)}\cdots$$

$$4\left(a^2-b^2\right)$$

$$\frac{b(e\sin(c+dx))^{3/2}}{2de\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\big\downarrow$ 3042

$$\frac{(3a^2+2b^2)\left(-\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b}-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\text{arctanh}\left(\cdots\right.}{2b^{3/2}\sqrt{\cdots}}\right.}{d}\cdots\right.}{2(a^2-b^2)}\cdots$$

$$4\left(a^2-b^2\right)$$

$$\frac{b(e\sin(c+dx))^{3/2}}{2de\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\big\downarrow$ 3286

$$\frac{(3a^2+2b^2)\left(-\frac{ae\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b\sqrt{e\sin(c+dx)}}+\frac{ae\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b\sqrt{e\sin(c+dx)}}-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+\cdots}{\sqrt[4]{b^2-\cdots}}\right.}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-\cdots}}\right.}{\cdots}\cdots\right.}{2(a^2-b^2)}\cdots$$

$$4\left(a^2-b^2\right)$$

$$\frac{b(e\sin(c+dx))^{3/2}}{2de\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\big\downarrow$ 3042

$$\frac{(3a^2+2b^2)\left(-\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b\sqrt{e\sin(c+dx)}}+\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b\sqrt{e\sin(c+dx)}}-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+}{\sqrt[4]{b^2-}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-}}\right.}{}\right.}{2(a^2-b^2)}$$

$$4\left(a^2-b^2\right)$$

$$\frac{b(e\sin(c+dx))^{3/2}}{2de\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3284$$

$$\frac{(3a^2+2b^2)\left(-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{d}+\frac{ae\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\sin(c+dx)}}+\frac{ae\sqrt{\sin(c+dx}}{b}\right.}{2(a^2-b^2)}$$

$$4\left(a^2-b^2\right)$$

$$\frac{b(e\sin(c+dx))^{3/2}}{2de\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

input

```
Int[Sqrt[e*Sin[c + d*x]]/(a + b*Cos[c + d*x])^3,x]
```

output
```
-1/2*(b*(e*Sin[c + d*x])^(3/2))/((a^2 - b^2)*d*e*(a + b*Cos[c + d*x])^2) +
 ((-5*a*b*(e*Sin[c + d*x])^(3/2))/((a^2 - b^2)*d*e*(a + b*Cos[c + d*x])) +
 ((10*a*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(d*Sqrt[Sin
[c + d*x]]) + (3*a^2 + 2*b^2)*((-2*b*e*(ArcTan[(Sqrt[b]*Sqrt[e]*Sin[c + d*
x])/(-a^2 + b^2)^(1/4)]/(2*b^(3/2)*(-a^2 + b^2)^(1/4)*Sqrt[e]) - ArcTanh[(
Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/(2*b^(3/2)*(-a^2 + b^2)^
(1/4)*Sqrt[e])))/d + (a*e*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi
/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(b*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin
[c + d*x]]) + (a*e*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*
x)/2, 2]*Sqrt[Sin[c + d*x]])/(b*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*
x]])))/(2*(a^2 - b^2)))/(4*(a^2 - b^2))
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3173
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Simp[1/((a^2 - b^2)*(m + 1))
   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m +
p + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

rule 3180
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[a*(g/(2*b))   Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Simp[a*(g/(2*b))   In
t[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Simp[b*(g/f)   Su
bst[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3343
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Simp[1/((a^2 - b^2)*(m + 1))   Int[(g*Cos[e + f*x])^p
*(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ
[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

## Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2364 vs. $2(463) = 926$.

Time = 3.89 (sec) , antiderivative size = 2365, normalized size of antiderivative = 4.47

| method | result | size |
|---|---|---|
| default | Expression too large to display | 2365 |

input
```
int((e*sin(d*x+c))^(1/2)/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)
```

output
```
(2*e^3*b*(-1/8/(a^4-2*a^2*b^2+b^4)*(e*sin(d*x+c))^(3/2)*(-3*cos(d*x+c)^2*a
^2*b^2-2*b^4*cos(d*x+c)^2+7*a^4-2*a^2*b^2)/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)
^2-1/64*(3*a^2+2*b^2)/(a^4-2*a^2*b^2+b^4)/e^2/b^2/(e^2*(a^2-b^2)/b^2)^(1/4
)*2^(1/2)*(ln((e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)
*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4
)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arctan(2^(1/2
)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2)/(e^2*
(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)))-(cos(d*x+c)^2*e*sin(d*x+c))
^(1/2)*e*a*(3/2*b^2/e/a^2/(a^2-b^2)*sin(d*x+c)*(cos(d*x+c)^2*e*sin(d*x+c))
^(1/2)/(-cos(d*x+c)^2*b^2+a^2)-3/2/a^2/(a^2-b^2)*(1-sin(d*x+c))^(1/2)*(2+2
*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*Elli
pticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))+3/4/a^2/(a^2-b^2)*(1-sin(d*x+c))^(
1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(
1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-9/8/(a^2-b^2)/b^2*(1-sin(
d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(
d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-
(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+3/4/a^2/(a^2-b^2)*(1-sin(d*x+c))^(1/2)*(2
+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1
-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)
/b),1/2*2^(1/2))-9/8/(a^2-b^2)/b^2*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c)...
```

## Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
Timed out
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))**(1/2)/(a+b*cos(d*x+c))**3,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{\sqrt{e\sin\left(dx+c\right)}}{\left(b\cos\left(dx+c\right)+a\right)^3}\,dx$$

input
```
integrate((e*sin(d*x+c))^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(e*sin(d*x + c))/(b*cos(d*x + c) + a)^3, x)
```

## Giac [F]

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{\sqrt{e\sin\left(dx+c\right)}}{\left(b\cos\left(dx+c\right)+a\right)^3}\,dx$$

input
```
integrate((e*sin(d*x+c))^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(sqrt(e*sin(d*x + c))/(b*cos(d*x + c) + a)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{\sqrt{e\,\sin\left(c+d\,x\right)}}{\left(a+b\,\cos\left(c+d\,x\right)\right)^3}\,dx$$

input
```
int((e*sin(c + d*x))^(1/2)/(a + b*cos(c + d*x))^3,x)
```

output
```
int((e*sin(c + d*x))^(1/2)/(a + b*cos(c + d*x))^3, x)
```

## Reduce [F]

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^3}\,dx = \sqrt{e}\left(\int \frac{\sqrt{\sin(dx+c)}}{\cos(dx+c)^3 b^3+3\cos(dx+c)^2 a b^2+3\cos(dx+c)a^2b+a^3}dx\right)$$

input
```
int((e*sin(d*x+c))^(1/2)/(a+b*cos(d*x+c))^3,x)
```

output
```
sqrt(e)*int(sqrt(sin(c + d*x))/(cos(c + d*x)**3*b**3 + 3*cos(c + d*x)**2*a
*b**2 + 3*cos(c + d*x)*a**2*b + a**3),x)
```

# 3.85 $\int \frac{1}{(a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 535

$$\begin{aligned}\int \frac{1}{(a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}}\,dx &= \frac{3\sqrt{b}(5a^2+2b^2)\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8\left(-a^2+b^2\right)^{11/4}d\sqrt{e}}+\frac{3\sqrt{b}(5a^2+2b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8\left(-a^2+b^2\right)^{11/4}d\sqrt{e}}\\
&\quad-\frac{7a\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{4\left(a^2-b^2\right)^2d\sqrt{e\sin(c+dx)}}\\
&\quad+\frac{3a(5a^2+2b^2)\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8\left(a^2-b^2\right)^2\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}}\\
&\quad+\frac{3a(5a^2+2b^2)\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8\left(a^2-b^2\right)^2\left(a^2-b\left(b+\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}}\\
&\quad-\frac{b\sqrt{e\sin(c+dx)}}{2\left(a^2-b^2\right)de(a+b\cos(c+dx))^2}-\frac{7ab\sqrt{e\sin(c+dx)}}{4\left(a^2-b^2\right)^2de(a+b\cos(c+dx))}\end{aligned}$$

output
```
3/8*b^(1/2)*(5*a^2+2*b^2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(
1/4)/e^(1/2))/(-a^2+b^2)^(11/4)/d/e^(1/2)+3/8*b^(1/2)*(5*a^2+2*b^2)*arctan
h(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a^2+b^2)^(11/4)
/d/e^(1/2)-7/4*a*InverseJacobiAM(1/2*c-1/4*Pi+1/2*d*x,2^(1/2))*sin(d*x+c)^
(1/2)/(a^2-b^2)^2/d/(e*sin(d*x+c))^(1/2)-3/8*a*(5*a^2+2*b^2)*EllipticPi(co
s(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)
/(a^2-b^2)^2/(a^2-b*(b-(-a^2+b^2)^(1/2)))/d/(e*sin(d*x+c))^(1/2)-3/8*a*(5*
a^2+2*b^2)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2
^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^2/(a^2-b*(b+(-a^2+b^2)^(1/2)))/d/(e*sin
(d*x+c))^(1/2)-1/2*b*(e*sin(d*x+c))^(1/2)/(a^2-b^2)/d/e/(a+b*cos(d*x+c))^2
-7/4*a*b*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^2/d/e/(a+b*cos(d*x+c))
```

## Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 13.92 (sec) , antiderivative size = 1226, normalized size of antiderivative = 2.29

$$\int \frac{1}{(a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}}\,dx = \text{Too large to display}$$

input
```
Integrate[1/((a + b*Cos[c + d*x])^3*Sqrt[e*Sin[c + d*x]]),x]
```

output
```
((-1/2*b/((a^2 - b^2)*(a + b*Cos[c + d*x])^2) - (7*a*b)/(4*(a^2 - b^2)^2*(
a + b*Cos[c + d*x])))*Sin[c + d*x])/(d*Sqrt[e*Sin[c + d*x]]) + (Sqrt[Sin[c
 + d*x]]*((-14*a*b*Cos[c + d*x]^2*(a + b*Sqrt[1 - Sin[c + d*x]^2])*((a*(-2
*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*Ar
cTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqr
t[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Si
n[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt
[Sin[c + d*x]] + b*Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)) +
 (5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c +
 d*x]^2)/(-a^2 + b^2)]*Sqrt[Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((-5*(
a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2
)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Sin[c + d*x]^2, (b^
2*Sin[c + d*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, S
in[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)])*Sin[c + d*x]^2)*(a^2 +
b^2*(-1 + Sin[c + d*x]^2)))))/((a + b*Cos[c + d*x])*(1 - Sin[c + d*x]^2))
+ (2*(8*a^2 + 6*b^2)*Cos[c + d*x]*(a + b*Sqrt[1 - Sin[c + d*x]^2])*(((-1/8
 + I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 +
 b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b
^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqr
t[Sin[c + d*x]] + I*b*Sin[c + d*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sq...
```

## Rubi [A] (warning: unable to verify)

Time = 2.24 (sec) , antiderivative size = 497, normalized size of antiderivative = 0.93, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.840$, Rules used = {3042, 3173, 27, 3042, 3343, 27, 3042, 3346, 3042, 3121, 3042, 3120, 3181, 266, 756, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3173}$$

$$-\frac{\int -\frac{4a-3b\cos(c+dx)}{2(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}}dx}{2\left(a^2-b^2\right)}-\frac{b\sqrt{e\sin(c+dx)}}{2de\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{4a-3b\cos(c+dx)}{(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}}dx}{4\left(a^2-b^2\right)}-\frac{b\sqrt{e\sin(c+dx)}}{2de\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{4a+3b\sin\left(c+dx-\frac{\pi}{2}\right)}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx}{4\left(a^2-b^2\right)}-\frac{b\sqrt{e\sin(c+dx)}}{2de\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{ 3343}$$

$$\frac{-\frac{\int -\frac{8a^2-7b\cos(c+dx)a+6b^2}{2(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{a^2-b^2}-\frac{7ab\sqrt{e\sin(c+dx)}}{de(a^2-b^2)(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b\sqrt{e\sin(c+dx)}}{2de\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\int \frac{2\left(4a^2+3b^2\right)-7ab\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{2(a^2-b^2)}-\frac{7ab\sqrt{e\sin(c+dx)}}{de(a^2-b^2)(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b\sqrt{e\sin(c+dx)}}{2de\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{\int \frac{2\left(4a^2+3b^2\right)+7ab\sin\left(c+dx-\frac{\pi}{2}\right)}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{2(a^2-b^2)}-\frac{7ab\sqrt{e\sin(c+dx)}}{de(a^2-b^2)(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b\sqrt{e\sin(c+dx)}}{2de\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{ 3346}$$

$$\frac{\frac{3(5a^2+2b^2)\int \frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx-7a\int \frac{1}{\sqrt{e\sin(c+dx)}}dx}{2(a^2-b^2)}-\frac{7ab\sqrt{e\sin(c+dx)}}{de(a^2-b^2)(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b\sqrt{e\sin(c+dx)}}{2de\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{3(5a^2+2b^2)\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx-7a\int \frac{1}{\sqrt{e\sin(c+dx)}}dx}{2(a^2-b^2)}-\frac{7ab\sqrt{e\sin(c+dx)}}{de(a^2-b^2)(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b\sqrt{e\sin(c+dx)}}{2de\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\downarrow$ 3121

$$\frac{\frac{3(5a^2+2b^2)\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx-\frac{7a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{\sqrt{e\sin(c+dx)}}}{2(a^2-b^2)}-\frac{7ab\sqrt{e\sin(c+dx)}}{de(a^2-b^2)(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b\sqrt{e\sin(c+dx)}}{2de\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\downarrow$ 3042

$$\frac{\frac{3(5a^2+2b^2)\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx-\frac{7a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{\sqrt{e\sin(c+dx)}}}{2(a^2-b^2)}-\frac{7ab\sqrt{e\sin(c+dx)}}{de(a^2-b^2)(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b\sqrt{e\sin(c+dx)}}{2de\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\downarrow$ 3120

$$\frac{\frac{3(5a^2+2b^2)\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx-\frac{14a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}}{2(a^2-b^2)}-\frac{7ab\sqrt{e\sin(c+dx)}}{de(a^2-b^2)(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b\sqrt{e\sin(c+dx)}}{2de\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\downarrow$ 3181

$$\frac{\frac{3(5a^2+2b^2)\left(-\frac{be\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b^2\sin^2(c+dx)e^2+\left(a^2-b^2\right)e^2\right)}d(e\sin(c+dx))}{d}-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+d\cdots\right.}}{2\sqrt{b^2-a^2}}\cdots\right.}{2(a^2-b^2)}\cdots}{4\left(a^2-b^2\right)}-\frac{b\sqrt{e\sin(c+dx)}}{2de\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\downarrow$ 266

$$\frac{\frac{3(5a^2+2b^2)\left(-\frac{2be\int\frac{1}{b^2e^4\sin^4(c+dx)+\left(a^2-b^2\right)e^2}d\sqrt{e\sin(c+dx)}}{d}-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}\cdots}{2\sqrt{b^2-a^2}}\cdots\right.}{2(a^2-b^2)}\cdots}{4\left(a^2-b^2\right)}-\frac{b\sqrt{e\sin(c+dx)}}{2de\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 756$$

$$\frac{3(5a^2+2b^2)\left(-\frac{2be\left(-\frac{\int \frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2e\sqrt{b^2-a^2}}-\frac{\int \frac{1}{be^2\sin^2(c+dx)+\sqrt{b^2-a^2}e}d\sqrt{e\sin(c+dx)}}{2e\sqrt{b^2-a^2}}\right)}{d}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}}{2\sqrt{b^2-a^2}}\right.}{2(a^2-b^2)} \qquad 4\left(a^2-b^2\right)$$

$$\frac{b\sqrt{e\sin(c+dx)}}{2de\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}$$

$$\Big\downarrow \; 218$$

$$\frac{3(5a^2+2b^2)\left(-\frac{2be\left(-\frac{\int \frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2e\sqrt{b^2-a^2}}-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}\right)}{d}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int \frac{}{\sqrt{e\sin}}}{}\right.}{2(a^2-b^2)} \qquad 4\left(a^2-b^2\right)$$

$$\frac{b\sqrt{e\sin(c+dx)}}{2de\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}$$

$$\Big\downarrow \; 221$$

$$\frac{3(5a^2+2b^2)\left(-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}}-\frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}-\frac{\text{arctanh}}{2\sqrt{b}e^{3}}\right.}{d}\right.}{2(a^2-b^2)} \qquad 4\left(a^2-b^2\right)$$

$$\frac{b\sqrt{e\sin(c+dx)}}{2de\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{3\left(5a^2+2b^2\right)\left(-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}}-\frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}-\frac{\text{arctanh}}{2\sqrt{b}e^{3}}\right.}{d}\right.}{2(a^2-b^2)}$$

$$4\left(a^2-b^2\right)$$

$$\frac{b\sqrt{e\sin(c+dx)}}{2de\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}$$

$\downarrow$ 3286

$$\frac{3\left(5a^2+2b^2\right)\left(-\frac{a\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}}-\frac{a\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}}-\frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c}{\sqrt[4]{b^2-}}\right.}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right.}\right.}{}\right.}{2(a^2-b^2)}$$

$$4\left(a^2-b^2\right)$$

$$\frac{b\sqrt{e\sin(c+dx)}}{2de\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}$$

$\downarrow$ 3042

$$\frac{3\left(5a^2+2b^2\right)\left(-\frac{a\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}}-\frac{a\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}}-\frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c}{\sqrt[4]{b^2-}}\right.}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right.}\right.}{}\right.}{2(a^2-b^2)}$$

$$4\left(a^2-b^2\right)$$

$$\frac{b\sqrt{e\sin(c+dx)}}{2de\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}$$

$\downarrow$ 3284

$$\frac{3(5a^2+2b^2)\left(-\frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}\right)}{d}+\frac{a\sqrt{\sin(c+dx)}\operatorname{EllipticPi}\left(\frac{2b}{b-\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{b^2-a^2}\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\sin(c+dx)}}-\frac{a\sqrt{\sin(c+dx)}\ldots}{d\sqrt{\ldots}}\right)}{2(a^2-b^2)}$$

$$4\left(a^2-b^2\right)$$

$$\frac{b\sqrt{e\sin(c+dx)}}{2de\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

input
```
Int[1/((a + b*Cos[c + d*x])^3*Sqrt[e*Sin[c + d*x]]),x]
```

output
```
-1/2*(b*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)*d*e*(a + b*Cos[c + d*x])^2) + (
(-7*a*b*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)*d*e*(a + b*Cos[c + d*x])) + ((-
14*a*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(d*Sqrt[e*Sin[c
+ d*x]]) + 3*(5*a^2 + 2*b^2)*((-2*b*e*(-1/2*ArcTan[(Sqrt[b]*Sqrt[e]*Sin[c
+ d*x])/(-a^2 + b^2)^(1/4)]/(Sqrt[b]*(-a^2 + b^2)^(3/4)*e^(3/2)) - ArcTanh
[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/(2*Sqrt[b]*(-a^2 + b^2
)^(3/4)*e^(3/2))))/d + (a*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi
/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(Sqrt[-a^2 + b^2]*(b - Sqrt[-a^2 + b^2
])*d*Sqrt[e*Sin[c + d*x]]) - (a*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (
c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(Sqrt[-a^2 + b^2]*(b + Sqrt[-a^2
 + b^2])*d*Sqrt[e*Sin[c + d*x]])))/(2*(a^2 - b^2)))/(4*(a^2 - b^2))
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3173
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Simp[1/((a^2 - b^2)*(m + 1))
   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m +
p + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

rule 3181
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[-a/(2*q)   Int[1/(
Sqrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Simp[b*(g/f)   Subst[
Int[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - S
imp[a/(2*q)   Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3343
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Simp[1/((a^2 - b^2)*(m + 1))   Int[(g*Cos[e + f*x])^p
*(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ
[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

## Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2219 vs. $2(468) = 936$.

Time = 3.27 (sec) , antiderivative size = 2220, normalized size of antiderivative = 4.15

| method | result | size |
|---|---|---|
| default | Expression too large to display | 2220 |

input

```
int(1/(a+cos(d*x+c)*b)^3/(e*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output

```
(2*b*e^3*(-1/8/(a^4-2*a^2*b^2+b^4)*(e*sin(d*x+c))^(1/2)*(-5*cos(d*x+c)^2*a
^2*b^2-2*b^4*cos(d*x+c)^2+9*a^4-2*a^2*b^2)/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)
^2-3/64*(5*a^2+2*b^2)/(a^4-2*a^2*b^2+b^4)/e^2*(e^2*(a^2-b^2)/b^2)^(1/4)/(a
^2*e^2-b^2*e^2)*2^(1/2)*(ln((e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin
(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)-(e^2*(a^2-
b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2
*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan
(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)))-(cos(d*x+c)^2
*e*sin(d*x+c))^(1/2)*a*(3/2*b^2/e/a^2/(a^2-b^2)*(cos(d*x+c)^2*e*sin(d*x+c)
)^(1/2)/(-cos(d*x+c)^2*b^2+a^2)+3/4/a^2/(a^2-b^2)*(1-sin(d*x+c))^(1/2)*(2+
2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*Ell
ipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-15/8/(a^2-b^2)/b/(-a^2+b^2)^(1/2)
*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^
2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/
2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+3/4/a^2/(a^2-b^2)*b/(-a^2+b^2)^(1
/2)*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+
c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^
(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+15/8/(a^2-b^2)/b/(-a^2+b^2)^(1
/2)*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+
c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c...
```

## Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))**3/(e*sin(d*x+c))**(1/2),x)
```

output
```
Timed out
```

## Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
Timed out
```

## Giac [F]

$$\int \frac{1}{(a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}}\,dx=\int \frac{1}{(b\cos(dx+c)+a)^3\sqrt{e\sin(dx+c)}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^3*sqrt(e*sin(d*x + c))), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}}\,dx=\int \frac{1}{\sqrt{e\,\sin(c+d\,x)}\,(a+b\,\cos(c+d\,x))^3}\,dx$$

input
```
int(1/((e*sin(c + d*x))^(1/2)*(a + b*cos(c + d*x))^3),x)
```

output
```
int(1/((e*sin(c + d*x))^(1/2)*(a + b*cos(c + d*x))^3), x)
```

## Reduce [F]

$$\int \frac{1}{(a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}}\,dx=\frac{\sqrt{e}\left(\int \frac{\sqrt{\sin(dx+c)}}{\cos(dx+c)^3\sin(dx+c)b^3+3\cos(dx+c)^2\sin(dx+c)a\,b^2+3\cos(dx+c)\sin(dx+c)a^2b+\sin(dx+c)a^3}dx\right)}{e}$$

input
```
int(1/(a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(1/2),x)
```

output
```
(sqrt(e)*int(sqrt(sin(c + d*x))/(cos(c + d*x)**3*sin(c + d*x)*b**3 + 3*cos
(c + d*x)**2*sin(c + d*x)*a*b**2 + 3*cos(c + d*x)*sin(c + d*x)*a**2*b + si
n(c + d*x)*a**3),x))/e
```

## 3.86 $\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}}\,dx$

## Optimal result

Integrand size = 25, antiderivative size = 611

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}}\,dx = -\frac{5b^{3/2}(7a^2+2b^2)\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8\left(-a^2+b^2\right)^{13/4}de^{3/2}} + \frac{5b^{3/2}(7a^2+2b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8\left(-a^2+b^2\right)^{13/4}de^{3/2}} - \frac{b}{2\left(a^2-b^2\right)de(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}} - \frac{9ab}{4\left(a^2-b^2\right)^2de(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}} + \frac{5b(7a^2+2b^2)-a(8a^2+37b^2)\cos(c+dx)}{4\left(a^2-b^2\right)^3de\sqrt{e\sin(c+dx)}} - \frac{5ab(7a^2+2b^2)\operatorname{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8\left(a^2-b^2\right)^3\left(b-\sqrt{-a^2+b^2}\right)de\sqrt{e\sin(c+dx)}} - \frac{5ab(7a^2+2b^2)\operatorname{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8\left(a^2-b^2\right)^3\left(b+\sqrt{-a^2+b^2}\right)de\sqrt{e\sin(c+dx)}} - \frac{a(8a^2+37b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{4\left(a^2-b^2\right)^3de^2\sqrt{\sin(c+dx)}}$$

output
```
-5/8*b^(3/2)*(7*a^2+2*b^2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^
(1/4)/e^(1/2))/(-a^2+b^2)^(13/4)/d/e^(3/2)+5/8*b^(3/2)*(7*a^2+2*b^2)*arcta
nh(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a^2+b^2)^(13/4
)/d/e^(3/2)-1/2*b/(a^2-b^2)/d/e/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(1/2)-9/
4*a*b/(a^2-b^2)^2/d/e/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(1/2)+1/4*(5*b*(7*a^
2+2*b^2)-a*(8*a^2+37*b^2)*cos(d*x+c))/(a^2-b^2)^3/d/e/(e*sin(d*x+c))^(1/2)
+5/8*a*b*(7*a^2+2*b^2)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b
^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^3/(b-(-a^2+b^2)^(1/2))/d/e/
(e*sin(d*x+c))^(1/2)+5/8*a*b*(7*a^2+2*b^2)*EllipticPi(cos(1/2*c+1/4*Pi+1/2
*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^3/(b+(-
a^2+b^2)^(1/2))/d/e/(e*sin(d*x+c))^(1/2)+1/4*a*(8*a^2+37*b^2)*EllipticE(co
s(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^3/d/e^2/si
n(d*x+c)^(1/2)
```

## Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 7.25 (sec) , antiderivative size = 922, normalized size of antiderivative = 1.51

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[1/((a + b*Cos[c + d*x])^3*(e*Sin[c + d*x])^(3/2)),x]
```

output
```
(Sin[c + d*x]^2*((-2*(-3*a^2*b - b^3 + a^3*Cos[c + d*x] + 3*a*b^2*Cos[c +
d*x])*Csc[c + d*x])/(a^2 - b^2)^3 + (b^3*Sin[c + d*x])/(2*(a^2 - b^2)^2*(a
 + b*Cos[c + d*x])^2) + (13*a*b^3*Sin[c + d*x])/(4*(a^2 - b^2)^3*(a + b*Co
s[c + d*x]))))/(d*(e*Sin[c + d*x])^(3/2)) - (Sin[c + d*x]^(3/2)*(((8*a^3*b
 + 37*a*b^3)*Cos[c + d*x]^2*(3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (
Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqr
t[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2]
- Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]] +
 Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]
] + b*Sin[c + d*x]]) + 8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin[c + d*x]^
2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2))*(a + b*Sqrt[1 -
Sin[c + d*x]^2]))/(12*b^(3/2)*(-a^2 + b^2)*(a + b*Cos[c + d*x])*(1 - Sin[c
 + d*x]^2)) + (2*(8*a^4 + 72*a^2*b^2 + 10*b^4)*Cos[c + d*x]*(((1/8 + I/8)*
(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2
*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log
[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]]
+ I*b*Sin[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^
(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)
) + (a*AppellF1[3/4, 1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a
^2 + b^2)]*Sin[c + d*x]^(3/2))/(3*(a^2 - b^2)))*(a + b*Sqrt[1 - Sin[c +...
```

## Rubi [A] (warning: unable to verify)

Time = 2.88 (sec) , antiderivative size = 575, normalized size of antiderivative = 0.94, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.960$, Rules used = {3042, 3173, 27, 3042, 3343, 27, 3042, 3345, 27, 3042, 3346, 3042, 3121, 3042, 3119, 3180, 266, 827, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3173}$$

$$-\frac{\int -\frac{4a-5b\cos(c+dx)}{2(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}}dx}{2\left(a^2-b^2\right)}-\frac{b}{2de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{4a-5b\cos(c+dx)}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}}dx}{4\left(a^2-b^2\right)}-\frac{b}{2de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{4a+5b\sin\left(c+dx-\frac{\pi}{2}\right)}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx}{4\left(a^2-b^2\right)}-\frac{b}{2de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{ 3343}$$

$$\frac{-\frac{\int -\frac{8a^2-27b\cos(c+dx)a+10b^2}{2(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}dx}{a^2-b^2}-\frac{9ab}{de(a^2-b^2)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b}{2de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\int \frac{2\left(4a^2+5b^2\right)-27ab\cos(c+dx)}{(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}dx}{2(a^2-b^2)}-\frac{9ab}{de(a^2-b^2)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b}{2de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{\int \frac{2\left(4a^2+5b^2\right)+27ab\sin\left(c+dx-\frac{\pi}{2}\right)}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{2(a^2-b^2)}-\frac{9ab}{de(a^2-b^2)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b}{2de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{ 3345}$$

$$\frac{\frac{\frac{2\left(5b\left(7a^2+2b^2\right)-a\left(8a^2+37b^2\right)\cos(c+dx)\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{2\int \frac{\left(8a^4+72b^2a^2+b\left(8a^2+37b^2\right)\cos(c+dx)a+10b^4\right)\sqrt{e\sin(c+dx)}}{2(a+b\cos(c+dx))}dx}{e^2\left(a^2-b^2\right)}}{2(a^2-b^2)}-\frac{9ab}{de(a^2-b^2)\sqrt{e\sin(c+dx)}(a+b\cos(c+}}{4\left(a^2-b^2\right)}-\frac{b}{2de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}$$

$\downarrow$ 27

$$\frac{\frac{\frac{2\left(5b\left(7a^2+2b^2\right)-a\left(8a^2+37b^2\right)\cos(c+dx)\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{\int\frac{\left(2\left(4a^4+36b^2a^2+5b^4\right)+ab\left(8a^2+37b^2\right)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx}{e^2\left(a^2-b^2\right)}}{2(a^2-b^2)}-\frac{9ab}{de(a^2-b^2)\sqrt{e\sin(c+dx)}(a+b\cos(c\ldots}}{4\left(a^2-b^2\right)}$$
$$\frac{b}{2de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{2\left(5b\left(7a^2+2b^2\right)-a\left(8a^2+37b^2\right)\cos(c+dx)\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{\int\frac{\sqrt{-e\cos\left(c+dx+\frac{\pi}{2}\right)}\left(2\left(4a^4+36b^2a^2+5b^4\right)+ab\left(8a^2+37b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{e^2\left(a^2-b^2\right)}}{2(a^2-b^2)}-\frac{9ab}{de(a^2-b^2)\sqrt{e\sin(c+dx)}(\ldots}}{4\left(a^2-b^2\right)}$$
$$\frac{b}{2de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}$$

$\downarrow$ 3346

$$\frac{\frac{\frac{2\left(5b\left(7a^2+2b^2\right)-a\left(8a^2+37b^2\right)\cos(c+dx)\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{a\left(8a^2+37b^2\right)\int\sqrt{e\sin(c+dx)}dx+5b^2\left(7a^2+2b^2\right)\int\frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx}{e^2\left(a^2-b^2\right)}}{2(a^2-b^2)}-\frac{9ab}{de(a^2-b^2)\sqrt{e\sin(c+dx)}(a+b\cos(c\ldots}}{4\left(a^2-b^2\right)}$$
$$\frac{b}{2de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{2\left(5b\left(7a^2+2b^2\right)-a\left(8a^2+37b^2\right)\cos(c+dx)\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{a\left(8a^2+37b^2\right)\int\sqrt{e\sin(c+dx)}dx+5b^2\left(7a^2+2b^2\right)\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{e^2\left(a^2-b^2\right)}}{2(a^2-b^2)}-\frac{9ab}{de(a^2-b^2)\sqrt{e\sin(c+dx)}(a+b\,\mathrm{co}\ldots}}{4\left(a^2-b^2\right)}$$
$$\frac{b}{2de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}$$

$\downarrow$ 3121

$$\frac{\frac{\frac{2\left(5b\left(7a^2+2b^2\right)-a\left(8a^2+37b^2\right)\cos(c+dx)\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{\frac{a\left(8a^2+37b^2\right)\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}+5b^2\left(7a^2+2b^2\right)\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{e^2\left(a^2-b^2\right)}}{2(a^2-b^2)}-\frac{9a\cdots}{de(a^2-b^2)\sqrt{e\sin(c+\cdots}}}{4\left(a^2-b^2\right)}$$

$$\frac{b}{2de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{2\left(5b\left(7a^2+2b^2\right)-a\left(8a^2+37b^2\right)\cos(c+dx)\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{\frac{a\left(8a^2+37b^2\right)\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}+5b^2\left(7a^2+2b^2\right)\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{e^2\left(a^2-b^2\right)}}{2(a^2-b^2)}-\frac{9a\cdots}{de(a^2-b^2)\sqrt{e\sin(c+\cdots}}}{4\left(a^2-b^2\right)}$$

$$\frac{b}{2de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}$$

$\downarrow$ 3119

$$\frac{\frac{\frac{2\left(5b\left(7a^2+2b^2\right)-a\left(8a^2+37b^2\right)\cos(c+dx)\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{5b^2\left(7a^2+2b^2\right)\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx+\frac{2a\left(8a^2+37b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}}{e^2\left(a^2-b^2\right)}}{2(a^2-b^2)}-\frac{\cdots}{de(a^2-b^2)\sqrt{e\sin\cdots}}}{4\left(a^2-b^2\right)}$$

$$\frac{b}{2de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}$$

$\downarrow$ 3180

$$\frac{\frac{\frac{2\left(5b\left(7a^2+2b^2\right)-a\left(8a^2+37b^2\right)\cos(c+dx)\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{5b^2\left(7a^2+2b^2\right)\left(-\frac{be\int\frac{\sqrt{e\sin(c+dx)}}{b^2\sin^2(c+dx)e^2+\left(a^2-b^2\right)e^2}d(e\sin(c+dx))}{d}-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}\cdots}{2b}\cdots\right.}{e^2\left(a^2-b^2\right)\cdots}}{2(a^2-b^2)\cdots}}{4\left(a^2-\cdots\right.}$$

$$\frac{b}{2de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}$$

$\downarrow$ 266

$$\frac{\frac{2\left(5b\left(7a^2+2b^2\right)-a\left(8a^2+37b^2\right)\cos(c+dx)\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{5b^2\left(7a^2+2b^2\right)\left(-\frac{2be\int\frac{e^2\sin^2(c+dx)}{b^2e^4\sin^4(c+dx)+\left(a^2-b^2\right)e^2}d\sqrt{e\sin(c+dx)}}{d}-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}}{2b}\right.}{e^2\left(a^2-b^2\right.}}{2(a^2-b^2)}}{4\left(a^2-\right.}$$

$$\frac{b}{2de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}$$

$\Big\downarrow$ 827

$$\frac{\frac{2\left(5b\left(7a^2+2b^2\right)-a\left(8a^2+37b^2\right)\cos(c+dx)\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{5b^2\left(7a^2+2b^2\right)\left(-\frac{2be\left(\frac{\int\frac{1}{be^2\sin^2(c+dx)+\sqrt{b^2-a^2}e}d\sqrt{e\sin(c+dx)}}{2b}-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2b}\right.}{d}\right.}{}}{2(a^2-b^2}$$

$$\frac{b}{2de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}$$

$\Big\downarrow$ 218

$$\frac{\frac{2\left(5b\left(7a^2+2b^2\right)-a\left(8a^2+37b^2\right)\cos(c+dx)\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{5b^2\left(7a^2+2b^2\right)\left(-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2b}\right)}{d}-\frac{ae\int}{\sqrt{e\sin(}}\right.}{}}{2(a^2-b^2)}$$

$$\frac{b}{2de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}$$

$\Big\downarrow$ 221

$$\frac{\frac{2\left(5b\left(7a^2+2b^2\right)-a\left(8a^2+37b^2\right)\cos(c+dx)\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{5b^2\left(7a^2+2b^2\right)\left(-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b}-\cdots\right)}{e\cdots}}{2(a^2-b^2)}$$

$$\frac{b}{2de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}$$

$\downarrow$ 3042

$$\frac{\frac{2\left(5b\left(7a^2+2b^2\right)-a\left(8a^2+37b^2\right)\cos(c+dx)\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{5b^2\left(7a^2+2b^2\right)\left(-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b}-\cdots\right)}{e\cdots}}{2(a^2-b^2)}$$

$$\frac{b}{2de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}$$

$\downarrow$ 3286

$$\frac{\frac{2\left(5b\left(7a^2+2b^2\right)-a\left(8a^2+37b^2\right)\cos(c+dx)\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{5b^2\left(7a^2+2b^2\right)\left(-\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b\sqrt{e\sin(c+dx)}}+\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c\cdots\right.}}{2b\sqrt{e\sin(c+dx)}}\cdots\right.}{\cdots}}{2(a^2-b^2)\cdots}$$

$$\frac{b}{2de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}$$

$\downarrow$ 3042

$$\frac{\frac{2\left(5b\left(7a^2+2b^2\right)-a\left(8a^2+37b^2\right)\cos(c+dx)\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{5b^2\left(7a^2+2b^2\right)\left(-\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b\sqrt{e\sin(c+dx)}}+\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c\cdots\right.}}{2b\sqrt{e\sin(c+dx)}}\cdots\right.}{\cdots}}{2(a^2-b^2)\cdots}$$

$$\frac{b}{2de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}$$

$\downarrow$ 3284

$$\frac{\frac{2\left(5b\left(7a^2+2b^2\right)-a\left(8a^2+37b^2\right)\cos(c+dx)\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{5b^2\left(7a^2+2b^2\right)\left(-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{d}+\frac{ae\sqrt{\sin(c+dx)}\,\text{EllipticPi}}{bd\left(b-\sqrt{b^2\right.}}\right.}{}}{2(a^2-b^2)}$$

$$\frac{b}{2de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}$$

input
```
Int[1/((a + b*Cos[c + d*x])^3*(e*Sin[c + d*x])^(3/2)),x]
```

output
```
-1/2*b/((a^2 - b^2)*d*e*(a + b*Cos[c + d*x])^2*Sqrt[e*Sin[c + d*x]]) + ((-
9*a*b)/((a^2 - b^2)*d*e*(a + b*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]]) + ((2*(
5*b*(7*a^2 + 2*b^2) - a*(8*a^2 + 37*b^2)*Cos[c + d*x]))/((a^2 - b^2)*d*e*S
qrt[e*Sin[c + d*x]]) - ((2*a*(8*a^2 + 37*b^2)*EllipticE[(c - Pi/2 + d*x)/2
, 2]*Sqrt[e*Sin[c + d*x]])/(d*Sqrt[Sin[c + d*x]]) + 5*b^2*(7*a^2 + 2*b^2)*
((-2*b*e*(ArcTan[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/(2*b^(
3/2)*(-a^2 + b^2)^(1/4)*Sqrt[e]) - ArcTanh[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/
(-a^2 + b^2)^(1/4)]/(2*b^(3/2)*(-a^2 + b^2)^(1/4)*Sqrt[e])))/d + (a*e*Elli
pticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d
*x]])/(b*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]]) + (a*e*EllipticPi[
(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(
b*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]])))/((a^2 - b^2)*e^2))/(2*(
a^2 - b^2)))/(4*(a^2 - b^2))
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3173
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Simp[1/((a^2 - b^2)*(m + 1))
   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m +
p + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

rule 3180
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[a*(g/(2*b))   Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Simp[a*(g/(2*b))   In
t[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Simp[b*(g/f)   Su
bst[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3343
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Simp[1/((a^2 - b^2)*(m + 1))   Int[(g*Cos[e + f*x])^p
*(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ
[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 3345
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Simp[1/(g^2*(a^2 - b^2)*(p +
 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
 2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[p, -1] && IntegerQ[2*m]
```

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

## Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2835 vs. $2(541) = 1082$.

Time = 4.20 (sec) , antiderivative size = 2836, normalized size of antiderivative = 4.64

| method | result | size |
|---|---|---|
| default | Expression too large to display | 2836 |

input
```
int(1/(a+cos(d*x+c)*b)^3/(e*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(2*e^3*b*(b^2/e^4/(a-b)^3/(a+b)^3*(1/8*(e*sin(d*x+c))^(3/2)*e^2*(-11*cos(d
*x+c)^2*a^2*b^2-2*b^4*cos(d*x+c)^2+15*a^4-2*a^2*b^2)/(-b^2*cos(d*x+c)^2*e^
2+a^2*e^2)^2+1/8*(35/8*a^2+5/4*b^2)/b^2/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*
(ln((e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(
e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d
*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arctan(2^(1/2)/(e^2*(a^
2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/
b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)))-(-3*a^2-b^2)/e^4/(a^2-b^2)^3/(e*sin(d
*x+c))^(1/2))-(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/e*a*(-(a^2+3*b^2)/(a^2-b^2
)^3*(2*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*Ellipt
icE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c)
)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-2*cos
(d*x+c)^2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)+b^2*(a^2+3*b^2)/(a-b)^2/(a+b)
^2*(1/2*b^2/e/a^2/(a^2-b^2)*sin(d*x+c)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(
-cos(d*x+c)^2*b^2+a^2)-1/2/a^2/(a^2-b^2)*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x
+c))^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticE((1
-sin(d*x+c))^(1/2),1/2*2^(1/2))+1/4/a^2/(a^2-b^2)*(1-sin(d*x+c))^(1/2)*(2+
2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*Ell
ipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-3/8/(a^2-b^2)/b^2*(1-sin(d*x+c))^
(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c...
```

## Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))**3/(e*sin(d*x+c))**(3/2),x)
```

output
```
Timed out
```

## Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

## Giac [F]

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\cos(dx+c)+a)^3(e\sin(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^3*(e*sin(d*x + c))^(3/2)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}}\,dx = \int \frac{1}{(e\,\sin(c+d\,x))^{3/2}\,(a+b\,\cos(c+d\,x))^3}\,dx$$

input
```
int(1/((e*sin(c + d*x))^(3/2)*(a + b*cos(c + d*x))^3),x)
```

output
```
int(1/((e*sin(c + d*x))^(3/2)*(a + b*cos(c + d*x))^3), x)
```

## Reduce [F]

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}}\,dx = \frac{\sqrt{e}\left(\int \frac{\sqrt{\sin(dx+c)}}{\cos(dx+c)^3\sin(dx+c)^2b^3+3\cos(dx+c)^2\sin(dx+c)^2ab^2+3\cos(dx+c)\sin}\right.}{e^2}$$

input
```
int(1/(a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(3/2),x)
```

output
```
(sqrt(e)*int(sqrt(sin(c + d*x))/(cos(c + d*x)**3*sin(c + d*x)**2*b**3 + 3*
cos(c + d*x)**2*sin(c + d*x)**2*a*b**2 + 3*cos(c + d*x)*sin(c + d*x)**2*a*
*2*b + sin(c + d*x)**2*a**3),x))/e**2
```

# 3.87 $\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 629

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}}\,dx = \frac{7b^{5/2}(9a^2+2b^2)\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8\left(-a^2+b^2\right)^{15/4}de^{5/2}}$$
$$+\frac{7b^{5/2}(9a^2+2b^2)\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8\left(-a^2+b^2\right)^{15/4}de^{5/2}}$$
$$-\frac{b}{2\left(a^2-b^2\right)de(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}}$$
$$-\frac{11ab}{4\left(a^2-b^2\right)^2de(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}$$
$$+\frac{7b(9a^2+2b^2)-a(8a^2+69b^2)\cos(c+dx)}{12\left(a^2-b^2\right)^3de(e\sin(c+dx))^{3/2}}$$
$$+\frac{a(8a^2+69b^2)\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{12\left(a^2-b^2\right)^3de^2\sqrt{e\sin(c+dx)}}$$
$$-\frac{7ab^2(9a^2+2b^2)\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8\left(a^2-b^2\right)^3\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)de^2\sqrt{e\sin(c+dx)}}$$
$$-\frac{7ab^2(9a^2+2b^2)\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8\left(a^2-b^2\right)^3\left(a^2-b\left(b+\sqrt{-a^2+b^2}\right)\right)de^2\sqrt{e\sin(c+dx)}}$$

output
```
7/8*b^(5/2)*(9*a^2+2*b^2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(
1/4)/e^(1/2))/(-a^2+b^2)^(15/4)/d/e^(5/2)+7/8*b^(5/2)*(9*a^2+2*b^2)*arctan
h(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a^2+b^2)^(15/4)
/d/e^(5/2)-1/2*b/(a^2-b^2)/d/e/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(3/2)-11/
4*a*b/(a^2-b^2)^2/d/e/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(3/2)+1/12*(7*b*(9*a
^2+2*b^2)-a*(8*a^2+69*b^2)*cos(d*x+c))/(a^2-b^2)^3/d/e/(e*sin(d*x+c))^(3/2
)+1/12*a*(8*a^2+69*b^2)*InverseJacobiAM(1/2*c-1/4*Pi+1/2*d*x,2^(1/2))*sin(
d*x+c)^(1/2)/(a^2-b^2)^3/d/e^2/(e*sin(d*x+c))^(1/2)+7/8*a*b^2*(9*a^2+2*b^2
)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*s
in(d*x+c)^(1/2)/(a^2-b^2)^3/(a^2-b*(b-(-a^2+b^2)^(1/2)))/d/e^2/(e*sin(d*x+
c))^(1/2)+7/8*a*b^2*(9*a^2+2*b^2)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b
/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^3/(a^2-b*(b+(-a^
2+b^2)^(1/2)))/d/e^2/(e*sin(d*x+c))^(1/2)
```

## Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 16.61 (sec) , antiderivative size = 1308, normalized size of antiderivative = 2.08

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[1/((a + b*Cos[c + d*x])^3*(e*Sin[c + d*x])^(5/2)),x]
```

output
```
((b^3/(2*(a^2 - b^2)^2*(a + b*Cos[c + d*x])^2) + (15*a*b^3)/(4*(a^2 - b^2)
^3*(a + b*Cos[c + d*x])) - (2*(-3*a^2*b - b^3 + a^3*Cos[c + d*x] + 3*a*b^2
*Cos[c + d*x])*Csc[c + d*x]^2)/(3*(a^2 - b^2)^3))*Sin[c + d*x]^3)/(d*(e*Si
n[c + d*x])^(5/2)) + (Sin[c + d*x]^(5/2)*((2*(8*a^3*b + 69*a*b^3)*Cos[c +
d*x]^2*(a + b*Sqrt[1 - Sin[c + d*x]^2])*((a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b
]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*S
qrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt
[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]] + Log[Sqrt[a^2
- b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c +
d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)) + (5*b*(a^2 - b^2)*AppellF1[
1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sqrt
[Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -1
/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*
AppellF1[5/4, -1/2, 2, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b
^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (b^2*Sin[c +
 d*x]^2)/(-a^2 + b^2)])*Sin[c + d*x]^2)*(a^2 + b^2*(-1 + Sin[c + d*x]^2)))
))/((a + b*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) + (2*(8*a^4 - 120*a^2*b^2 -
 42*b^4)*Cos[c + d*x]*(a + b*Sqrt[1 - Sin[c + d*x]^2])*(((-1/8 + I/8)*Sqrt
[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)]
 - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)...
```

## Rubi [A] (warning: unable to verify)

Time = 2.90 (sec) , antiderivative size = 598, normalized size of antiderivative = 0.95, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.960$, Rules used = {3042, 3173, 27, 3042, 3343, 27, 3042, 3345, 27, 3042, 3346, 3042, 3121, 3042, 3120, 3181, 266, 756, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{ 3173}$$

$$-\frac{\int -\frac{4a-7b\cos(c+dx)}{2(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}}dx}{2\left(a^2-b^2\right)}-\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{4a-7b\cos(c+dx)}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}}dx}{4\left(a^2-b^2\right)}-\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{4a+7b\sin\left(c+dx-\frac{\pi}{2}\right)}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx}{4\left(a^2-b^2\right)}-\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3343}$$

$$\frac{-\frac{\int -\frac{8a^2-55b\cos(c+dx)a+14b^2}{2(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}dx}{a^2-b^2}-\frac{11ab}{de(a^2-b^2)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{2\left(4a^2+7b^2\right)-55ab\cos(c+dx)}{(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}dx}{2(a^2-b^2)}-\frac{11ab}{de(a^2-b^2)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{2\left(4a^2+7b^2\right)+55ab\sin\left(c+dx-\frac{\pi}{2}\right)}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{2(a^2-b^2)}-\frac{11ab}{de(a^2-b^2)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3345}$$

$$\frac{\frac{\frac{2\left(7b\left(9a^2+2b^2\right)-a\left(8a^2+69b^2\right)\cos(c+dx)\right)}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}-\frac{2\int -\frac{8a^4-120b^2a^2+b\left(8a^2+69b^2\right)\cos(c+dx)a-42b^4}{2(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{3e^2\left(a^2-b^2\right)}}{2(a^2-b^2)}-\frac{11ab}{de(a^2-b^2)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{\int \frac{2\left(4a^4-60b^2a^2-21b^4\right)+ab\left(8a^2+69b^2\right)\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{3e^2\left(a^2-b^2\right)}+\frac{2\left(7b\left(9a^2+2b^2\right)-a\left(8a^2+69b^2\right)\cos(c+dx)\right)}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}}{2(a^2-b^2)}-\frac{11ab}{de(a^2-b^2)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{\int \frac{2\left(4a^4-60b^2a^2-21b^4\right)-ab\left(8a^2+69b^2\right)\sin\left(c+dx-\frac{\pi}{2}\right)}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{3e^2\left(a^2-b^2\right)}+\frac{2\left(7b\left(9a^2+2b^2\right)-a\left(8a^2+69b^2\right)\cos(c+dx)\right)}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}}{2(a^2-b^2)}-\frac{11ab}{de(a^2-b^2)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}$$

$$\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3346$$

$$\frac{\frac{\frac{a\left(8a^2+69b^2\right)\int \frac{1}{\sqrt{e\sin(c+dx)}}dx-21b^2\left(9a^2+2b^2\right)\int \frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{3e^2\left(a^2-b^2\right)}+\frac{2\left(7b\left(9a^2+2b^2\right)-a\left(8a^2+69b^2\right)\cos(c+dx)\right)}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}}{2(a^2-b^2)}-\frac{1}{de(a^2-b^2)(e\sin(c+d}}{4\left(a^2-b^2\right)}$$

$$\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{a\left(8a^2+69b^2\right)\int \frac{1}{\sqrt{e\sin(c+dx)}}dx-21b^2\left(9a^2+2b^2\right)\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{3e^2\left(a^2-b^2\right)}+\frac{2\left(7b\left(9a^2+2b^2\right)-a\left(8a^2+69b^2\right)\cos(c+dx)\right)}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}}{2(a^2-b^2)}-\frac{}{de(a^2-b^2)(}}{4\left(a^2-b^2\right)}$$

$$\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3121$$

$$\frac{\frac{\frac{\frac{a\left(8a^2+69b^2\right)\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}}dx}{\sqrt{e\sin(c+dx)}}-21b^2\left(9a^2+2b^2\right)\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{3e^2\left(a^2-b^2\right)}+\frac{2\left(7b\left(9a^2+2b^2\right)-a\left(8a^2+69b^2\right)\cos(c+dx)\right)}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}}{2(a^2-b^2)}-\frac{}{d}}{4\left(a^2-b^2\right)}$$

$$\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{\frac{a\left(8a^2+69b^2\right)\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}}dx}{\sqrt{e\sin(c+dx)}}-21b^2\left(9a^2+2b^2\right)\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{3e^2\left(a^2-b^2\right)}+\frac{2\left(7b\left(9a^2+2b^2\right)-a\left(8a^2+69b^2\right)\cos(c+dx)\right)}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}}{2(a^2-b^2)}-\frac{}{d}}{4\left(a^2-b^2\right)}$$

$$\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}$$

$\Big\downarrow$ 3120

$$\frac{\frac{\frac{\frac{2a\left(8a^2+69b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-21b^2\left(9a^2+2b^2\right)\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{3e^2\left(a^2-b^2\right)}+\frac{2\left(7b\left(9a^2+2b^2\right)-a\left(8a^2+69b^2\right)\cos(c+\right.}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}}{2(a^2-b^2)}}{4\left(a^2-b^2\right)}$$

$$\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}$$

$\Big\downarrow$ 3181

$$\frac{\frac{\frac{2a\left(8a^2+69b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-21b^2\left(9a^2+2b^2\right)\left(-\frac{be\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b^2\sin^2(c+dx)e^2+\left(a^2-b^2\right)e^2\right)}d(e\sin(c+dx))}{d}-\frac{a\int \frac{}{\sqrt{e\sin(c+dx}}}{}\right.}{3e^2\left(a^2-b^2\right)}}{2(a^2-b^2)}$$

$$\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}$$

$\Big\downarrow$ 266

$$\frac{\frac{\frac{\frac{2a\left(8a^2+69b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-21b^2\left(9a^2+2b^2\right)\left(-\frac{2be\int\frac{1}{b^2e^4\sin^4(c+dx)+\left(a^2-b^2\right)e^2}d\sqrt{e\sin(c+dx)}}{d}-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\right.}}{2\sqrt{b^2-a^2}}\right.}{3e^2\left(a^2-b^2\right)}}{2(a^2-b^2)}}{4\,(a^2}$$

$$\frac{b}{2de\,(a^2-b^2)\,(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}$$

$\downarrow$ 756

$$\frac{\frac{\frac{2a\left(8a^2+69b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-21b^2\left(9a^2+2b^2\right)\left(-\frac{2be\left(-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2e\sqrt{b^2-a^2}}-\frac{\int\frac{1}{be^2\sin^2(c+dx)+\sqrt{b^2-a^2}e}d}{2e\sqrt{b^2-a^2}}\right.}{d}\right.}{3e^2\left(a^2-b^2\right)}}{2(a^2-}$$

$$\frac{b}{2de\,(a^2-b^2)\,(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}$$

$\downarrow$ 218

$$\frac{\frac{\frac{2a\left(8a^2+69b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-21b^2\left(9a^2+2b^2\right)\left(-\frac{2be\left(-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2e\sqrt{b^2-a^2}}-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}\right)}{d}-\right.}{3e^2\left(a^2-b^2\right)}}{2(a^2-b^2)}$$

$$\frac{b}{2de\,(a^2-b^2)\,(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}$$

$\downarrow$ 221

$$\frac{\frac{\frac{2a\left(8a^2+69b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-21b^2\left(9a^2+2b^2\right)\left(-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-\cdots}\right.}\cdots}{2\sqrt{b^2-a^2}}\cdots\right.}{3e^2\left(a^2-b^2\right)}\cdots}{2(a^2-b^2)}\cdots$$

$$\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{2a\left(8a^2+69b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-21b^2\left(9a^2+2b^2\right)\left(-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-\cdots}\right.}\cdots}{2\sqrt{b^2-a^2}}\cdots\right.}{3e^2\left(a^2-b^2\right)}\cdots}{2(a^2-b^2)}\cdots$$

$$\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3286$$

$$\frac{\frac{2a\left(8a^2+69b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-21b^2\left(9a^2+2b^2\right)\left(-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}}-\frac{a\sqrt{\sin(c+dx)}\int\frac{}{\sqrt{\sin(c+dx}}}{2\sqrt{b^2-a^2}}\right.}{3e^2\left(a^2-b^2\right)}$$

$$2(a^2-b^2)$$

$$\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}$$

$\downarrow$ 3042

$$\frac{\frac{2a\left(8a^2+69b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-21b^2\left(9a^2+2b^2\right)\left(-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}}-\frac{a\sqrt{\sin(c+dx)}\int\frac{}{\sqrt{\sin(c+dx}}}{2\sqrt{b^2-a^2}}\right.}{3e^2\left(a^2-b^2\right)}$$

$$2(a^2-b^2)$$

$$\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}$$

$\downarrow$ 3284

$$\frac{\frac{\frac{2a(8a^2+69b^2)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-21b^2(9a^2+2b^2)\left(-\frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}(b^2-a^2)^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}(b^2-a^2)^{3/4}}\right)}{d}+\frac{a\sqrt{\sin(c+d}}{d}\right.}{3e^2(a^2-b^2)}}{2(a^2-b^2)}$$

$$\frac{b}{2de\,(a^2-b^2)\,(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}$$

input
```
Int[1/((a + b*Cos[c + d*x])^3*(e*Sin[c + d*x])^(5/2)),x]
```

output
```
-1/2*b/((a^2 - b^2)*d*e*(a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(3/2)) + (
(-11*a*b)/((a^2 - b^2)*d*e*(a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2)) +
((2*(7*b*(9*a^2 + 2*b^2) - a*(8*a^2 + 69*b^2)*Cos[c + d*x]))/(3*(a^2 - b^2
)*d*e*(e*Sin[c + d*x])^(3/2)) + ((2*a*(8*a^2 + 69*b^2)*EllipticF[(c - Pi/2
 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(d*Sqrt[e*Sin[c + d*x]]) - 21*b^2*(9*a^2
 + 2*b^2)*((-2*b*e*(-1/2*ArcTan[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(-a^2 + b^2
)^(1/4)]/(Sqrt[b]*(-a^2 + b^2)^(3/4)*e^(3/2)) - ArcTanh[(Sqrt[b]*Sqrt[e]*S
in[c + d*x])/(-a^2 + b^2)^(1/4)]/(2*Sqrt[b]*(-a^2 + b^2)^(3/4)*e^(3/2))))/
d + (a*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqr
t[Sin[c + d*x]])/(Sqrt[-a^2 + b^2]*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c +
 d*x]]) - (a*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2,
2]*Sqrt[Sin[c + d*x]])/(Sqrt[-a^2 + b^2]*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*S
in[c + d*x]]))))/(3*(a^2 - b^2)*e^2))/(2*(a^2 - b^2)))/(4*(a^2 - b^2))
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3173
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Simp[1/((a^2 - b^2)*(m + 1))
   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m +
p + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

rule 3181
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[-a/(2*q)   Int[1/(
Sqrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Simp[b*(g/f)   Subst[
Int[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - S
imp[a/(2*q)   Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3343
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Simp[1/((a^2 - b^2)*(m + 1))   Int[(g*Cos[e + f*x])^p
*(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ
[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 3345
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Simp[1/(g^2*(a^2 - b^2)*(p +
 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
 2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[p, -1] && IntegerQ[2*m]
```

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

## Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2678 vs. $2(558) = 1116$.

Time = 4.41 (sec) , antiderivative size = 2679, normalized size of antiderivative = 4.26

| method | result | size |
|---|---|---|
| default | Expression too large to display | 2679 |

input
```
int(1/(a+cos(d*x+c)*b)^3/(e*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(2*e^3*b*(b^2/e^4/(a-b)^3/(a+b)^3*(1/8*(e*sin(d*x+c))^(1/2)*e^2*(-13*cos(d
*x+c)^2*a^2*b^2-2*b^4*cos(d*x+c)^2+17*a^4-2*a^2*b^2)/(-b^2*cos(d*x+c)^2*e^
2+a^2*e^2)^2+7/64*(9*a^2+2*b^2)*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2
)*2^(1/2)*(ln((e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)
*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4
)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arctan(2^(1/2
)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2)/(e^2*
(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)))-1/3*(-3*a^2-b^2)/e^4/(a^2-b
^2)^3/(e*sin(d*x+c))^(3/2))-(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/e^2*a*(-1/3*
(a^2+3*b^2)/(a^2-b^2)^3/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(cos(d*x+c)^2-1)
*((1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(5/2)*EllipticF((
1-sin(d*x+c))^(1/2),1/2*2^(1/2))+2*cos(d*x+c)^2*sin(d*x+c))+b^2*(a^2+3*b^2
)/(a+b)^2/(a-b)^2*(1/2*b^2/e/a^2/(a^2-b^2)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/
2)/(-cos(d*x+c)^2*b^2+a^2)+1/4/a^2/(a^2-b^2)*(1-sin(d*x+c))^(1/2)*(2+2*sin
(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*Elliptic
F((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-5/8/(a^2-b^2)/b/(-a^2+b^2)^(1/2)*(1-si
n(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*si
n(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(
1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/4/a^2/(a^2-b^2)*b/(-a^2+b^2)^(1/2)*(1
-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^...
```

## Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))**3/(e*sin(d*x+c))**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

## Giac [F]

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\cos(dx+c)+a)^3(e\sin(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^3*(e*sin(d*x + c))^(5/2)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}}\,dx = \int \frac{1}{(e\,\sin(c+d\,x))^{5/2}\,(a+b\,\cos(c+d\,x))^3}\,dx$$

input
```
int(1/((e*sin(c + d*x))^(5/2)*(a + b*cos(c + d*x))^3),x)
```

output
```
int(1/((e*sin(c + d*x))^(5/2)*(a + b*cos(c + d*x))^3), x)
```

## Reduce [F]

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}}\,dx = \frac{\sqrt{e}\left(\int \frac{\sqrt{\sin(dx+c)}}{\cos(dx+c)^3\sin(dx+c)^3b^3+3\cos(dx+c)^2\sin(dx+c)^3a\,b^2+3\cos(dx+c)\sin}\right.}{e^3}$$

input
```
int(1/(a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(5/2),x)
```

output
```
(sqrt(e)*int(sqrt(sin(c + d*x))/(cos(c + d*x)**3*sin(c + d*x)**3*b**3 + 3*
cos(c + d*x)**2*sin(c + d*x)**3*a*b**2 + 3*cos(c + d*x)*sin(c + d*x)**3*a*
*2*b + sin(c + d*x)**3*a**3),x))/e**3
```

# 3.88 $\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}}\,dx$

## Optimal result

Integrand size = 25, antiderivative size = 700

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}}\,dx = -\frac{9b^{7/2}(11a^2+2b^2)\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8\left(-a^2+b^2\right)^{17/4}de^{7/2}} + \frac{9b^{7/2}(11a^2+2b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8\left(-a^2+b^2\right)^{17/4}de^{7/2}} - \frac{b}{2\left(a^2-b^2\right)de(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}} - \frac{13ab}{4\left(a^2-b^2\right)^2de(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}} + \frac{9b(11a^2+2b^2)-a(8a^2+109b^2)\cos(c+dx)}{20\left(a^2-b^2\right)^3de(e\sin(c+dx))^{5/2}} - \frac{3(15b^3(11a^2+2b^2)+a(8a^4-64a^2b^2-139b^4)\cos(c+dx))}{20\left(a^2-b^2\right)^4de^3\sqrt{e\sin(c+dx)}} + \frac{9ab^3(11a^2+2b^2)\operatorname{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8\left(a^2-b^2\right)^4\left(b-\sqrt{-a^2+b^2}\right)de^3\sqrt{e\sin(c+dx)}} + \frac{9ab^3(11a^2+2b^2)\operatorname{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8\left(a^2-b^2\right)^4\left(b+\sqrt{-a^2+b^2}\right)de^3\sqrt{e\sin(c+dx)}} - \frac{3a(8a^4-64a^2b^2-139b^4)E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{20\left(a^2-b^2\right)^4de^4\sqrt{\sin(c+dx)}}$$

output
```
-9/8*b^(7/2)*(11*a^2+2*b^2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)
^(1/4)/e^(1/2))/(-a^2+b^2)^(17/4)/d/e^(7/2)+9/8*b^(7/2)*(11*a^2+2*b^2)*arc
tanh(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a^2+b^2)^(17
/4)/d/e^(7/2)-1/2*b/(a^2-b^2)/d/e/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(5/2)-
13/4*a*b/(a^2-b^2)^2/d/e/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(5/2)+1/20*(9*b*(
11*a^2+2*b^2)-a*(8*a^2+109*b^2)*cos(d*x+c))/(a^2-b^2)^3/d/e/(e*sin(d*x+c))
^(5/2)-3/20*(15*b^3*(11*a^2+2*b^2)+a*(8*a^4-64*a^2*b^2-139*b^4)*cos(d*x+c)
)/(a^2-b^2)^4/d/e^3/(e*sin(d*x+c))^(1/2)-9/8*a*b^3*(11*a^2+2*b^2)*Elliptic
Pi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^
(1/2)/(a^2-b^2)^4/(b-(-a^2+b^2)^(1/2))/d/e^3/(e*sin(d*x+c))^(1/2)-9/8*a*b^
3*(11*a^2+2*b^2)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1
/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^4/(b+(-a^2+b^2)^(1/2))/d/e^3/(e*s
in(d*x+c))^(1/2)+3/20*a*(8*a^4-64*a^2*b^2-139*b^4)*EllipticE(cos(1/2*c+1/4
*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^4/d/e^4/sin(d*x+c)^(1
/2)
```

## Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 7.62 (sec) , antiderivative size = 1014, normalized size of antiderivative = 1.45

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[1/((a + b*Cos[c + d*x])^3*(e*Sin[c + d*x])^(7/2)),x]
```

output
```
(Sin[c + d*x]^4*((-2*(50*a^2*b^3 + 10*b^5 + 3*a^5*Cos[c + d*x] - 24*a^3*b^
2*Cos[c + d*x] - 39*a*b^4*Cos[c + d*x])*Csc[c + d*x])/(5*(a^2 - b^2)^4) -
(2*(-3*a^2*b - b^3 + a^3*Cos[c + d*x] + 3*a*b^2*Cos[c + d*x])*Csc[c + d*x]
^3)/(5*(a^2 - b^2)^3) - (b^5*Sin[c + d*x])/(2*(a^2 - b^2)^3*(a + b*Cos[c +
 d*x])^2) - (21*a*b^5*Sin[c + d*x])/(4*(a^2 - b^2)^4*(a + b*Cos[c + d*x]))
))/(d*(e*Sin[c + d*x])^(7/2)) - (3*Sin[c + d*x]^(7/2)*(((8*a^5*b - 64*a^3*
b^3 - 139*a*b^5)*Cos[c + d*x]^2*(3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1
 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 +
(Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b
^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x
]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c +
d*x]] + b*Sin[c + d*x]]) + 8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin[c + d
*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2))*(a + b*Sqrt[
1 - Sin[c + d*x]^2]))/(12*b^(3/2)*(-a^2 + b^2)*(a + b*Cos[c + d*x])*(1 - S
in[c + d*x]^2)) + (2*(8*a^6 - 64*a^4*b^2 - 304*a^2*b^4 - 30*b^6)*Cos[c + d
*x]*(((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2
 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 +
 b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*S
qrt[Sin[c + d*x]] + I*b*Sin[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqr
t[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]]))/(Sqrt[...
```

## Rubi [A] (warning: unable to verify)

Time = 3.64 (sec) , antiderivative size = 679, normalized size of antiderivative = 0.97, number of steps used = 28, number of rules used = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.080, Rules used = {3042, 3173, 27, 3042, 3343, 27, 3042, 3345, 27, 3042, 3345, 27, 3042, 3346, 3042, 3121, 3042, 3119, 3180, 266, 827, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(e\sin(c+dx))^{7/2}(a+b\cos(c+dx))^3}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{7/2}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$
$$\Big\downarrow \text{ 3173}$$

$$-\frac{\int -\frac{4a-9b\cos(c+dx)}{2(a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}}dx}{2\left(a^2-b^2\right)}-\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{4a-9b\cos(c+dx)}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}}dx}{4\left(a^2-b^2\right)}-\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{4a+9b\sin\left(c+dx-\frac{\pi}{2}\right)}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{7/2}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx}{4\left(a^2-b^2\right)}-\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3343$$

$$\frac{-\frac{\int -\frac{8a^2-91b\cos(c+dx)a+18b^2}{2(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}dx}{a^2-b^2}-\frac{13ab}{de(a^2-b^2)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{2\left(4a^2+9b^2\right)-91ab\cos(c+dx)}{(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}dx}{2(a^2-b^2)}-\frac{13ab}{de(a^2-b^2)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{2\left(4a^2+9b^2\right)+91ab\sin\left(c+dx-\frac{\pi}{2}\right)}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{7/2}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{2(a^2-b^2)}-\frac{13ab}{de(a^2-b^2)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3345$$

$$\frac{\frac{\frac{2\left(9b\left(11a^2+2b^2\right)-a\left(8a^2+109b^2\right)\cos(c+dx)\right)}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}}-\frac{2\int -\frac{3\left(2\left(4a^4-28b^2a^2-15b^4\right)+ab\left(8a^2+109b^2\right)\cos(c+dx)\right)}{2(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}dx}{5e^2\left(a^2-b^2\right)}}{2(a^2-b^2)}-\frac{13ab}{de(a^2-b^2)(e\sin(c+dx))^{5/2}(a+b\cos(c+}}{4\left(a^2-b^2\right)} \frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{3\int \frac{2\left(4a^4-28b^2a^2-15b^4\right)+ab\left(8a^2+109b^2\right)\cos(c+dx)}{(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}dx}{5e^2\left(a^2-b^2\right)}+\frac{2\left(9b\left(11a^2+2b^2\right)-a\left(8a^2+109b^2\right)\cos(c+dx)\right)}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}}}{2(a^2-b^2)}-\frac{13ab}{de(a^2-b^2)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}$$

$$\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{3\int \frac{2\left(4a^4-28b^2a^2-15b^4\right)-ab\left(8a^2+109b^2\right)\sin\left(c+dx-\frac{\pi}{2}\right)}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{5e^2\left(a^2-b^2\right)}+\frac{2\left(9b\left(11a^2+2b^2\right)-a\left(8a^2+109b^2\right)\cos(c+dx)\right)}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}}}{2(a^2-b^2)}-\frac{13ab}{de(a^2-b^2)(e\sin(c+dx))^{5/2}(a+b\cos(c+\cdots}}{4\left(a^2-b^2\right)}$$

$$\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3345$$

$$\frac{\frac{3\left(-\frac{2\int \frac{\left(8a^6-64b^2a^4-304b^4a^2+b\left(8a^4-64b^2a^2-139b^4\right)\cos(c+dx)a-30b^6\right)\sqrt{e\sin(c+dx)}}{2(a+b\cos(c+dx))}dx}{e^2\left(a^2-b^2\right)}-\frac{2\left(15b^3\left(11a^2+2b^2\right)+a\left(8a^4-64a^2b^2-139b^4\right)\cos(c+dx)\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}\right)}{5e^2\left(a^2-b^2\right)}+\frac{2(\cdots}{\cdots}}{2(a^2-b^2)} \cdots}{4\left(a^2-b^2\right)}$$

$$\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{3\left(-\frac{\int \frac{\left(2\left(4a^6-32b^2a^4-152b^4a^2-15b^6\right)+ab\left(8a^4-64b^2a^2-139b^4\right)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx}{e^2\left(a^2-b^2\right)}-\frac{2\left(15b^3\left(11a^2+2b^2\right)+a\left(8a^4-64a^2b^2-139b^4\right)\cos(c+dx)\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}\right)}{5e^2\left(a^2-b^2\right)}+\cdots}{2(a^2-b^2)} \cdots}{4\left(a^2-b^2\right)}$$

$$\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{3\left(-\frac{\int \frac{\sqrt{-e\cos\left(c+dx+\frac{\pi}{2}\right)}\left(2\left(4a^6-32b^2a^4-152b^4a^2-15b^6\right)+ab\left(8a^4-64b^2a^2-139b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{e^2\left(a^2-b^2\right)}-\frac{2\left(15b^3\left(11a^2+2b^2\right)+a\left(8a^4-64a^2b^2-139b^4\right)\cos(c\cdots\right.}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}\cdots\right.}{5e^2\left(a^2-b^2\right)}\cdots}{2(a^2-b^2)}\cdots}{4\left(a^2-b^2\right)}\cdots$$

$$\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}$$

$\big\downarrow$ 3346

$$\frac{\frac{3\left(-\frac{a\left(8a^4-64a^2b^2-139b^4\right)\int\sqrt{e\sin(c+dx)}dx-15b^4\left(11a^2+2b^2\right)\int\frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx}{e^2\left(a^2-b^2\right)}-\frac{2\left(15b^3\left(11a^2+2b^2\right)+a\left(8a^4-64a^2b^2-139b^4\right)\cos(c+dx)\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}\right)}{5e^2\left(a^2-b^2\right)}+\frac{2\left(9b\left(11a\cdots\right.\right.}{5e\cdots}}{2(a^2-b^2)}\cdots}{4\left(a^2-b^2\right)}\cdots$$

$$\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}$$

$\big\downarrow$ 3042

$$\frac{\frac{3\left(-\frac{a\left(8a^4-64a^2b^2-139b^4\right)\int\sqrt{e\sin(c+dx)}dx-15b^4\left(11a^2+2b^2\right)\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{e^2\left(a^2-b^2\right)}-\frac{2\left(15b^3\left(11a^2+2b^2\right)+a\left(8a^4-64a^2b^2-139b^4\right)\cos(c+dx)\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}\right)}{5e^2\left(a^2-b^2\right)}+\frac{2\left(9b\cdots\right.}{\cdots}}{2(a^2-b^2)}\cdots}{4\left(a^2-b^2\right)}\cdots$$

$$\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}$$

$\big\downarrow$ 3121

$$\frac{\frac{3\left(-\frac{\frac{a\left(8a^4-64a^2b^2-139b^4\right)\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}-15b^4\left(11a^2+2b^2\right)\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{e^2\left(a^2-b^2\right)}-\frac{2\left(15b^3\left(11a^2+2b^2\right)+a\left(8a^4-64a^2b^2-139b^4\right)\cos(c+dx\cdots\right.}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}\cdots\right.}{5e^2\left(a^2-b^2\right)}\cdots}{2(a^2-b^2)}\cdots}{4\left(a^2-b^2\right)}\cdots$$

$$\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}$$

$\big\downarrow$ 3042

$$3\left(-\frac{\frac{a\left(8a^4-64a^2b^2-139b^4\right)\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}-15b^4\left(11a^2+2b^2\right)\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{e^2\left(a^2-b^2\right)}-\frac{2\left(15b^3\left(11a^2+2b^2\right)+a\left(8a^4-64a^2b^2-139b^4\right)\cos(c+da\ldots\right.}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}\ldots\right.$$

$$\overline{5e^2\left(a^2-b^2\right)}\ldots$$

$$\overline{2(a^2-b^2)}\ldots$$

$$\overline{4\left(a^2-b^2\right)}\ldots$$

$$\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{ 3119}$$

$$3\left(-\frac{\frac{2a\left(8a^4-64a^2b^2-139b^4\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-15b^4\left(11a^2+2b^2\right)\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{e^2\left(a^2-b^2\right)}-\frac{2\left(15b^3\left(11a^2+2b^2\right)+a\left(8a^4-64a^2b^2-139b^4\right)\cos(\ldots\right.}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}\ldots\right.$$

$$\overline{5e^2\left(a^2-b^2\right)}\ldots$$

$$\overline{2(a^2-b^2)}\ldots$$

$$\overline{4\left(a^2-b^2\right)}\ldots$$

$$\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{ 3180}$$

$$3\left(-\frac{\frac{2a\left(8a^4-64a^2b^2-139b^4\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-15b^4\left(11a^2+2b^2\right)\left(-\frac{be\int\frac{\sqrt{e\sin(c+dx)}}{b^2\sin^2(c+dx)e^2+\left(a^2-b^2\right)e^2}d(e\sin(c+dx))}{d}-\frac{ae\int\frac{\ldots}{\sqrt{e\sin(c+dx)}\left(\sqrt{\ldots}\right.}\ldots}{\ldots}\right.}{e^2\left(a^2-b^2\right)}\ldots\right.$$

$$\overline{5e^2\left(a^2-b^2\right)}\ldots$$

$$\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{ 266}$$

$$3\left(-\frac{\frac{2a\left(8a^4-64a^2b^2-139b^4\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-15b^4\left(11a^2+2b^2\right)\left(-\frac{2be\int\frac{e^2\sin^2(c+dx)}{b^2e^4\sin^4(c+dx)+\left(a^2-b^2\right)e^2}d\sqrt{e\sin(c+dx)}}{d}-\frac{ae\int\frac{}{\sqrt{e\sin(c+dx)}\left(\right.}}{}\right.}{e^2\left(a^2-b^2\right)}\right.$$

$$5e^2\left(a^2-b^2\right)$$

$$\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}$$

$\downarrow$ 827

$$3\left(-\frac{\frac{2a\left(8a^4-64a^2b^2-139b^4\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-15b^4\left(11a^2+2b^2\right)\left(-\frac{2be\left(\frac{\int\frac{1}{be^2\sin^2(c+dx)+\sqrt{b^2-a^2}e}d\sqrt{e\sin(c+dx)}}{2b}-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2}}{}\right.}{d}\right.}{e^2\left(a^2-b^2\right)}\right.$$

$$\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}$$

$\downarrow$ 218

$$3\left(-\frac{\frac{2a\left(8a^4-64a^2b^2-139b^4\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-15b^4\left(11a^2+2b^2\right)\left(-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c}}{2b}\right.}{d}\right.}{e^2\left(a^2-b^2\right)}\right.$$

$$\frac{\phantom{x}}{5e}$$

$$\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 221$$

$$3\left(-\frac{\frac{2a\left(8a^4-64a^2b^2-139b^4\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-15b^4\left(11a^2+2b^2\right)\left(-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(\right.}}{2b}\right.}{e^2\left(a^2-b^2\right)}\right.$$

$$\overline{5e^2\left(a^2-\right.}$$

$$\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}$$

$\downarrow$ 3042

$$3\left(-\frac{\frac{2a\left(8a^4-64a^2b^2-139b^4\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-15b^4\left(11a^2+2b^2\right)\left(-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(\right.}}{2b}\right.}{e^2\left(a^2-b^2\right)}\right.$$

$$5e^2\left(a^2-\right.$$

$$\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}$$

$\downarrow$ 3286

$$3\left(-\frac{\frac{2a\left(8a^4-64a^2b^2-139b^4\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-15b^4\left(11a^2+2b^2\right)\left(-\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b\sqrt{e\sin(c+dx)}}+\frac{ae\sqrt{\sin(c+dx)}}{}\right.}{e^2\left(a^2-b^2\right)}\right.$$

$$\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$3\left(-\frac{\frac{2a\left(8a^4-64a^2b^2-139b^4\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-15b^4\left(11a^2+2b^2\right)\left(-\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b\sqrt{e\sin(c+dx)}}+\frac{ae\sqrt{\sin(c+dx)}}{}\right.}{e^2\left(a^2-b^2\right)}\right.$$

$$\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3284$$

$$\frac{\frac{2\left(9b\left(11a^2+2b^2\right)-a\left(8a^2+109b^2\right)\cos(c+dx)\right)}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}}+\frac{3\left(-\frac{\frac{2a\left(8a^4-64a^2b^2-139b^4\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-15b^4\left(11a^2+2b^2\right)\left(-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c}{\sqrt[4]{b^2-}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2}}\right.}{}\right.}{}\right.}{}}{}$$

$$\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}$$

input

```
Int[1/((a + b*Cos[c + d*x])^3*(e*Sin[c + d*x])^(7/2)),x]
```

output

```
-1/2*b/((a^2 - b^2)*d*e*(a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(5/2)) + (
(-13*a*b)/((a^2 - b^2)*d*e*(a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(5/2)) +
((2*(9*b*(11*a^2 + 2*b^2) - a*(8*a^2 + 109*b^2)*Cos[c + d*x]))/(5*(a^2 - b
^2)*d*e*(e*Sin[c + d*x])^(5/2)) + (3*((-2*(15*b^3*(11*a^2 + 2*b^2) + a*(8*
a^4 - 64*a^2*b^2 - 139*b^4)*Cos[c + d*x]))/((a^2 - b^2)*d*e*Sqrt[e*Sin[c +
 d*x]]) - ((2*a*(8*a^4 - 64*a^2*b^2 - 139*b^4)*EllipticE[(c - Pi/2 + d*x)/
2, 2]*Sqrt[e*Sin[c + d*x]])/(d*Sqrt[Sin[c + d*x]]) - 15*b^4*(11*a^2 + 2*b^
2)*((-2*b*e*(ArcTan[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/(2*
b^(3/2)*(-a^2 + b^2)^(1/4)*Sqrt[e]) - ArcTanh[(Sqrt[b]*Sqrt[e]*Sin[c + d*x
])/(-a^2 + b^2)^(1/4)]/(2*b^(3/2)*(-a^2 + b^2)^(1/4)*Sqrt[e])))/d + (a*e*E
llipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c
+ d*x]])/(b*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]]) + (a*e*Elliptic
Pi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]]
)/(b*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]])))/((a^2 - b^2)*e^2)))/
(5*(a^2 - b^2)*e^2))/(2*(a^2 - b^2)))/(4*(a^2 - b^2))
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3173
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Simp[1/((a^2 - b^2)*(m + 1))
   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m +
p + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

rule 3180
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[a*(g/(2*b))   Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Simp[a*(g/(2*b))   In
t[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Simp[b*(g/f)   Su
bst[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3343
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Simp[1/((a^2 - b^2)*(m + 1))   Int[(g*Cos[e + f*x])^p
*(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ
[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 3345
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Simp[1/(g^2*(a^2 - b^2)*(p +
 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
 2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[p, -1] && IntegerQ[2*m]
```

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

## Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 3076 vs. $2(626) = 1252$.

Time = 4.96 (sec) , antiderivative size = 3077, normalized size of antiderivative = 4.40

| method | result | size |
|---|---|---|
| default | Expression too large to display | 3077 |

input
```
int(1/(a+cos(d*x+c)*b)^3/(e*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
(2*e^3*b*(-b^4/e^6/(a-b)^4/(a+b)^4*(1/8*(e*sin(d*x+c))^(3/2)*e^2*(-19*cos(
d*x+c)^2*a^2*b^2-2*b^4*cos(d*x+c)^2+23*a^4-2*a^2*b^2)/(-b^2*cos(d*x+c)^2*e
^2+a^2*e^2)^2+1/8*(99/8*a^2+9/4*b^2)/b^2/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)
*(ln((e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+
(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(
d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arctan(2^(1/2)/(e^2*(a
^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2)/(e^2*(a^2-b^2)
/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)))-2*b^2*(5*a^2+b^2)/e^6/(a-b)^4/(a+b)^
4/(e*sin(d*x+c))^(1/2)-1/5*(-3*a^2-b^2)/e^4/(a+b)^3/(a-b)^3/(e*sin(d*x+c))
^(5/2))-(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/e^3*a*(1/5*(a^2+3*b^2)/(a^2-b^2)
^3/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/sin(d*x+c)/(cos(d*x+c)^2-1)*(6*(1-sin
(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(7/2)*EllipticE((1-sin(d*
x+c))^(1/2),1/2*2^(1/2))-3*(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin
(d*x+c)^(7/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))+6*cos(d*x+c)^4*s
in(d*x+c)-8*cos(d*x+c)^2*sin(d*x+c))+6*b^2*(a^2+b^2)/(a^2-b^2)^4*(2*(1-sin
(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticE((1-sin(d*
x+c))^(1/2),1/2*2^(1/2))-(1-sin(d*x+c))^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d
*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-2*cos(d*x+c)^2)/(c
os(d*x+c)^2*e*sin(d*x+c))^(1/2)-4*a^2*b^4/(a+b)^2/(a-b)^2*(1/4*b^2/e/a^2/(
a^2-b^2)*sin(d*x+c)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-cos(d*x+c)^2*b^...
```

## Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
Timed out
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))**3/(e*sin(d*x+c))**(7/2),x)
```

output
```
Timed out
```

## Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
Timed out
```

## Giac [F]

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}}\,dx = \int \frac{1}{(b\cos(dx+c)+a)^3\,(e\sin(dx+c))^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^3*(e*sin(d*x + c))^(7/2)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}}\,dx = \int \frac{1}{(e\,\sin(c+d\,x))^{7/2}\,(a+b\,\cos(c+d\,x))^3}\,dx$$

input
```
int(1/((e*sin(c + d*x))^(7/2)*(a + b*cos(c + d*x))^3),x)
```

output
```
int(1/((e*sin(c + d*x))^(7/2)*(a + b*cos(c + d*x))^3), x)
```

## Reduce [F]

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}}\,dx = \frac{\sqrt{e}\left(\int \frac{\sqrt{\sin(dx+c)}}{\cos(dx+c)^3\sin(dx+c)^4b^3+3\cos(dx+c)^2\sin(dx+c)^4a\,b^2+3\cos(dx+c)\sin}\right.}{e^4}$$

input
```
int(1/(a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(7/2),x)
```

output
```
(sqrt(e)*int(sqrt(sin(c + d*x))/(cos(c + d*x)**3*sin(c + d*x)**4*b**3 + 3*
cos(c + d*x)**2*sin(c + d*x)**4*a*b**2 + 3*cos(c + d*x)*sin(c + d*x)**4*a*
*2*b + sin(c + d*x)**4*a**3),x))/e**4
```

# 3.89 $\int (a+b\cos(c+dx))^m(g\sin(c+dx))^p\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 159

$$\int (a+b\cos(c+dx))^m(g\sin(c+dx))^p\,dx = -\frac{g\,\text{AppellF1}\left(1+m,\frac{1-p}{2},\frac{1-p}{2},2+m,\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(c+dx)}{a+b}\right)(a+b\cos(c+dx))^{1+m}\left(1-\frac{a+b\cos(c+dx}{a-b}\right.}{bd(1+m)}$$

output
```
-g*AppellF1(1+m,1/2-1/2*p,1/2-1/2*p,2+m,(a+b*cos(d*x+c))/(a-b),(a+b*cos(d*
x+c))/(a+b))*(a+b*cos(d*x+c))^(1+m)*(1-(a+b*cos(d*x+c))/(a-b))^(1/2-1/2*p)
*(1-(a+b*cos(d*x+c))/(a+b))^(1/2-1/2*p)*(g*sin(d*x+c))^(-1+p)/b/d/(1+m)
```

## Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 676 vs. $2(159) = 318$.

Time = 4.11 (sec) , antiderivative size = 676, normalized size of antiderivative = 4.25

$$\int (a+b\cos(c+dx))^m (g\sin(c+dx))^p\,dx$$

$$= \frac{}{d\left(\operatorname{AppellF1}\left(\frac{1+p}{2},1+m+p,-m,\frac{3+p}{2},-\tan^2\left(\frac{1}{2}(c+dx)\right),\frac{(-a+b)\tan^2\left(\frac{1}{2}(c+dx)\right)}{a+b}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)-\frac{4bm}{}\right.}$$

input
```
Integrate[(a + b*Cos[c + d*x])^m*(g*Sin[c + d*x])^p,x]
```

output
```
(2*AppellF1[(1 + p)/2, 1 + m + p, -m, (3 + p)/2, -Tan[(c + d*x)/2]^2, ((-a
 + b)*Tan[(c + d*x)/2]^2)/(a + b)]*(a + b*Cos[c + d*x])^m*(g*Sin[c + d*x])
^p*Tan[(c + d*x)/2])/(d*(AppellF1[(1 + p)/2, 1 + m + p, -m, (3 + p)/2, -Ta
n[(c + d*x)/2]^2, ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)]*Sec[(c + d*x)/2]^
2 - (4*b*m*AppellF1[(1 + p)/2, 1 + m + p, -m, (3 + p)/2, -Tan[(c + d*x)/2]
^2, ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)]*Sin[(c + d*x)/2]^2)/(a + b*Cos[
c + d*x]) + 2*p*AppellF1[(1 + p)/2, 1 + m + p, -m, (3 + p)/2, -Tan[(c + d*
x)/2]^2, ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)]*Cot[c + d*x]*Tan[(c + d*x)
/2] + 2*(m + p)*AppellF1[(1 + p)/2, 1 + m + p, -m, (3 + p)/2, -Tan[(c + d*
x)/2]^2, ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)]*Tan[(c + d*x)/2]^2 + (2*(-
a + b)*m*AppellF1[(1 + p)/2, 1 + m + p, -m, (3 + p)/2, -Tan[(c + d*x)/2]^2
, ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)]*Tan[(c + d*x)/2]^2)/(a + b*Cos[c
+ d*x]) + (2*(1 + p)*(((a - b)*m*AppellF1[(3 + p)/2, 1 + m + p, 1 - m, (5
+ p)/2, -Tan[(c + d*x)/2]^2, ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)])/(a +
b) - (1 + m + p)*AppellF1[(3 + p)/2, 2 + m + p, -m, (5 + p)/2, -Tan[(c + d
*x)/2]^2, ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)])*Sec[(c + d*x)/2]^2*Tan[(
c + d*x)/2]^2)/(3 + p)))
```

## Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 3183, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g\sin(c+dx))^p(a+b\cos(c+dx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(g\cos\left(c+dx-\frac{\pi}{2}\right)\right)^p\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^m dx$$

$$\Big\downarrow \text{3183}$$

$$\frac{g(g\sin(c+dx))^{p-1}\left(1-\frac{a+b\cos(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1-\frac{a+b\cos(c+dx)}{a+b}\right)^{\frac{1-p}{2}}\int(a+b\cos(c+dx))^m\left(-\frac{\cos(c+dx)b}{a-b}-\frac{b}{a-b}\right)^{\frac{p-1}{2}}}{d}$$

$$\Big\downarrow \text{155}$$

$$-\frac{g(g\sin(c+dx))^{p-1}(a+b\cos(c+dx))^{m+1}\left(1-\frac{a+b\cos(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1-\frac{a+b\cos(c+dx)}{a+b}\right)^{\frac{1-p}{2}}\text{AppellF1}\left(m+1,\frac{1-p}{2}\right.}{bd(m+1)}$$

input
```
Int[(a + b*Cos[c + d*x])^m*(g*Sin[c + d*x])^p,x]
```

output
```
-((g*AppellF1[1 + m, (1 - p)/2, (1 - p)/2, 2 + m, (a + b*Cos[c + d*x])/(a
- b), (a + b*Cos[c + d*x])/(a + b)]*(a + b*Cos[c + d*x])^(1 + m)*(1 - (a +
 b*Cos[c + d*x])/(a - b))^((1 - p)/2)*(1 - (a + b*Cos[c + d*x])/(a + b))^(
(1 - p)/2)*(g*Sin[c + d*x])^(-1 + p))/(b*d*(1 + m)))
```

### Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] && !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) && !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3183
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(f*(1 - (a + b*Sin
[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1)
/2)))   Subst[Int[(-b/(a - b) - b*(x/(a - b)))^((p - 1)/2)*(b/(a + b) - b*(
x/(a + b)))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a,
b, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] && !IGtQ[m, 0]
```

## Maple [F]

$$\int (a+\cos (dx+c)\, b)^m (g\sin (dx+c))^p \, dx$$

input
```
int((a+cos(d*x+c)*b)^m*(g*sin(d*x+c))^p,x)
```

output
```
int((a+cos(d*x+c)*b)^m*(g*sin(d*x+c))^p,x)
```

## Fricas [F]

$$\int (a+b\cos(c+dx))^m (g\sin(c+dx))^p\,dx = \int (b\cos(dx+c)+a)^m (g\sin(dx+c))^p\,dx$$

input
```
integrate((a+b*cos(d*x+c))^m*(g*sin(d*x+c))^p,x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c) + a)^m*(g*sin(d*x + c))^p, x)
```

## Sympy [F]

$$\int (a+b\cos(c+dx))^m (g\sin(c+dx))^p\,dx = \int (g\sin(c+dx))^p (a+b\cos(c+dx))^m\,dx$$

input
```
integrate((a+b*cos(d*x+c))**m*(g*sin(d*x+c))**p,x)
```

output
```
Integral((g*sin(c + d*x))**p*(a + b*cos(c + d*x))**m, x)
```

## Maxima [F]

$$\int (a+b\cos(c+dx))^m (g\sin(c+dx))^p\,dx = \int (b\cos(dx+c)+a)^m (g\sin(dx+c))^p\,dx$$

input
```
integrate((a+b*cos(d*x+c))^m*(g*sin(d*x+c))^p,x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^m*(g*sin(d*x + c))^p, x)
```

## Giac [F]

$$\int (a+b\cos(c+dx))^m(g\sin(c+dx))^p\,dx = \int (b\cos(dx+c)+a)^m(g\sin(dx+c))^p\,dx$$

input
```
integrate((a+b*cos(d*x+c))^m*(g*sin(d*x+c))^p,x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^m*(g*sin(d*x + c))^p, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^m(g\sin(c+dx))^p\,dx = \int (g\,\sin(c+d\,x))^p\,(a+b\,\cos(c+d\,x))^m\,dx$$

input
```
int((g*sin(c + d*x))^p*(a + b*cos(c + d*x))^m,x)
```

output
```
int((g*sin(c + d*x))^p*(a + b*cos(c + d*x))^m, x)
```

## Reduce [F]

$$\int (a+b\cos(c+dx))^m(g\sin(c+dx))^p\,dx = g^p\left(\int \sin(dx+c)^p\,(\cos(dx+c)\,b+a)^m\,dx\right)$$

input
```
int((a+b*cos(d*x+c))^m*(g*sin(d*x+c))^p,x)
```

output
```
g**p*int(sin(c + d*x)**p*(cos(c + d*x)*b + a)**m,x)
```

# CHAPTER 4

## Appendix



## 4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

### Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*           GradeAntiderivative[result,optimal] returns*)
(*   "F" if the result fails to integrate an expression that*)
(*       is integrable*)
(*   "C" if result involves higher level functions than necessary*)
(*   "B" if result is more than twice the size of the optimal*)
(*       antiderivative*)
(*   "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                   ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

## Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                     return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

## Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):   #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expn
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

## SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
            'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
            'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
            'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
            'elliptic_pi','exp_integral_e','log_integral',
            'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
            'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
              #is checked before calling the grading function that is passed.
              #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

## 4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file